Computer Algebra Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/24-1.1.2.8-P-x-c-x-^m-a+b-x^2-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [174]. This is test number [24].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (174)	0.00 (0)
Mathematica	100.00 (174)	0.00 (0)
Maple	97.70 (170)	2.30 (4)
Fricas	97.70 (170)	2.30 (4)
Maxima	97.70 (170)	2.30 (4)
Giac	95.40 (166)	4.60 (8)
Sympy	88.51 (154)	11.49 (20)
Mupad	74.14 (129)	25.86 (45)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

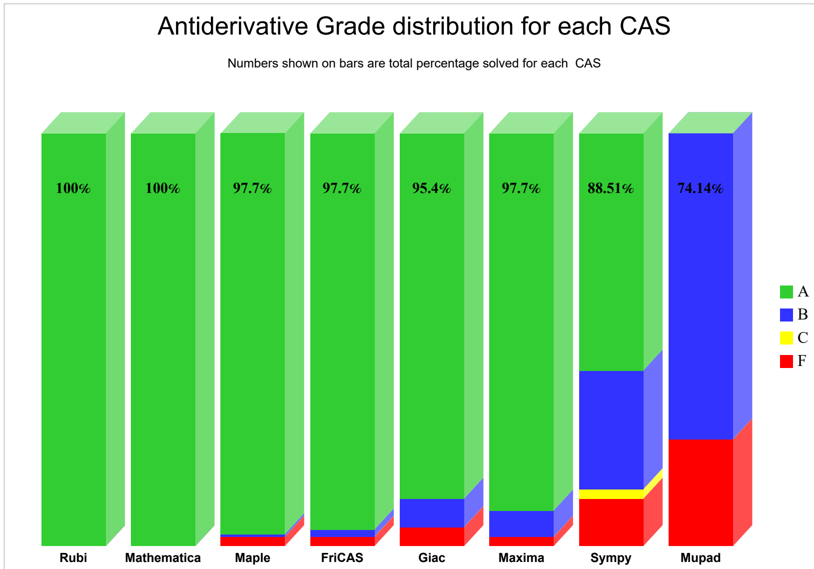
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

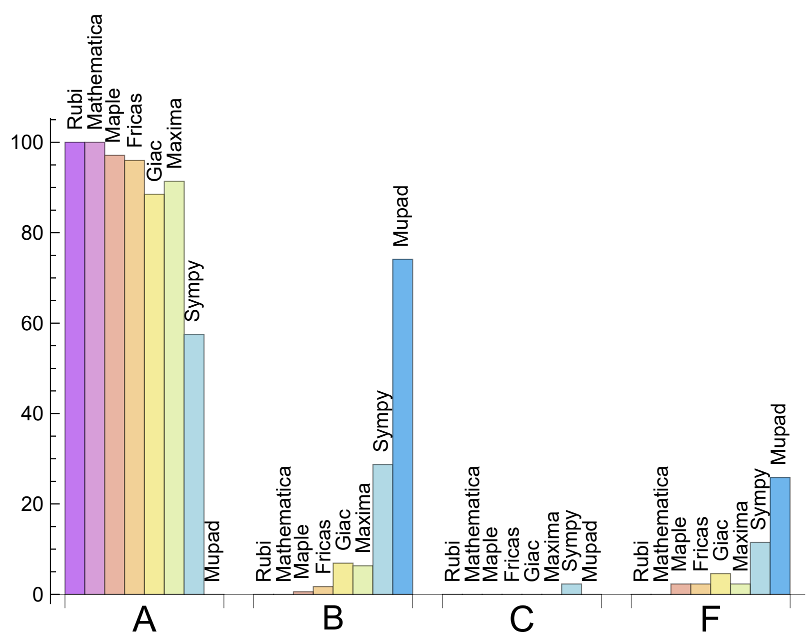
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maple	97.126	0.575	0.000	2.299
Fricas	95.977	1.724	0.000	2.299
Maxima	91.379	6.322	0.000	2.299
Giac	88.506	6.897	0.000	4.598
Sympy	57.471	28.736	2.299	11.494
Mupad	0.000	74.138	0.000	25.862

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	4	100.00	0.00	0.00
Maple	4	100.00	0.00	0.00
Maxima	4	100.00	0.00	0.00
Giac	8	50.00	0.00	50.00
Sympy	20	0.00	100.00	0.00
Mupad	45	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Mathematica	0.22
Maxima	0.23
Giac	0.30
Fricas	0.30
Rubi	0.38
Maple	3.32
Mupad	5.45
Sympy	11.87

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	113.44	0.91	103.00	0.91
Maple	115.09	0.90	102.00	0.90
Mupad	129.83	1.02	108.00	0.98
Rubi	134.73	1.05	128.00	1.03
Giac	151.84	1.09	125.00	0.98
Maxima	164.95	1.14	122.00	0.99
Fricas	279.44	2.09	223.50	2.10
Sympy	647.16	4.11	199.00	1.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

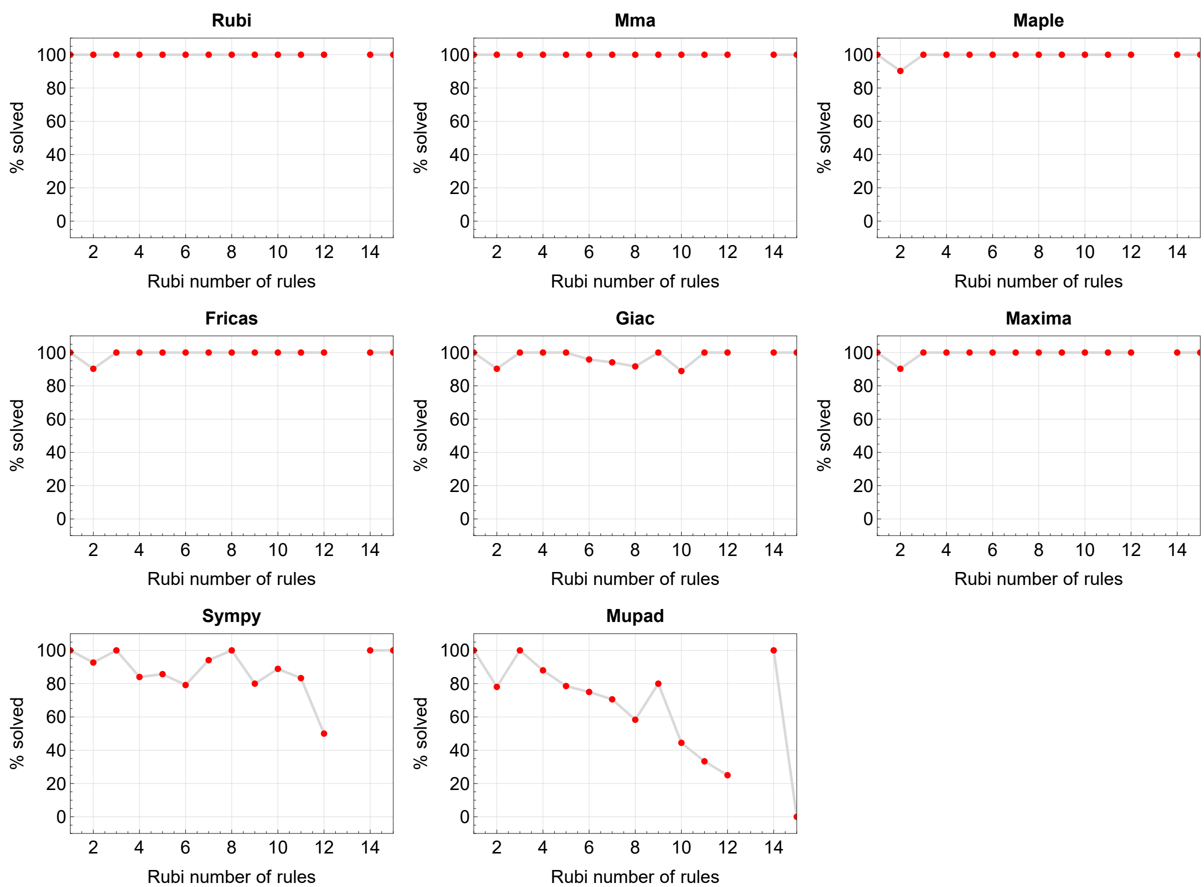


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

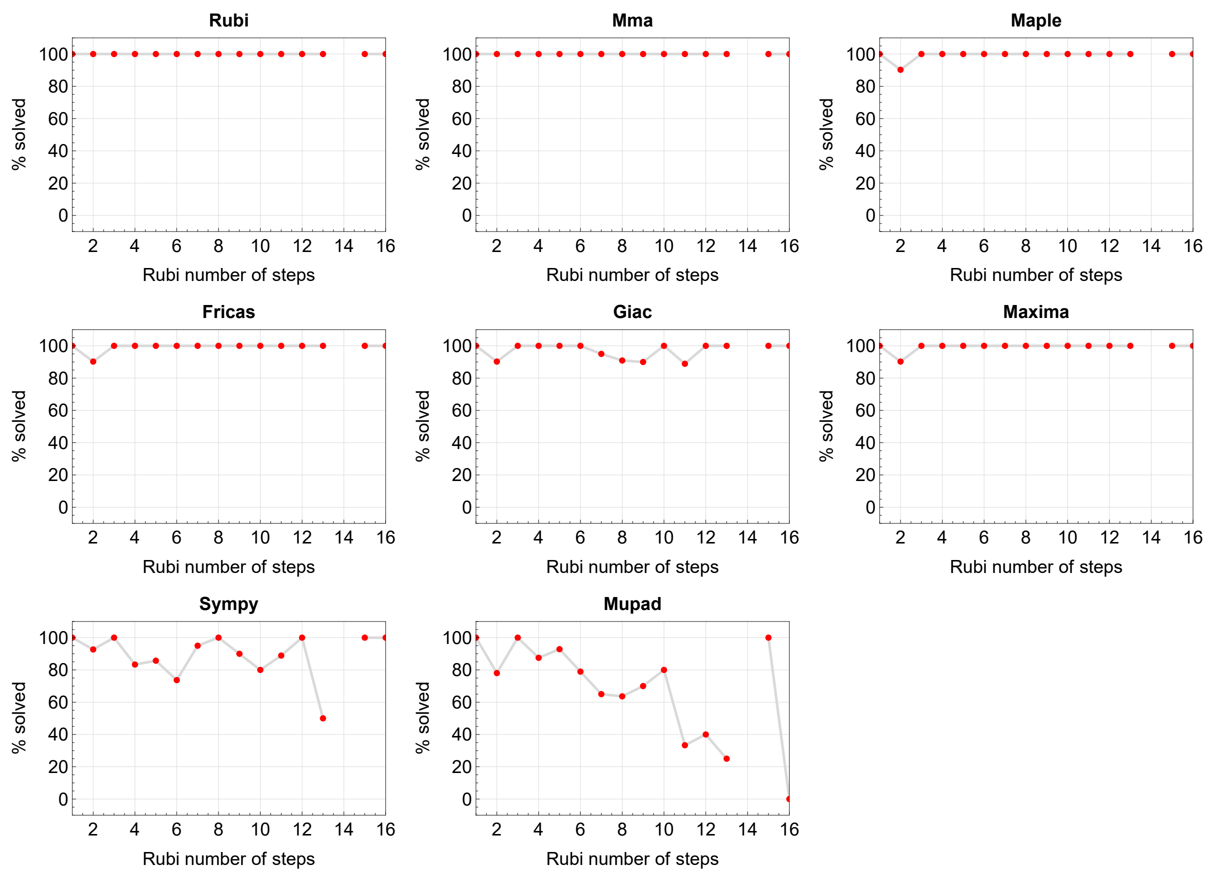


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

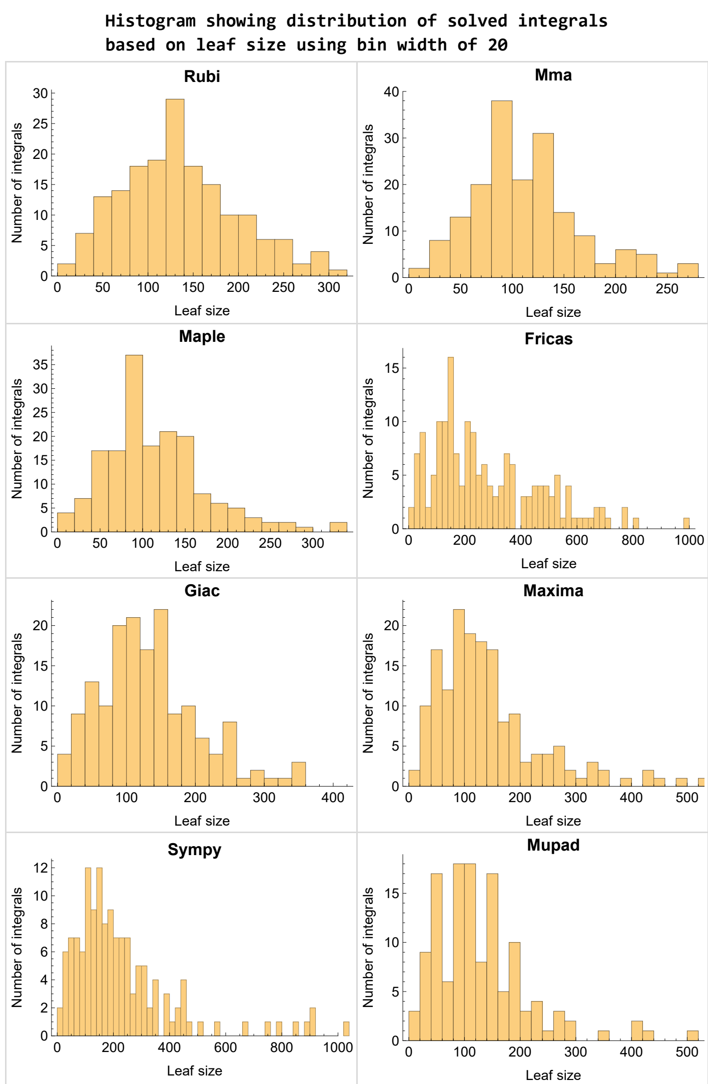


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

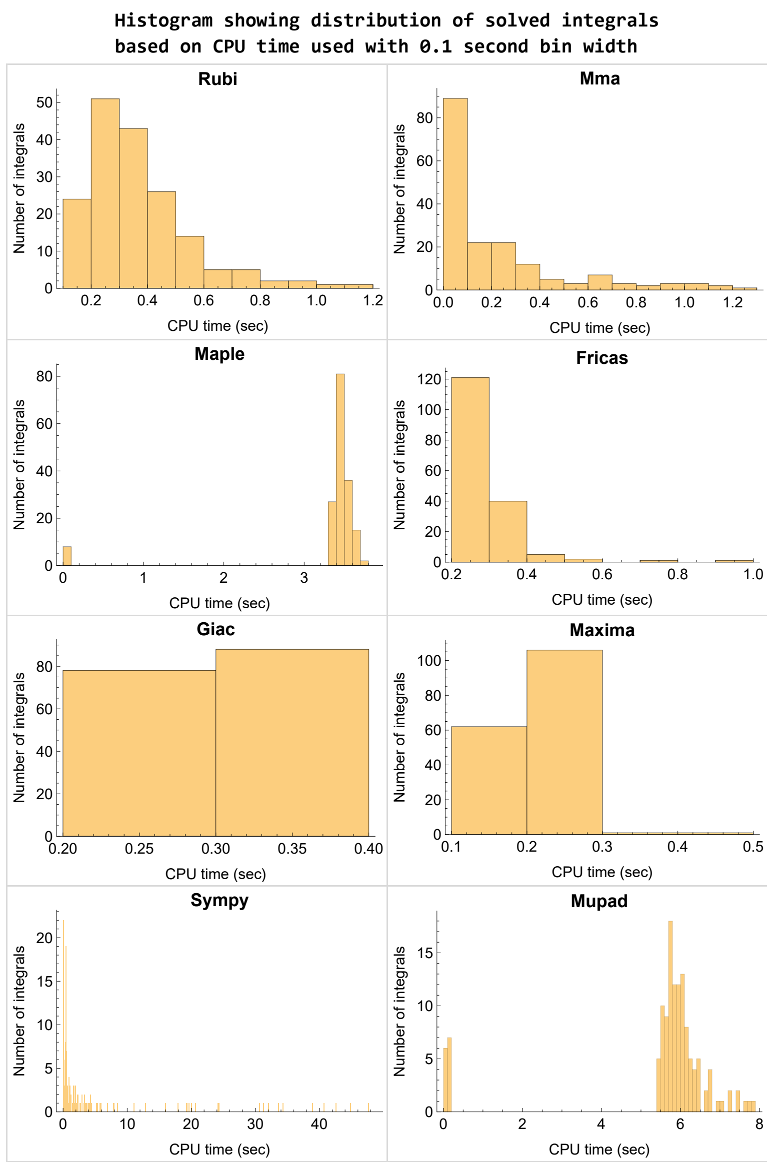


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

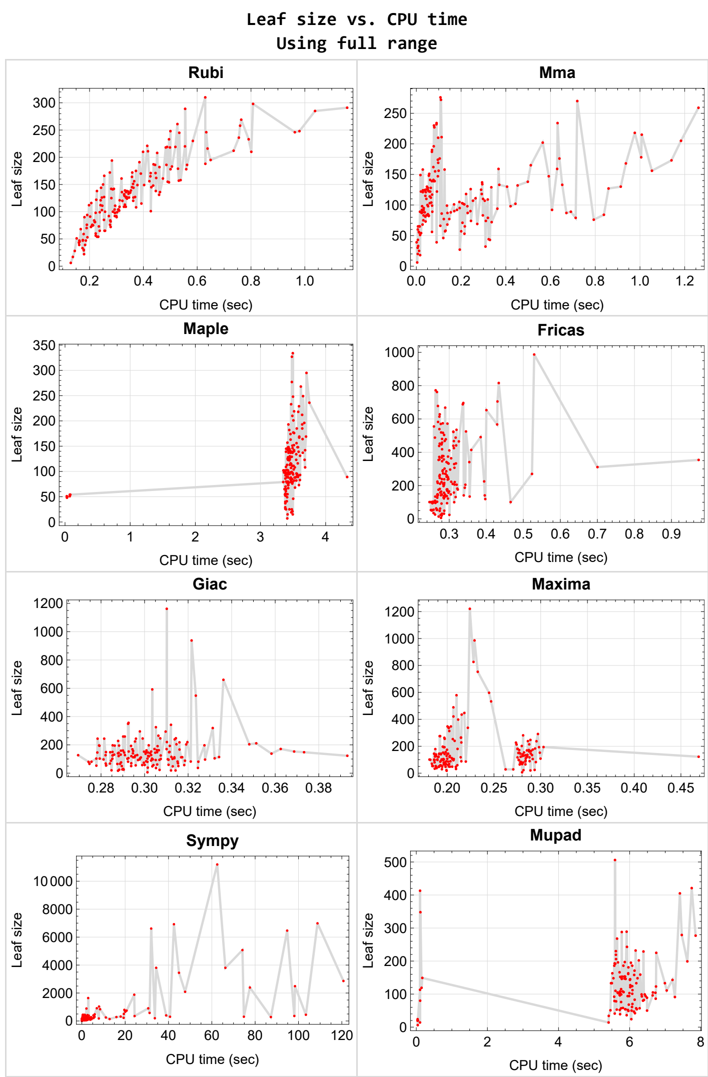


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{147, 148, 149, 150\}$

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

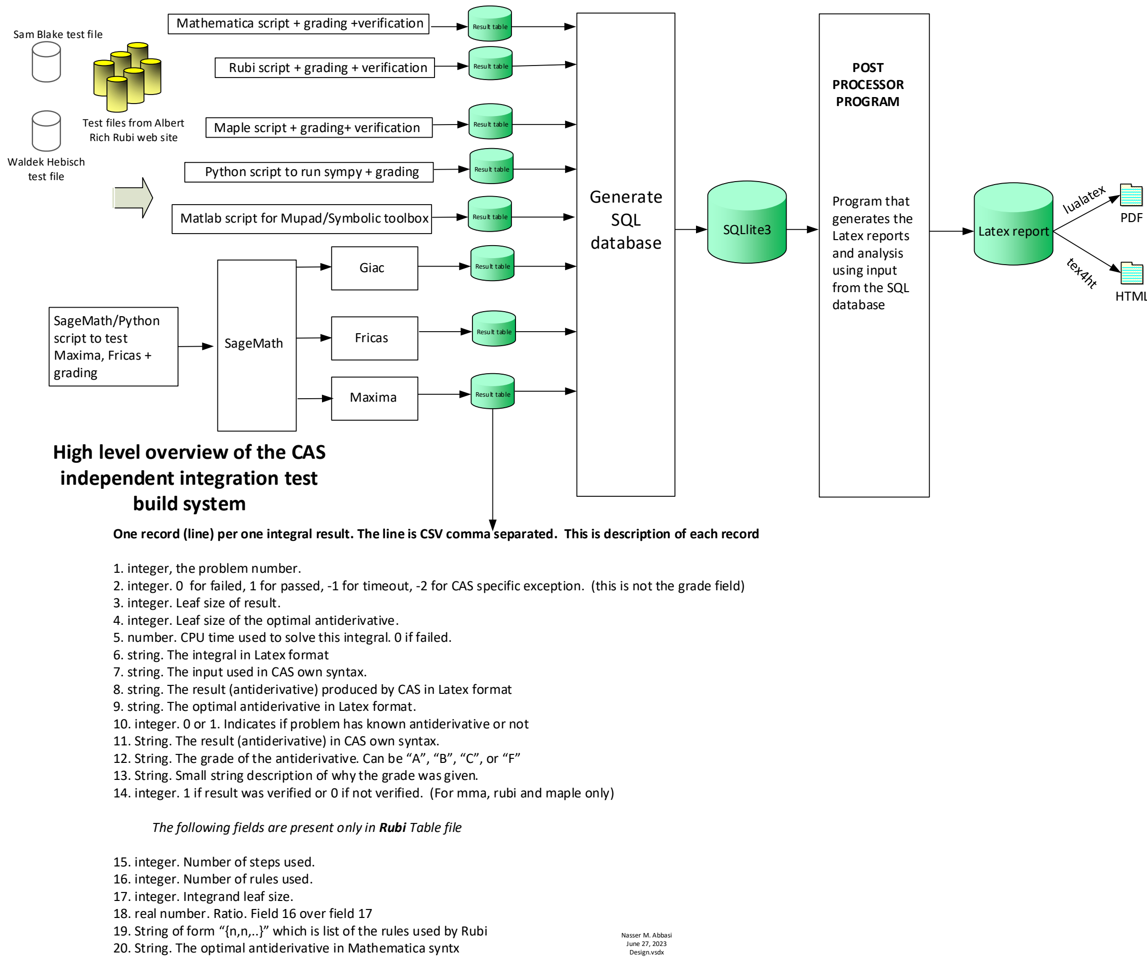
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 48 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174 }

B grade { 107, 108, 109 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 164, 165, 166, 167, 168, 169, 170, 171 }

B grade { 47, 48, 49, 159, 160, 161, 162, 163, 172, 173, 174 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 13, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 27, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 159, 160, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

B grade { 7, 14, 28, 35, 45, 150, 156, 157, 158, 166, 167, 168 }

C grade { }

F normal fail { 58, 59, 60, 61 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 12, 19, 26 }

2.1.7 Mupad

A grade { }

B grade { 4, 5, 6, 7, 11, 12, 13, 14, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 90, 92, 93, 96, 98, 100, 101, 102, 104, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 155, 156, 157, 158, 166, 167, 169, 170, 171 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 8, 9, 10, 15, 16, 17, 22, 29, 36, 47, 48, 58, 59, 60, 61, 86, 87, 88, 89, 91, 94, 95, 97, 99, 103, 105, 107, 151, 152, 153, 154, 159, 160, 161, 162, 163, 164, 165, 168, 172, 173, 174 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 35, 36, 38, 43, 44, 45, 46, 47, 50, 51, 53, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 105, 106, 110, 111, 112, 113, 114, 116, 117, 120, 123, 124, 125, 126, 127, 128, 129, 130, 133, 134, 135, 136, 137, 138, 146, 147, 148, 151, 152, 153, 154, 155, 156 }

B grade { 33, 34, 37, 39, 40, 41, 42, 48, 49, 52, 54, 55, 56, 57, 86, 87, 88, 89, 90, 94, 95, 96, 97, 98, 102, 103, 104, 115, 118, 119, 121, 122, 143, 144, 145, 149, 150, 157, 158, 161, 162, 163, 164, 165, 169, 170, 171, 172, 173, 174 }

C grade { 58, 59, 60, 61 }

F normal fail { }

F(-1) timedout fail { 91, 92, 93, 99, 100, 101, 107, 108, 109, 131, 132, 139, 140, 141, 142, 159, 160, 166, 167, 168 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	150	101	89	107	206	138	93	0
N.S.	1	1.18	0.80	0.70	0.84	1.62	1.09	0.73	0.00
time (sec)	N/A	0.259	0.169	4.330	0.200	0.314	0.444	0.305	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	122	87	80	86	175	122	81	0
N.S.	1	1.17	0.84	0.77	0.83	1.68	1.17	0.78	0.00
time (sec)	N/A	0.222	0.139	3.394	0.201	0.271	0.421	0.292	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	94	77	65	67	157	107	68	0
N.S.	1	1.18	0.96	0.81	0.84	1.96	1.34	0.85	0.00
time (sec)	N/A	0.196	0.123	3.376	0.197	0.286	0.428	0.284	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	68	68	54	45	128	90	55	52
N.S.	1	1.01	1.01	0.81	0.67	1.91	1.34	0.82	0.78
time (sec)	N/A	0.173	0.139	3.375	0.188	0.285	0.346	0.296	5.720

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	81	90	79	59	341	129	0	68
N.S.	1	1.03	1.14	1.00	0.75	4.32	1.63	0.00	0.86
time (sec)	N/A	0.223	0.170	3.349	0.193	0.293	1.966	0.000	5.976

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	88	78	59	333	124	102	89
N.S.	1	1.00	1.17	1.04	0.79	4.44	1.65	1.36	1.19
time (sec)	N/A	0.226	0.154	3.405	0.211	0.318	1.947	0.285	6.487

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	84	90	73	83	377	107	163	94
N.S.	1	1.05	1.12	0.91	1.04	4.71	1.34	2.04	1.18
time (sec)	N/A	0.225	0.242	3.427	0.198	0.284	1.992	0.306	6.447

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	172	118	113	126	254	168	115	0
N.S.	1	1.15	0.79	0.75	0.84	1.69	1.12	0.77	0.00
time (sec)	N/A	0.278	0.222	3.450	0.188	0.294	0.518	0.312	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	144	107	104	105	223	150	103	0
N.S.	1	1.13	0.84	0.82	0.83	1.76	1.18	0.81	0.00
time (sec)	N/A	0.250	0.253	3.510	0.207	0.305	0.473	0.303	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	116	101	89	86	205	134	89	0
N.S.	1	1.13	0.98	0.86	0.83	1.99	1.30	0.86	0.00
time (sec)	N/A	0.212	0.215	3.409	0.220	0.344	0.449	0.303	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	90	87	70	61	176	119	76	54
N.S.	1	1.03	1.00	0.80	0.70	2.02	1.37	0.87	0.62
time (sec)	N/A	0.182	0.229	3.384	0.201	0.323	0.400	0.289	6.202

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	112	110	109	88	439	274	0	83
N.S.	1	1.06	1.04	1.03	0.83	4.14	2.58	0.00	0.78
time (sec)	N/A	0.249	0.298	3.391	0.203	0.311	4.260	0.000	6.140

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	111	111	113	88	411	243	124	86
N.S.	1	1.03	1.03	1.05	0.81	3.81	2.25	1.15	0.80
time (sec)	N/A	0.254	0.265	3.408	0.199	0.300	2.276	0.296	6.732

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	103	109	102	112	425	224	191	91
N.S.	1	0.93	0.98	0.92	1.01	3.83	2.02	1.72	0.82
time (sec)	N/A	0.251	0.319	3.424	0.205	0.313	2.910	0.306	7.272

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	194	137	137	145	302	199	140	0
N.S.	1	1.12	0.79	0.79	0.84	1.75	1.15	0.81	0.00
time (sec)	N/A	0.291	0.294	3.430	0.186	0.287	0.626	0.295	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	166	125	128	124	271	184	128	0
N.S.	1	1.11	0.83	0.85	0.83	1.81	1.23	0.85	0.00
time (sec)	N/A	0.262	0.290	3.422	0.202	0.290	0.593	0.303	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	138	118	108	105	253	168	114	0
N.S.	1	1.10	0.94	0.86	0.83	2.01	1.33	0.90	0.00
time (sec)	N/A	0.228	0.292	3.527	0.196	0.293	0.552	0.334	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	112	107	86	77	224	150	101	54
N.S.	1	1.05	1.00	0.80	0.72	2.09	1.40	0.94	0.50
time (sec)	N/A	0.197	0.306	3.382	0.196	0.268	0.460	0.313	5.873

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	141	130	139	119	539	474	0	101
N.S.	1	1.07	0.98	1.05	0.90	4.08	3.59	0.00	0.77
time (sec)	N/A	0.287	0.406	3.391	0.190	0.273	5.944	0.000	6.073

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	142	133	157	120	519	420	150	104
N.S.	1	1.04	0.98	1.15	0.88	3.82	3.09	1.10	0.76
time (sec)	N/A	0.289	0.371	3.413	0.203	0.278	2.623	0.311	6.724

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	142	132	145	143	535	381	219	111
N.S.	1	1.01	0.94	1.03	1.01	3.79	2.70	1.55	0.79
time (sec)	N/A	0.298	0.453	3.394	0.200	0.320	3.210	0.320	7.041

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	126	77	65	88	158	121	74	0
N.S.	1	1.21	0.74	0.62	0.85	1.52	1.16	0.71	0.00
time (sec)	N/A	0.252	0.147	3.391	0.215	0.276	0.399	0.300	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	98	74	56	67	127	102	61	93
N.S.	1	1.21	0.91	0.69	0.83	1.57	1.26	0.75	1.15
time (sec)	N/A	0.220	0.311	3.384	0.199	0.268	0.393	0.306	6.343

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	70	58	46	47	109	87	50	82
N.S.	1	1.25	1.04	0.82	0.84	1.95	1.55	0.89	1.46
time (sec)	N/A	0.193	0.131	3.502	0.209	0.262	0.411	0.294	6.400

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	37	29	92	71	39	36
N.S.	1	1.00	1.07	0.86	0.67	2.14	1.65	0.91	0.84
time (sec)	N/A	0.164	0.121	3.381	0.194	0.279	0.317	0.301	5.915

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	66	52	33	273	70	0	42
N.S.	1	1.00	1.25	0.98	0.62	5.15	1.32	0.00	0.79
time (sec)	N/A	0.193	0.107	3.383	0.187	0.283	1.204	0.000	6.148

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	49	37	101	41	65	39
N.S.	1	1.00	1.21	1.04	0.79	2.15	0.87	1.38	0.83
time (sec)	N/A	0.178	0.128	3.409	0.189	0.466	1.079	0.287	5.730

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	74	65	55	56	123	66	146	58
N.S.	1	1.03	0.90	0.76	0.78	1.71	0.92	2.03	0.81
time (sec)	N/A	0.215	0.217	3.415	0.194	0.286	1.652	0.302	5.810

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	100	74	77	85	197	117	70	0
N.S.	1	1.23	0.91	0.95	1.05	2.43	1.44	0.86	0.00
time (sec)	N/A	0.294	0.244	3.428	0.193	0.285	3.633	0.294	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	72	69	68	64	164	83	58	61
N.S.	1	1.09	1.05	1.03	0.97	2.48	1.26	0.88	0.92
time (sec)	N/A	0.207	0.272	3.530	0.196	0.278	2.714	0.300	6.065

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	53	55	46	147	66	48	53
N.S.	1	1.00	1.10	1.15	0.96	3.06	1.38	1.00	1.10
time (sec)	N/A	0.179	0.215	3.378	0.191	0.304	2.345	0.313	5.638

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	27	26	31	35	46	23	24
N.S.	1	1.00	0.96	0.93	1.11	1.25	1.64	0.82	0.86
time (sec)	N/A	0.147	0.194	3.381	0.209	0.258	1.874	0.307	6.046

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	61	48	146	206	59	50
N.S.	1	1.00	1.21	1.30	1.02	3.11	4.38	1.26	1.06
time (sec)	N/A	0.192	0.201	3.390	0.203	0.289	3.302	0.311	6.489

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	76	71	80	68	169	235	96	70
N.S.	1	1.09	1.01	1.14	0.97	2.41	3.36	1.37	1.00
time (sec)	N/A	0.226	0.206	3.421	0.202	0.294	4.145	0.328	6.077

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	102	83	88	89	211	124	171	94
N.S.	1	1.07	0.87	0.93	0.94	2.22	1.31	1.80	0.99
time (sec)	N/A	0.330	0.297	3.523	0.193	0.275	3.961	0.305	5.942

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	85	72	97	102	239	400	70	0
N.S.	1	1.08	0.91	1.23	1.29	3.03	5.06	0.89	0.00
time (sec)	N/A	0.279	0.337	3.412	0.194	0.280	5.333	0.315	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	59	44	41	70	63	141	36	51
N.S.	1	1.11	0.83	0.77	1.32	1.19	2.66	0.68	0.96
time (sec)	N/A	0.189	0.323	3.404	0.184	0.265	4.252	0.305	5.459

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	47	32	29	51	49	95	26	34
N.S.	1	0.94	0.64	0.58	1.02	0.98	1.90	0.52	0.68
time (sec)	N/A	0.166	0.309	3.380	0.192	0.287	3.876	0.290	5.412

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	43	40	48	62	146	37	41
N.S.	1	1.00	0.84	0.78	0.94	1.22	2.86	0.73	0.80
time (sec)	N/A	0.154	0.329	3.381	0.198	0.275	3.593	0.325	5.580

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	81	79	98	80	239	840	82	80
N.S.	1	1.07	1.04	1.29	1.05	3.14	11.05	1.08	1.05
time (sec)	N/A	0.223	0.320	3.460	0.186	0.298	8.428	0.315	6.374

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	113	94	122	100	264	910	119	96
N.S.	1	1.09	0.90	1.17	0.96	2.54	8.75	1.14	0.92
time (sec)	N/A	0.341	0.362	3.442	0.182	0.275	6.905	0.302	6.645

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	138	102	146	122	307	1034	197	123
N.S.	1	1.07	0.79	1.13	0.95	2.38	8.02	1.53	0.95
time (sec)	N/A	0.467	0.443	3.492	0.191	0.314	7.939	0.302	6.740

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	39	39	25	28	31	24	19	20
N.S.	1	1.44	1.44	0.93	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.161	0.091	3.481	0.271	0.286	0.083	0.314	0.043

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	39	39	25	28	31	24	19	20
N.S.	1	1.44	1.44	0.93	1.04	1.15	0.89	0.70	0.74
time (sec)	N/A	0.187	0.001	3.392	0.262	0.280	0.082	0.312	0.033

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	33	15	22	26	27	30	14
N.S.	1	1.00	1.94	0.88	1.29	1.53	1.59	1.76	0.82
time (sec)	N/A	0.139	0.008	3.416	0.282	0.262	0.036	0.290	5.407

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	5	6	6
N.S.	1	1.00	1.00	1.17	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.134	0.004	3.411	0.280	0.278	0.036	0.301	0.041

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	233	156	295	435	522	3806	204	0
N.S.	1	1.09	0.73	1.38	2.04	2.45	17.87	0.96	0.00
time (sec)	N/A	0.816	1.053	3.706	0.215	0.311	34.303	0.348	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	178	127	334	447	467	3448	138	0
N.S.	1	1.19	0.85	2.23	2.98	3.11	22.99	0.92	0.00
time (sec)	N/A	0.578	0.859	3.496	0.218	0.312	44.840	0.358	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	154	89	95	240	137	740	112	196
N.S.	1	1.17	0.67	0.72	1.82	1.04	5.61	0.85	1.48
time (sec)	N/A	0.501	0.691	3.557	0.208	0.293	20.663	0.314	5.672

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	162	79	76	253	122	575	81	186
N.S.	1	1.09	0.53	0.51	1.70	0.82	3.86	0.54	1.25
time (sec)	N/A	0.448	0.712	3.488	0.206	0.297	31.270	0.325	5.567

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	145	84	85	179	131	660	95	133
N.S.	1	1.04	0.60	0.61	1.29	0.94	4.75	0.68	0.96
time (sec)	N/A	0.363	0.839	3.485	0.187	0.317	20.049	0.311	5.499

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	87	88	197	134	904	94	133
N.S.	1	1.00	0.63	0.63	1.42	0.96	6.50	0.68	0.96
time (sec)	N/A	0.334	0.671	3.426	0.195	0.355	30.654	0.307	5.475

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	125	76	73	123	119	796	82	99
N.S.	1	1.05	0.64	0.61	1.03	1.00	6.69	0.69	0.83
time (sec)	N/A	0.281	0.793	3.590	0.187	0.397	19.626	0.321	5.763

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	121	92	96	153	137	1880	112	115
N.S.	1	0.95	0.72	0.76	1.20	1.08	14.80	0.88	0.91
time (sec)	N/A	0.245	0.607	3.494	0.196	0.285	24.192	0.317	5.759

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	156	130	193	157	465	6613	152	159
N.S.	1	1.13	0.94	1.40	1.14	3.37	47.92	1.10	1.15
time (sec)	N/A	0.362	0.914	3.428	0.198	0.326	32.006	0.300	6.299

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	212	168	277	228	525	6922	239	225
N.S.	1	1.13	0.89	1.47	1.21	2.79	36.82	1.27	1.20
time (sec)	N/A	0.756	0.937	3.481	0.215	0.345	42.510	0.314	6.747

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	246	173	327	265	688	11198	325	279
N.S.	1	1.12	0.79	1.49	1.21	3.14	51.13	1.48	1.27
time (sec)	N/A	0.986	1.141	3.482	0.215	0.337	62.494	0.305	7.465

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	45	45	43	0	0	0	97	0	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	2.16	0.00	0.00
time (sec)	N/A	0.170	0.007	0.000	0.000	0.000	0.614	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	91	91	82	0	0	0	189	0	0
N.S.	1	1.00	0.90	0.00	0.00	0.00	2.08	0.00	0.00
time (sec)	N/A	0.205	0.055	0.000	0.000	0.000	1.674	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	76	76	56	0	0	0	201	0	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	2.64	0.00	0.00
time (sec)	N/A	0.192	0.070	0.000	0.000	0.000	1.817	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	C	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	121	121	99	0	0	0	291	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	2.40	0.00	0.00
time (sec)	N/A	0.315	0.137	0.000	0.000	0.000	2.309	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.260	0.015	0.081	0.188	0.250	0.017	0.304	5.760

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.251	0.012	0.078	0.185	0.253	0.017	0.318	6.280

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	65	54	53	53	60	57	57
N.S.	1	1.00	1.00	0.83	0.82	0.82	0.92	0.88	0.88
time (sec)	N/A	0.246	0.007	0.074	0.184	0.267	0.019	0.299	6.182

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	51	50	50	56	54	54
N.S.	1	1.00	1.00	0.85	0.83	0.83	0.93	0.90	0.90
time (sec)	N/A	0.225	0.006	0.073	0.191	0.253	0.017	0.282	5.885

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	51	48	48	54	53	52
N.S.	1	1.00	1.00	0.91	0.86	0.86	0.96	0.95	0.93
time (sec)	N/A	0.227	0.011	0.027	0.198	0.274	0.061	0.278	5.950

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	50	48	55	49	50	49
N.S.	1	1.00	1.00	0.93	0.89	1.02	0.91	0.93	0.91
time (sec)	N/A	0.233	0.021	0.031	0.195	0.258	0.069	0.290	5.964

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	51	48	48	55	51	48	47
N.S.	1	1.00	0.94	0.89	0.89	1.02	0.94	0.89	0.87
time (sec)	N/A	0.230	0.017	0.030	0.205	0.267	0.146	0.304	6.118

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	55	49	49	55	54	50	50
N.S.	1	1.00	1.02	0.91	0.91	1.02	1.00	0.93	0.93
time (sec)	N/A	0.225	0.013	0.030	0.202	0.268	0.370	0.316	6.052

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	98	102	101	101	110	105	108
N.S.	1	1.00	0.90	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.326	0.026	3.360	0.189	0.253	0.023	0.332	5.804

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	92	102	101	101	110	105	108
N.S.	1	1.00	0.84	0.94	0.93	0.93	1.01	0.96	0.99
time (sec)	N/A	0.323	0.032	3.382	0.181	0.247	0.023	0.314	5.729

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	92	102	101	101	110	105	107
N.S.	1	1.00	0.88	0.98	0.97	0.97	1.06	1.01	1.03
time (sec)	N/A	0.299	0.025	3.351	0.186	0.250	0.025	0.298	5.729

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	88	99	98	98	107	102	105
N.S.	1	1.00	0.89	1.00	0.99	0.99	1.08	1.03	1.06
time (sec)	N/A	0.301	0.021	3.381	0.190	0.258	0.026	0.292	5.658

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	88	96	96	96	104	100	103
N.S.	1	1.00	0.96	1.04	1.04	1.04	1.13	1.09	1.12
time (sec)	N/A	0.294	0.028	3.383	0.186	0.263	0.089	0.307	5.728

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	88	98	96	103	99	98	92
N.S.	1	1.00	0.98	1.09	1.07	1.14	1.10	1.09	1.02
time (sec)	N/A	0.312	0.038	3.360	0.196	0.267	0.101	0.311	5.565

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	95	96	103	100	97	103
N.S.	1	1.00	0.89	0.97	0.98	1.05	1.02	0.99	1.05
time (sec)	N/A	0.289	0.024	3.448	0.207	0.256	0.182	0.304	5.671

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	92	97	103	100	97	106
N.S.	1	1.00	0.85	0.94	0.99	1.05	1.02	0.99	1.08
time (sec)	N/A	0.293	0.032	3.384	0.186	0.256	0.447	0.298	5.830

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	149	148	145	145	163	153	153
N.S.	1	1.00	1.00	0.99	0.97	0.97	1.09	1.03	1.03
time (sec)	N/A	0.396	0.018	3.576	0.191	0.269	0.027	0.293	5.871

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	125	148	145	145	165	153	153
N.S.	1	1.00	0.84	0.99	0.97	0.97	1.11	1.03	1.03
time (sec)	N/A	0.359	0.041	3.431	0.201	0.268	0.027	0.303	5.845

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	124	148	145	145	163	153	153
N.S.	1	1.00	0.90	1.07	1.05	1.05	1.18	1.11	1.11
time (sec)	N/A	0.342	0.036	3.424	0.193	0.268	0.027	0.369	5.785

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	121	144	142	142	158	149	149
N.S.	1	1.00	0.91	1.08	1.07	1.07	1.19	1.12	1.12
time (sec)	N/A	0.344	0.029	3.446	0.197	0.278	0.028	0.282	5.770

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	121	142	140	140	158	148	147
N.S.	1	1.00	0.94	1.10	1.09	1.09	1.22	1.15	1.14
time (sec)	N/A	0.325	0.042	3.421	0.193	0.269	0.122	0.301	5.853

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	123	145	139	147	150	145	121
N.S.	1	1.00	0.99	1.17	1.12	1.19	1.21	1.17	0.98
time (sec)	N/A	0.348	0.054	3.428	0.187	0.268	0.135	0.305	6.016

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	142	139	147	151	144	143
N.S.	1	1.00	0.92	1.05	1.03	1.09	1.12	1.07	1.06
time (sec)	N/A	0.341	0.040	3.430	0.188	0.280	0.221	0.288	6.006

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	124	141	142	147	155	146	148
N.S.	1	1.00	0.89	1.01	1.02	1.06	1.12	1.05	1.06
time (sec)	N/A	0.349	0.029	3.430	0.191	0.269	0.481	0.291	6.221

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	130	141	145	332	316	161	0
N.S.	1	1.00	0.86	0.93	0.96	2.20	2.09	1.07	0.00
time (sec)	N/A	0.368	0.047	3.441	0.278	0.298	0.545	0.301	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	128	127	270	274	137	0
N.S.	1	1.00	0.88	0.98	0.98	2.08	2.11	1.05	0.00
time (sec)	N/A	0.328	0.060	3.438	0.280	0.290	0.526	0.300	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	95	95	98	238	245	112	0
N.S.	1	1.00	0.86	0.86	0.88	2.14	2.21	1.01	0.00
time (sec)	N/A	0.300	0.033	3.412	0.276	0.287	0.484	0.295	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	81	85	82	180	211	88	0
N.S.	1	1.00	0.88	0.92	0.89	1.96	2.29	0.96	0.00
time (sec)	N/A	0.272	0.040	3.459	0.295	0.275	0.453	0.319	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	68	65	64	157	219	66	79
N.S.	1	1.00	0.93	0.89	0.88	2.15	3.00	0.90	1.08
time (sec)	N/A	0.241	0.028	3.369	0.278	0.298	0.420	0.289	5.572

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	73	73	65	158	0	66	0
N.S.	1	1.00	1.01	1.01	0.90	2.19	0.00	0.92	0.00
time (sec)	N/A	0.280	0.041	3.423	0.284	0.313	0.000	0.275	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	75	67	67	165	0	68	78
N.S.	1	1.00	0.99	0.88	0.88	2.17	0.00	0.89	1.03
time (sec)	N/A	0.281	0.031	3.425	0.280	0.300	0.000	0.282	5.618

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	84	89	76	205	0	80	97
N.S.	1	1.00	0.91	0.97	0.83	2.23	0.00	0.87	1.05
time (sec)	N/A	0.299	0.050	3.417	0.279	0.320	0.000	0.301	5.715

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	179	139	134	150	468	335	159	0
N.S.	1	1.02	0.79	0.76	0.85	2.66	1.90	0.90	0.00
time (sec)	N/A	0.533	0.082	3.549	0.276	0.288	1.866	0.289	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	155	128	124	127	372	289	131	0
N.S.	1	1.01	0.83	0.81	0.82	2.42	1.88	0.85	0.00
time (sec)	N/A	0.507	0.052	3.449	0.286	0.273	1.712	0.286	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	131	100	103	108	357	284	111	152
N.S.	1	0.98	0.75	0.77	0.81	2.66	2.12	0.83	1.13
time (sec)	N/A	0.468	0.050	3.406	0.300	0.274	1.602	0.298	5.735

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	109	92	78	84	287	212	81	0
N.S.	1	1.08	0.91	0.77	0.83	2.84	2.10	0.80	0.00
time (sec)	N/A	0.360	0.032	3.435	0.279	0.282	1.250	0.276	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	99	83	88	89	257	233	88	110
N.S.	1	1.06	0.89	0.95	0.96	2.76	2.51	0.95	1.18
time (sec)	N/A	0.252	0.053	3.466	0.288	0.273	0.988	0.285	5.600

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	104	85	99	87	296	0	93	0
N.S.	1	1.09	0.89	1.04	0.92	3.12	0.00	0.98	0.00
time (sec)	N/A	0.316	0.051	3.479	0.284	0.303	0.000	0.297	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	115	110	96	105	336	0	103	133
N.S.	1	1.05	1.00	0.87	0.95	3.05	0.00	0.94	1.21
time (sec)	N/A	0.399	0.045	3.552	0.291	0.296	0.000	0.279	6.186

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	136	112	127	117	441	0	126	158
N.S.	1	1.01	0.83	0.94	0.87	3.27	0.00	0.93	1.17
time (sec)	N/A	0.473	0.065	3.450	0.285	0.319	0.000	0.285	6.127

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	186	139	140	165	574	357	157	232
N.S.	1	1.01	0.75	0.76	0.89	3.10	1.93	0.85	1.25
time (sec)	N/A	0.577	0.070	3.463	0.290	0.283	98.075	0.295	6.166

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	161	126	115	136	480	282	122	0
N.S.	1	1.04	0.81	0.74	0.88	3.10	1.82	0.79	0.00
time (sec)	N/A	0.500	0.049	3.471	0.286	0.281	87.259	0.302	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	148	122	123	146	447	304	128	195
N.S.	1	1.09	0.90	0.90	1.07	3.29	2.24	0.94	1.43
time (sec)	N/A	0.468	0.062	3.421	0.289	0.279	74.789	0.291	6.034

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	127	99	97	111	357	178	97	0
N.S.	1	1.07	0.83	0.82	0.93	3.00	1.50	0.82	0.00
time (sec)	N/A	0.355	0.070	3.493	0.284	0.285	7.835	0.281	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	127	104	98	122	346	184	106	163
N.S.	1	1.09	0.90	0.84	1.05	2.98	1.59	0.91	1.41
time (sec)	N/A	0.266	0.067	3.433	0.279	0.281	3.412	0.293	5.960

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	144	117	130	133	488	0	128	0
N.S.	1	1.11	0.90	1.00	1.02	3.75	0.00	0.98	0.00
time (sec)	N/A	0.382	0.070	3.450	0.284	0.317	0.000	0.297	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	157	141	125	152	524	0	141	202
N.S.	1	1.09	0.98	0.87	1.06	3.64	0.00	0.98	1.40
time (sec)	N/A	0.569	0.062	3.416	0.299	0.311	0.000	0.303	6.257

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	188	147	169	172	696	0	162	229
N.S.	1	1.08	0.84	0.97	0.99	4.00	0.00	0.93	1.32
time (sec)	N/A	0.646	0.096	3.595	0.287	0.338	0.000	0.307	6.387

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	22	20	19	18	24	15	25	20
N.S.	1	1.10	1.00	0.95	0.90	1.20	0.75	1.25	1.00
time (sec)	N/A	0.187	0.006	3.472	0.199	0.300	0.035	0.290	0.038

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	29	18	15	22	14	22	17	14
N.S.	1	1.26	0.78	0.65	0.96	0.61	0.96	0.74	0.61
time (sec)	N/A	0.187	0.017	3.495	0.199	0.271	0.110	0.288	0.105

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	33	25	21	20	20	20	20	20
N.S.	1	1.32	1.00	0.84	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.177	0.012	3.491	0.289	0.280	0.037	0.285	0.041

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	22	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.73	0.80	0.80
time (sec)	N/A	0.181	0.006	3.444	0.293	0.277	0.041	0.288	0.038

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	210	233	213	452	384	244	289
N.S.	1	1.00	1.00	1.11	1.01	2.15	1.83	1.16	1.38
time (sec)	N/A	0.405	0.097	3.470	0.274	0.274	0.463	0.281	5.914

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	162	185	172	368	337	195	243
N.S.	1	1.00	0.94	1.08	1.00	2.14	1.96	1.13	1.41
time (sec)	N/A	0.358	0.076	3.557	0.277	0.280	0.446	0.288	5.924

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	128	137	133	286	185	148	193
N.S.	1	1.00	0.94	1.01	0.98	2.10	1.36	1.09	1.42
time (sec)	N/A	0.322	0.065	3.491	0.285	0.292	0.405	0.284	5.576

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	98	94	94	236	160	103	96
N.S.	1	1.00	0.98	0.94	0.94	2.36	1.60	1.03	0.96
time (sec)	N/A	0.264	0.052	3.501	0.284	0.296	0.365	0.301	5.724

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	83	79	80	211	150	84	76
N.S.	1	1.00	0.99	0.94	0.95	2.51	1.79	1.00	0.90
time (sec)	N/A	0.285	0.045	3.468	0.278	0.284	0.515	0.301	5.758

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	83	79	79	216	151	80	80
N.S.	1	1.00	1.01	0.96	0.96	2.63	1.84	0.98	0.98
time (sec)	N/A	0.283	0.055	3.493	0.281	0.287	0.989	0.274	0.109

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	103	94	97	246	167	103	94
N.S.	1	1.00	0.99	0.90	0.93	2.37	1.61	0.99	0.90
time (sec)	N/A	0.296	0.060	3.425	0.277	0.274	2.246	0.278	5.986

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	139	129	134	292	301	148	127
N.S.	1	1.00	1.01	0.94	0.98	2.13	2.20	1.08	0.93
time (sec)	N/A	0.349	0.078	3.480	0.291	0.283	5.860	0.311	6.017

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	174	163	175	374	354	197	161
N.S.	1	1.00	0.99	0.93	1.00	2.14	2.02	1.13	0.92
time (sec)	N/A	0.372	0.096	3.462	0.292	0.297	24.351	0.327	5.959

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	211	201	214	458	398	244	197
N.S.	1	1.00	1.00	0.95	1.01	2.17	1.89	1.16	0.93
time (sec)	N/A	0.422	0.113	3.505	0.288	0.281	38.915	0.296	5.863

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	248	227	229	227	572	444	246	413
N.S.	1	1.03	0.95	0.95	0.95	2.38	1.85	1.02	1.72
time (sec)	N/A	0.527	0.080	3.591	0.274	0.295	1.155	0.307	0.106

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	208	187	182	183	478	257	196	288
N.S.	1	1.03	0.93	0.90	0.91	2.37	1.27	0.97	1.43
time (sec)	N/A	0.472	0.069	3.441	0.294	0.291	1.078	0.279	5.778

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	165	148	139	140	418	221	148	153
N.S.	1	1.01	0.91	0.85	0.86	2.56	1.36	0.91	0.94
time (sec)	N/A	0.459	0.056	3.458	0.281	0.281	0.977	0.373	5.866

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	126	122	114	117	364	201	123	113
N.S.	1	1.07	1.03	0.97	0.99	3.08	1.70	1.04	0.96
time (sec)	N/A	0.351	0.058	3.465	0.275	0.305	0.804	0.393	0.104

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	120	115	107	117	354	197	120	112
N.S.	1	1.07	1.03	0.96	1.04	3.16	1.76	1.07	1.00
time (sec)	N/A	0.368	0.040	3.469	0.288	0.271	2.034	0.294	6.079

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	127	125	116	130	378	212	121	119
N.S.	1	1.05	1.03	0.96	1.07	3.12	1.75	1.00	0.98
time (sec)	N/A	0.388	0.051	3.462	0.282	0.281	5.298	0.288	0.149

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	158	151	137	151	438	226	148	145
N.S.	1	1.04	0.99	0.90	0.99	2.88	1.49	0.97	0.95
time (sec)	N/A	0.570	0.058	3.461	0.277	0.273	19.290	0.306	6.013

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	195	190	174	194	488	0	197	181
N.S.	1	1.03	1.01	0.92	1.03	2.58	0.00	1.04	0.96
time (sec)	N/A	0.668	0.070	3.460	0.303	0.261	0.000	0.285	5.964

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	236	230	210	238	582	0	247	219
N.S.	1	1.03	1.00	0.91	1.03	2.53	0.00	1.07	0.95
time (sec)	N/A	0.761	0.079	3.489	0.286	0.270	0.000	0.314	5.613

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	298	272	268	281	762	503	294	506
N.S.	1	1.04	0.95	0.93	0.98	2.66	1.75	1.02	1.76
time (sec)	N/A	0.859	0.110	3.619	0.281	0.267	19.388	0.310	5.587

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	258	232	219	237	668	316	244	348
N.S.	1	1.04	0.94	0.89	0.96	2.70	1.28	0.99	1.41
time (sec)	N/A	0.782	0.090	3.516	0.295	0.289	17.925	0.278	0.116

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	216	176	177	193	614	280	195	206
N.S.	1	1.04	0.85	0.86	0.93	2.97	1.35	0.94	1.00
time (sec)	N/A	0.643	0.105	3.503	0.298	0.273	15.945	0.286	5.616

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	178	156	151	169	555	260	169	163
N.S.	1	1.07	0.93	0.90	1.01	3.32	1.56	1.01	0.98
time (sec)	N/A	0.524	0.100	3.489	0.285	0.259	5.710	0.294	5.512

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	164	141	139	154	504	243	146	148
N.S.	1	1.12	0.96	0.95	1.05	3.43	1.65	0.99	1.01
time (sec)	N/A	0.367	0.076	3.499	0.282	0.286	3.400	0.278	5.543

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	170	155	140	161	517	250	150	149
N.S.	1	1.11	1.01	0.92	1.05	3.38	1.63	0.98	0.97
time (sec)	N/A	0.423	0.084	3.516	0.288	0.283	11.076	0.285	0.170

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	186	169	152	181	570	0	167	166
N.S.	1	1.11	1.01	0.90	1.08	3.39	0.00	0.99	0.99
time (sec)	N/A	0.511	0.098	3.513	0.275	0.277	0.000	0.316	5.715

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	210	196	176	202	628	0	194	192
N.S.	1	1.07	1.00	0.90	1.03	3.20	0.00	0.99	0.98
time (sec)	N/A	0.836	0.078	3.516	0.280	0.274	0.000	0.279	5.595

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	234	248	234	212	247	678	0	245	230
N.S.	1	1.06	1.00	0.91	1.06	2.90	0.00	1.05	0.98
time (sec)	N/A	1.056	0.090	3.639	0.289	0.270	0.000	0.299	5.612

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	291	276	248	291	772	0	295	268
N.S.	1	1.05	1.00	0.90	1.05	2.79	0.00	1.06	0.97
time (sec)	N/A	1.196	0.108	3.499	0.297	0.263	0.000	0.298	5.645

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	219	158	142	347	177	442	259	186
N.S.	1	1.02	0.74	0.66	1.62	0.83	2.07	1.21	0.87
time (sec)	N/A	0.503	0.126	3.515	0.204	0.295	0.461	0.290	5.823

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	171	122	111	263	134	340	193	146
N.S.	1	1.02	0.73	0.66	1.57	0.80	2.04	1.16	0.87
time (sec)	N/A	0.444	0.092	3.487	0.197	0.262	0.378	0.312	5.778

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	89	82	180	94	238	127	103
N.S.	1	1.04	0.74	0.68	1.49	0.78	1.97	1.05	0.85
time (sec)	N/A	0.348	0.071	3.470	0.275	0.254	0.294	0.269	5.721

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	108	86	82	122	205	134	125	99
N.S.	1	1.05	0.83	0.80	1.18	1.99	1.30	1.21	0.96
time (sec)	N/A	0.362	0.113	3.516	0.469	0.286	4.479	0.285	6.335

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	101	92	88	104	210	138	113	99
N.S.	1	1.01	0.92	0.88	1.04	2.10	1.38	1.13	0.99
time (sec)	N/A	0.440	0.182	3.537	0.206	0.309	12.825	0.295	6.423

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	145	102	95	128	221	194	140	133
N.S.	1	1.27	0.89	0.83	1.12	1.94	1.70	1.23	1.17
time (sec)	N/A	0.437	0.210	3.561	0.203	0.315	33.693	0.288	6.990

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	182	126	116	193	261	303	228	199
N.S.	1	1.25	0.86	0.79	1.32	1.79	2.08	1.56	1.36
time (sec)	N/A	0.463	0.243	3.572	0.196	0.294	40.705	0.288	7.622

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	218	159	148	275	341	444	356	277
N.S.	1	1.12	0.82	0.76	1.41	1.75	2.28	1.83	1.42
time (sec)	N/A	0.497	0.367	3.568	0.192	0.354	103.418	0.293	7.854

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	221	218	169	339	414	243	218	0
N.S.	1	0.90	0.89	0.69	1.38	1.69	0.99	0.89	0.00
time (sec)	N/A	0.443	0.977	3.699	0.201	0.360	0.613	0.316	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	191	159	137	255	329	199	170	0
N.S.	1	0.98	0.82	0.71	1.31	1.70	1.03	0.88	0.00
time (sec)	N/A	0.406	0.630	3.649	0.197	0.313	0.571	0.310	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	161	129	108	174	250	155	125	0
N.S.	1	1.11	0.89	0.74	1.20	1.72	1.07	0.86	0.00
time (sec)	N/A	0.333	0.335	3.685	0.200	0.297	0.407	0.307	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	132	105	93	118	216	246	113	0
N.S.	1	1.13	0.90	0.79	1.01	1.85	2.10	0.97	0.00
time (sec)	N/A	0.340	0.193	3.608	0.214	0.269	0.918	0.317	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	120	95	83	102	210	196	171	143
N.S.	1	1.09	0.86	0.75	0.93	1.91	1.78	1.55	1.30
time (sec)	N/A	0.336	0.194	3.605	0.196	0.284	1.088	0.363	7.205

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	136	98	90	128	221	427	319	105
N.S.	1	1.15	0.83	0.76	1.08	1.87	3.62	2.70	0.89
time (sec)	N/A	0.342	0.228	3.555	0.199	0.279	1.490	0.331	6.653

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	156	103	92	193	100	891	548	124
N.S.	1	1.11	0.74	0.66	1.38	0.71	6.36	3.91	0.89
time (sec)	N/A	0.438	0.205	3.583	0.197	0.313	2.123	0.324	6.072

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	186	134	123	275	141	1642	660	171
N.S.	1	0.98	0.71	0.65	1.46	0.75	8.69	3.49	0.90
time (sec)	N/A	0.464	0.295	3.677	0.192	0.396	2.922	0.336	6.136

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	310	259	236	1221	987	0	342	0
N.S.	1	0.81	0.68	0.62	3.20	2.59	0.00	0.90	0.00
time (sec)	N/A	0.650	1.262	3.750	0.224	0.529	0.000	0.312	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	245	215	196	986	816	0	265	0
N.S.	1	0.88	0.77	0.70	3.53	2.92	0.00	0.95	0.00
time (sec)	N/A	0.548	1.008	3.672	0.229	0.434	0.000	0.306	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	230	178	164	753	653	6467	203	0
N.S.	1	1.10	0.85	0.78	3.59	3.11	30.80	0.97	0.00
time (sec)	N/A	0.613	1.005	3.646	0.233	0.400	94.777	0.319	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	220	147	139	533	491	3803	160	0
N.S.	1	1.23	0.82	0.78	2.98	2.74	21.25	0.89	0.00
time (sec)	N/A	0.562	0.592	3.585	0.247	0.385	66.249	0.290	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	183	98	89	335	141	2088	131	0
N.S.	1	1.37	0.73	0.66	2.50	1.05	15.58	0.98	0.00
time (sec)	N/A	0.522	0.421	3.575	0.202	0.339	47.665	0.289	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	187	133	123	313	182	2392	211	0
N.S.	1	1.01	0.72	0.66	1.69	0.98	12.93	1.14	0.00
time (sec)	N/A	0.458	0.652	3.548	0.202	0.322	77.574	0.351	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	218	165	156	337	225	2861	349	0
N.S.	1	0.90	0.68	0.64	1.39	0.93	11.82	1.44	0.00
time (sec)	N/A	0.515	0.512	3.553	0.222	0.394	120.749	0.292	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	233	202	185	398	270	0	592	405
N.S.	1	0.83	0.72	0.66	1.42	0.96	0.00	2.11	1.44
time (sec)	N/A	0.514	0.565	3.641	0.212	0.523	0.000	0.304	7.412

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	334	261	234	218	489	311	0	938	421
N.S.	1	0.78	0.70	0.65	1.46	0.93	0.00	2.81	1.26
time (sec)	N/A	0.554	0.631	3.579	0.207	0.700	0.000	0.322	7.744

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	289	270	249	579	354	0	1162	0
N.S.	1	0.74	0.69	0.64	1.48	0.90	0.00	2.96	0.00
time (sec)	N/A	0.567	0.720	3.653	0.210	0.973	0.000	0.310	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	219	158	142	347	177	442	259	186
N.S.	1	1.02	0.74	0.66	1.62	0.83	2.07	1.21	0.87
time (sec)	N/A	0.527	0.029	3.685	0.200	0.293	0.458	0.291	6.072

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	171	122	111	263	134	340	193	146
N.S.	1	1.02	0.73	0.66	1.57	0.80	2.04	1.16	0.87
time (sec)	N/A	0.451	0.020	3.508	0.215	0.289	0.365	0.294	5.886

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	126	89	82	180	94	238	127	103
N.S.	1	1.04	0.74	0.68	1.49	0.78	1.97	1.05	0.85
time (sec)	N/A	0.374	0.016	3.550	0.211	0.294	0.291	0.282	5.992

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	285	205	193	826	705	6987	224	0
N.S.	1	1.09	0.79	0.74	3.16	2.70	26.77	0.86	0.00
time (sec)	N/A	1.097	1.183	3.640	0.228	0.430	108.770	0.296	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	269	176	167	597	567	5071	204	0
N.S.	1	1.26	0.82	0.78	2.79	2.65	23.70	0.95	0.00
time (sec)	N/A	0.806	0.640	3.594	0.245	0.429	74.123	0.315	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	246	138	131	421	187	2490	220	0
N.S.	1	1.27	0.72	0.68	2.18	0.97	12.90	1.14	0.00
time (sec)	N/A	0.641	0.498	3.607	0.207	0.342	98.387	0.310	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [57] had the largest ratio of [.560000000000000053]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	11	10	1.18	20	0.500
2	A	9	8	1.17	20	0.400
3	A	6	5	1.18	18	0.278
4	A	5	4	1.01	17	0.235
5	A	8	7	1.03	20	0.350
6	A	8	7	1.00	20	0.350
7	A	9	8	1.05	20	0.400
8	A	12	11	1.15	20	0.550
9	A	10	9	1.13	20	0.450
10	A	7	6	1.13	18	0.333
11	A	6	5	1.03	17	0.294
12	A	9	8	1.06	20	0.400
13	A	9	8	1.03	20	0.400
14	A	11	10	0.93	20	0.500
15	A	12	11	1.12	20	0.550
16	A	11	10	1.11	20	0.500
17	A	8	7	1.10	18	0.389
18	A	7	6	1.05	17	0.353
19	A	11	10	1.07	20	0.500
20	A	10	9	1.04	20	0.450
21	A	12	11	1.01	20	0.550
22	A	9	8	1.21	20	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	8	7	1.21	20	0.350
24	A	5	4	1.25	18	0.222
25	A	4	3	1.00	17	0.176
26	A	7	6	1.00	20	0.300
27	A	5	4	1.00	20	0.200
28	A	7	6	1.03	20	0.300
29	A	7	6	1.23	20	0.300
30	A	7	6	1.09	20	0.300
31	A	6	5	1.00	18	0.278
32	A	1	1	1.00	17	0.059
33	A	7	6	1.00	20	0.300
34	A	7	6	1.09	20	0.300
35	A	9	8	1.07	20	0.400
36	A	6	5	1.08	20	0.250
37	A	4	4	1.11	20	0.200
38	A	4	4	0.94	18	0.222
39	A	2	2	1.00	17	0.118
40	A	8	7	1.07	20	0.350
41	A	9	8	1.09	20	0.400
42	A	11	10	1.07	20	0.500
43	A	4	4	1.44	18	0.222
44	A	5	5	1.44	19	0.263
45	A	2	2	1.00	13	0.154
46	A	2	2	1.00	13	0.154
47	A	11	10	1.09	25	0.400
48	A	11	10	1.19	25	0.400
49	A	7	7	1.17	25	0.280
50	A	6	6	1.09	25	0.240
51	A	7	7	1.04	25	0.280
52	A	7	7	1.00	25	0.280
53	A	5	5	1.05	23	0.217
54	A	6	6	0.95	22	0.273
55	A	12	11	1.13	25	0.440
56	A	13	12	1.13	25	0.480

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	15	14	1.12	25	0.560
58	A	2	2	1.00	16	0.125
59	A	2	2	1.00	20	0.100
60	A	2	2	1.00	22	0.091
61	A	2	2	1.00	25	0.080
62	A	2	2	1.00	26	0.077
63	A	2	2	1.00	26	0.077
64	A	2	2	1.00	24	0.083
65	A	2	2	1.00	23	0.087
66	A	2	2	1.00	26	0.077
67	A	2	2	1.00	26	0.077
68	A	2	2	1.00	26	0.077
69	A	2	2	1.00	26	0.077
70	A	2	2	1.00	28	0.071
71	A	2	2	1.00	28	0.071
72	A	3	3	1.00	26	0.115
73	A	3	3	1.00	25	0.120
74	A	3	3	1.00	28	0.107
75	A	3	3	1.00	28	0.107
76	A	2	2	1.00	28	0.071
77	A	2	2	1.00	28	0.071
78	A	2	2	1.00	28	0.071
79	A	2	2	1.00	28	0.071
80	A	3	3	1.00	26	0.115
81	A	3	3	1.00	25	0.120
82	A	3	3	1.00	28	0.107
83	A	3	3	1.00	28	0.107
84	A	2	2	1.00	28	0.071
85	A	2	2	1.00	28	0.071
86	A	2	2	1.00	28	0.071
87	A	2	2	1.00	28	0.071
88	A	2	2	1.00	28	0.071
89	A	2	2	1.00	26	0.077
90	A	2	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
91	A	2	2	1.00	28	0.071
92	A	2	2	1.00	28	0.071
93	A	2	2	1.00	28	0.071
94	A	4	4	1.02	28	0.143
95	A	4	4	1.01	28	0.143
96	A	4	4	0.98	28	0.143
97	A	4	4	1.08	26	0.154
98	A	6	6	1.06	25	0.240
99	A	5	5	1.09	28	0.179
100	A	4	4	1.05	28	0.143
101	A	4	4	1.01	28	0.143
102	A	6	6	1.01	28	0.214
103	A	7	7	1.04	28	0.250
104	A	8	8	1.09	28	0.286
105	A	6	6	1.07	26	0.231
106	A	5	5	1.09	25	0.200
107	A	7	7	1.11	28	0.250
108	A	6	6	1.09	28	0.214
109	A	6	6	1.08	28	0.214
110	A	6	5	1.10	17	0.294
111	A	6	5	1.26	17	0.294
112	A	4	4	1.32	21	0.190
113	A	3	3	1.00	15	0.200
114	A	2	2	1.00	30	0.067
115	A	2	2	1.00	30	0.067
116	A	2	2	1.00	30	0.067
117	A	2	2	1.00	27	0.074
118	A	2	2	1.00	30	0.067
119	A	2	2	1.00	30	0.067
120	A	2	2	1.00	30	0.067
121	A	2	2	1.00	30	0.067
122	A	2	2	1.00	30	0.067
123	A	2	2	1.00	30	0.067
124	A	4	4	1.03	30	0.133

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
125	A	4	4	1.03	30	0.133
126	A	4	4	1.01	30	0.133
127	A	4	4	1.07	27	0.148
128	A	4	4	1.07	30	0.133
129	A	4	4	1.05	30	0.133
130	A	4	4	1.04	30	0.133
131	A	4	4	1.03	30	0.133
132	A	4	4	1.03	30	0.133
133	A	5	5	1.04	30	0.167
134	A	6	6	1.04	30	0.200
135	A	5	5	1.04	30	0.167
136	A	7	7	1.07	30	0.233
137	A	7	7	1.12	27	0.259
138	A	6	6	1.11	30	0.200
139	A	5	5	1.11	30	0.167
140	A	6	6	1.07	30	0.200
141	A	6	6	1.06	30	0.200
142	A	6	6	1.05	30	0.200
143	A	4	3	1.02	32	0.094
144	A	4	3	1.02	32	0.094
145	A	4	3	1.04	30	0.100
146	A	4	3	1.05	32	0.094
147	A	8	7	1.01	32	0.219
148	A	9	8	1.27	32	0.250
149	A	10	9	1.25	32	0.281
150	A	10	9	1.12	32	0.281
151	A	8	7	0.90	32	0.219
152	A	7	6	0.98	32	0.188
153	A	7	6	1.11	29	0.207
154	A	9	8	1.13	32	0.250
155	A	8	7	1.09	32	0.219
156	A	7	6	1.15	32	0.188
157	A	5	5	1.11	32	0.156
158	A	7	7	0.98	32	0.219

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
159	A	13	12	0.81	32	0.375
160	A	13	12	0.88	32	0.375
161	A	13	12	1.10	32	0.375
162	A	12	11	1.23	32	0.344
163	A	8	8	1.37	29	0.276
164	A	6	6	1.01	32	0.188
165	A	7	7	0.90	32	0.219
166	A	9	9	0.83	32	0.281
167	A	10	10	0.78	32	0.312
168	A	11	11	0.74	32	0.344
169	A	5	4	1.02	33	0.121
170	A	5	4	1.02	33	0.121
171	A	5	4	1.04	31	0.129
172	A	16	15	1.09	37	0.405
173	A	11	10	1.26	34	0.294
174	A	8	8	1.27	37	0.216

CHAPTER 3

Listing of integrals

3.1 $\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$

3.1.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx=\frac{a^2Bx\sqrt{a+bx^2}}{16b^2}+\frac{Ax^2(a+bx^2)^{3/2}}{5b}+\frac{Bx^3(a+bx^2)^{3/2}}{6b}-\frac{a(16A+15Bx)\,(a+bx^2)^{3/2}}{120b^2}+\frac{a^3B\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{5/2}}$$

output
```
1/5*A*x^2*(b*x^2+a)^(3/2)/b+1/6*B*x^3*(b*x^2+a)^(3/2)/b-1/120*a*(15*B*x+16
*A)*(b*x^2+a)^(3/2)/b^2+1/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5
/2)+1/16*a^2*B*x*(b*x^2+a)^(1/2)/b^2
```

3.1.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(-32a^2A-15a^2Bx+16aAbx^2+10abBx^3+48Ab^2x^4+40b^2Bx^5)}{240b^2}-\frac{a^3B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{5/2}}$$

input
```
Integrate[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-32*a^2*A - 15*a^2*B*x + 16*a*A*b*x^2 + 10*a*b*B*x^3 + 4
8*A*b^2*x^4 + 40*b^2*B*x^5))/(240*b^2) - (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a
+ b*x^2]])/(16*b^(5/2))
```

3.1.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {533, 27, 533, 25, 27, 533, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\int 3x^2(aB-2Abx)\sqrt{bx^2+a}dx}{6b}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\int x^2(aB-2Abx)\sqrt{bx^2+a}dx}{2b}$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{-\frac{\int -abx(4A+5Bx)\sqrt{bx^2+a}dx}{5b}-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 25$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{\int abx(4A+5Bx)\sqrt{bx^2+a}dx}{5b}-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\int x(4A+5Bx)\sqrt{bx^2+a}dx-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\left(\frac{5Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{\int (5aB-16Abx)\sqrt{bx^2+a}dx}{4b}\right)-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 455$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\left(\frac{5Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{5aB\int\sqrt{bx^2+a}dx-\frac{16}{3}A\left(a+bx^2\right)^{3/2}}{4b}\right)-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 211$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\left(\frac{5Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{5aB\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{16}{3}A\left(a+bx^2\right)^{3/2}}{4b}\right)-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 224$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\left(\frac{5Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{5aB\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{16}{3}A\left(a+bx^2\right)^{3/2}}{4b}\right)-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

$$\Big\downarrow \; 219$$

$$\frac{Bx^3\left(a+bx^2\right)^{3/2}}{6b}-\frac{\frac{1}{5}a\left(\frac{5Bx\left(a+bx^2\right)^{3/2}}{4b}-\frac{5aB\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{16}{3}A\left(a+bx^2\right)^{3/2}}{4b}\right)-\frac{2}{5}Ax^2\left(a+bx^2\right)^{3/2}}{2b}$$

input
```
Int[x^3*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(B*x^3*(a + b*x^2)^(3/2))/(6*b) - ((-2*A*x^2*(a + b*x^2)^(3/2))/5 + (a*((5
*B*x*(a + b*x^2)^(3/2))/(4*b) - ((-16*A*(a + b*x^2)^(3/2))/3 + 5*a*B*((x*S
qrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))
/(4*b)))/5)/(2*b)
```

3.1.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.1.4 Maple [A] (verified)

Time = 4.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.70

method	result	si
risch	$-\frac{(-40b^2B\,x^5-48A\,b^2x^4-10Bab\,x^3-16aAb\,x^2+15a^2Bx+32a^2A)\sqrt{b\,x^2+a}}{240b^2}+\frac{a^3B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{16b^{\frac{5}{2}}}$	89
default	$B\left(\frac{x^3\left(b\,x^2+a\right)^{\frac{3}{2}}}{6b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)}{2b}\right)+A\left(\frac{x^2\left(b\,x^2+a\right)^{\frac{3}{2}}}{5b}-\frac{2a\left(b\,x^2+a\right)^{\frac{3}{2}}}{15b^2}\right)$	12

input
```
int(x^3*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/240*(-40*B*b^2*x^5-48*A*b^2*x^4-10*B*a*b*x^3-16*A*a*b*x^2+15*B*a^2*x+32
*A*a^2)/b^2*(b*x^2+a)^(1/2)+1/16*a^3*B/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2
))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.62

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$$

$$=\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15}{480\,b^3}\right.$$

$$-\frac{15\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(40\,Bb^3x^5+48\,Ab^3x^4+10\,Bab^2x^3+16\,Aab^2x^2-15\,Ba^2bx-32\,Aa^2b)}{240\,b^3}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*x^3 + 16*A*a*b^2*x^2 - 15*B*a
^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3, -1/240*(15*B*a^3*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 10*B*a*b^2*
x^3 + 16*A*a*b^2*x^2 - 15*B*a^2*b*x - 32*A*a^2*b)*sqrt(b*x^2 + a))/b^3]
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\begin{cases} \frac{Ba^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b^2}+\sqrt{a+bx^2}\left(-\frac{2Aa^2}{15b^2}+\frac{Aax^2}{15b}+\frac{Ax^4}{5}-\frac{Ba^2x}{16b^2}+\frac{Bax^3}{24b}+\frac{Bx^5}{6}\right) & \text{for } b\ \\ \sqrt{a}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) & \text{other}\end{cases}$$

input
```
integrate(x**3*(B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((B*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(
b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b**2) + sqrt(a + b*x**2)
*(-2*A*a**2/(15*b**2) + A*a*x**2/(15*b) + A*x**4/5 - B*a**2*x/(16*b**2) +
B*a*x**3/(24*b) + B*x**5/6), Ne(b, 0)), (sqrt(a)*(A*x**4/4 + B*x**5/5), Tr
ue))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx=\frac{(bx^2+a)^{\frac{3}{2}}Bx^3}{6\,b}+\frac{(bx^2+a)^{\frac{3}{2}}Ax^2}{5\,b}-\frac{(bx^2+a)^{\frac{3}{2}}Bax}{8\,b^2}+\frac{\sqrt{bx^2+a}Ba^2x}{16\,b^2}+\frac{Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{3}{2}}Aa}{15\,b^2}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(3/2)*B*x^3/b + 1/5*(b*x^2 + a)^(3/2)*A*x^2/b - 1/8*(b*x^2
 + a)^(3/2)*B*a*x/b^2 + 1/16*sqrt(b*x^2 + a)*B*a^2*x/b^2 + 1/16*B*a^3*arcs
inh(b*x/sqrt(a*b))/b^(5/2) - 2/15*(b*x^2 + a)^(3/2)*A*a/b^2
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx$$
$$= -\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{5}{2}}} + \frac{1}{240}\sqrt{bx^2+a}\left(\left(2\left(\left(4\,(5\,Bx+6\,A)x+\frac{5\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)x-\frac{15\,Ba^2}{b^2}\right)x-\frac{32\,Aa^2}{b^2}\right)$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) + 1/240*sqrt(b*
x^2 + a)*((2*((4*(5*B*x + 6*A)*x + 5*B*a/b)*x + 8*A*a/b)*x - 15*B*a^2/b^2)
*x - 32*A*a^2/b^2)
```

3.1.9 Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\sqrt{a+bx^2}\,dx = \int x^3\,\sqrt{b\,x^2+a}\,(A+B\,x)\,dx$$

input
```
int(x^3*(a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^3*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.2 $\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$

3.2.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx=-\frac{aAx\sqrt{a+bx^2}}{8b}+\frac{Bx^2(a+bx^2)^{3/2}}{5b}-\frac{(8aB-15Abx)\,(a+bx^2)^{3/2}}{60b^2}-\frac{a^2A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/5*B*x^2*(b*x^2+a)^(3/2)/b-1/60*(-15*A*b*x+8*B*a)*(b*x^2+a)^(3/2)/b^2-1/8
*a^2*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/8*a*A*x*(b*x^2+a)^(1/2
)/b
```

3.2.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(-16a^2B+6b^2x^3(5A+4Bx)+abx(15A+8Bx))+15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{120b^2}$$

input
```
Integrate[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a^2*B + 6*b^2*x^3*(5*A + 4*B*x) + a*b*x*(15*A + 8*B*
x)) + 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(120*b^2)
```

3.2.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {533, 533, 25, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\int x(2aB-5Abx)\sqrt{bx^2+a}dx}{5b}$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{-\frac{\int -ab(5A+8Bx)\sqrt{bx^2+a}dx}{4b}-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{25}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\frac{\int ab(5A+8Bx)\sqrt{bx^2+a}dx}{4b}-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\frac{1}{4}a\int(5A+8Bx)\sqrt{bx^2+a}dx-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\frac{1}{4}a\left(5A\int\sqrt{bx^2+a}dx+\frac{8B\left(a+bx^2\right)^{3/2}}{3b}\right)-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\frac{1}{4}a\left(5A\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{8B\left(a+bx^2\right)^{3/2}}{3b}\right)-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$\frac{Bx^2\left(a+bx^2\right)^{3/2}}{5b}-\frac{\frac{1}{4}a\left(5A\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{8B\left(a+bx^2\right)^{3/2}}{3b}\right)-\frac{5}{4}Ax\left(a+bx^2\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^2(a+bx^2)^{3/2}}{5b} - \frac{\frac{1}{4}a\left(5A\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{8B(a+bx^2)^{3/2}}{3b}\right)-\frac{5}{4}Ax(a+bx^2)^{3/2}}{5b}$$

input
```
Int[x^2*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(B*x^2*(a + b*x^2)^(3/2))/(5*b) - ((-5*A*x*(a + b*x^2)^(3/2))/4 + (a*((8*B
*(a + b*x^2)^(3/2))/(3*b) + 5*A*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))))/4)/(5*b)
```

3.2.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.2.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{\left(24 b^{2} B \,x^{4}+30 A \,b^{2} x^{3}+8 B a b \,x^{2}+15 a A b x -16 a^{2} B \right) \sqrt{b \,x^{2}+a}}{120 b^{2}}-\frac{a^{2} A \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{8 b^{\frac{3}{2}}}$	80
default	$B \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{5 b}-\frac{2 a \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{15 b^{2}}\right)+A \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4 b}-\frac{a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4 b}\right)$	96

input
```
int(x^2*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*(24*B*b^2*x^4+30*A*b^2*x^3+8*B*a*b*x^2+15*A*a*b*x-16*B*a^2)/b^2*(b*x
^2+a)^(1/2)-1/8*a^2*A/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.68

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$$

$$= \left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(24\,Bb^2x^4+30\,Ab^2x^3+8\,Babx^2+15\,Aabx-16\,Ba^2}{240\,b^2}\right.$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*A*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)*sqr
t(b*x^2 + a))/b^2, 1/120*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
 a)) + (24*B*b^2*x^4 + 30*A*b^2*x^3 + 8*B*a*b*x^2 + 15*A*a*b*x - 16*B*a^2)
*sqrt(b*x^2 + a))/b^2]
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx$$
$$= \begin{cases} -\frac{Aa^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b} + \sqrt{a+bx^2}\left(\frac{Aax}{8b}+\frac{Ax^3}{4}-\frac{2Ba^2}{15b^2}+\frac{Bax^2}{15b}+\frac{Bx^4}{5}\right) & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(x**2*(B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + sqrt(a + b*x**2)*(A
*a*x/(8*b) + A*x**3/4 - 2*B*a**2/(15*b**2) + B*a*x**2/(15*b) + B*x**4/5),
Ne(b, 0)), (sqrt(a)*(A*x**3/3 + B*x**4/4), True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx^2}{5\,b} + \frac{(bx^2+a)^{\frac{3}{2}}Ax}{4\,b} - \frac{\sqrt{bx^2+a}Aax}{8\,b} - \frac{Aa^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} - \frac{2\,(bx^2+a)^{\frac{3}{2}}Ba}{15\,b^2}$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(b*x^2 + a)^(3/2)*B*x^2/b + 1/4*(b*x^2 + a)^(3/2)*A*x/b - 1/8*sqrt(b*x
^2 + a)*A*a*x/b - 1/8*A*a^2*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/15*(b*x^2 +
 a)^(3/2)*B*a/b^2
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.78

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}} + \frac{1}{120}\sqrt{bx^2+a}\left(\left(2\left(3\,(4\,Bx+5\,A)x+\frac{4\,Ba}{b}\right)x+\frac{15\,Aa}{b}\right)x-\frac{16\,Ba^2}{b^2}\right)$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/120*sqrt(b*x^
2 + a)*((2*(3*(4*B*x + 5*A)*x + 4*B*a/b)*x + 15*A*a/b)*x - 16*B*a^2/b^2)
```

3.2.9 Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\sqrt{a+bx^2}\,dx = \int x^2\,\sqrt{b\,x^2+a}\,(A+B\,x)\,dx$$

input
```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^2*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.3 $\int x(A+Bx)\sqrt{a+bx^2}\,dx$

3.3.1 Optimal result

Integrand size = 18, antiderivative size = 80

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = -\frac{aBx\sqrt{a+bx^2}}{8b}+\frac{(4A+3Bx)\left(a+bx^2\right)^{3/2}}{12b}-\frac{a^2B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{3/2}}$$

output
```
1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)/b-1/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(3/2)-1/8*a*B*x*(b*x^2+a)^(1/2)/b
```

3.3.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \frac{\sqrt{a+bx^2}(8aA+3aBx+8Abx^2+6bBx^3)}{24b}+\frac{a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{3/2}}$$

input
```
Integrate[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(8*a*A + 3*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b) + (a^2*
B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(3/2))
```

3.3.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {533, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \ 533$$

$$\frac{Bx(a+bx^2)^{3/2}}{4b}-\frac{\int (aB-4Abx)\sqrt{bx^2+a}dx}{4b}$$

$$\Big\downarrow \ 455$$

$$\frac{Bx(a+bx^2)^{3/2}}{4b}-\frac{aB\int \sqrt{bx^2+a}dx-\frac{4}{3}A(a+bx^2)^{3/2}}{4b}$$

$$\Big\downarrow \ 211$$

$$\frac{Bx(a+bx^2)^{3/2}}{4b}-\frac{aB\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{4}{3}A(a+bx^2)^{3/2}}{4b}$$

$$\Big\downarrow \ 224$$

$$\frac{Bx(a+bx^2)^{3/2}}{4b}-\frac{aB\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{4}{3}A(a+bx^2)^{3/2}}{4b}$$

$$\Big\downarrow \ 219$$

$$\frac{Bx(a+bx^2)^{3/2}}{4b}-\frac{aB\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)-\frac{4}{3}A(a+bx^2)^{3/2}}{4b}$$

input
```
Int[x*(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(B*x*(a + b*x^2)^(3/2))/(4*b) - ((-4*A*(a + b*x^2)^(3/2))/3 + a*B*((x*Sqrt
[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(4
*b)
```

3.3.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.3.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.81

method	result	size
risch	$\frac{(6bB\,x^3+8Ab\,x^2+3Bax+8Aa)\sqrt{b\,x^2+a}}{24b}-\frac{B\,a^2\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{3}{2}}}$	65
default	$B\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4b}-\frac{a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4b}\right)+\frac{A(b\,x^2+a)^{\frac{3}{2}}}{3b}$	76

input
```
int(x*(B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*B*b*x^3+8*A*b*x^2+3*B*a*x+8*A*a)/b*(b*x^2+a)^(1/2)-1/8*B*a^2/b^(3/
2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.96

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\left[\frac{3\,Ba^2\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,Bb^2x^3+8\,Ab^2x^2+3\,Babx+8\,Aab)\sqrt{bx^2+a}}{48\,b^2},\frac{3\,Ba}{}\right.$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*B*a^2*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2, 1
/24*(3*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (6*B*b^2*x^3 +
8*A*b^2*x^2 + 3*B*a*b*x + 8*A*a*b)*sqrt(b*x^2 + a))/b^2]
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx$$
$$=\begin{cases}-\frac{Ba^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\sqrt{a+bx^2}\left(\frac{Aa}{3b}+\frac{Ax^2}{3}+\frac{Bax}{8b}+\frac{Bx^3}{4}\right) & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x*(B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-B*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + sqrt(a + b*x**2)*(A
*a/(3*b) + A*x**2/3 + B*a*x/(8*b) + B*x**3/4), Ne(b, 0)), (sqrt(a)*(A*x**2
/2 + B*x**3/3), True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.84

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \frac{(bx^2+a)^{\frac{3}{2}}Bx}{4\,b} - \frac{\sqrt{bx^2+a}Bax}{8\,b} - \frac{Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{3}{2}}A}{3\,b}$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*B*x/b - 1/8*sqrt(b*x^2 + a)*B*a*x/b - 1/8*B*a^2*arcs
inh(b*x/sqrt(a*b))/b^(3/2) + 1/3*(b*x^2 + a)^(3/2)*A/b
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx = \frac{Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{3}{2}}} + \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\,(3\,Bx+4\,A)x+\frac{3\,Ba}{b}\right)x+\frac{8\,Aa}{b}\right)$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/24*sqrt(b*x^2
 + a)*((2*(3*B*x + 4*A)*x + 3*B*a/b)*x + 8*A*a/b)
```

3.3.9 Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\sqrt{a+bx^2}\,dx=\int x\,\sqrt{b\,x^2+a}\,(A+B\,x)\;dx$$

input
```
int(x*(a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x*(a + b*x^2)^(1/2)*(A + B*x), x)
```

3.4 $\int (A+Bx)\sqrt{a+bx^2}\,dx$

3.4.1 Optimal result

Integrand size = 17, antiderivative size = 67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{1}{2}Ax\sqrt{a+bx^2}+\frac{B(a+bx^2)^{3/2}}{3b}+\frac{aA\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}$$

output
```
1/3*B*(b*x^2+a)^(3/2)/b+1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)
+1/2*A*x*(b*x^2+a)^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int (A+Bx)\sqrt{a+bx^2}\,dx=\frac{\sqrt{a+bx^2}(2aB+3Abx+2bBx^2)}{6b}-\frac{aA\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2\sqrt{b}}$$

input
```
Integrate[(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(2*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b) - (a*A*Log[-(Sqrt[b]
*x) + Sqrt[a + b*x^2]])/(2*Sqrt[b])
```

3.4.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+bx^2}(A+Bx)\,dx$$

$$\Big\downarrow \text{455}$$

$$A\int \sqrt{bx^2+a}dx+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{211}$$

$$A\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{224}$$

$$A\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

$$\Big\downarrow \text{219}$$

$$A\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{B\left(a+bx^2\right)^{3/2}}{3b}$$

input
```
Int[(A + B*x)*Sqrt[a + b*x^2],x]
```

output
```
(B*(a + b*x^2)^(3/2))/(3*b) + A*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[
b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))
```

3.4.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.4.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

method	result	size
default	$A\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)+\frac{B\left(b\,x^2+a\right)^{\frac{3}{2}}}{3b}$	54
risch	$\frac{\left(2bB\,x^2+3Abx+2Ba\right)\sqrt{b\,x^2+a}}{6b}+\frac{Aa\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$	56

input
```
int((B*x+A)*(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+1/3*
B*(b*x^2+a)^(3/2)/b
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{12\,b},\ -\frac{3\,Aa\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,Bbx^2+3\,Abx+2\,Ba)\sqrt{bx^2+a}}{6\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2 + a))/b, -1/6*(3*A*a*sqrt(-b)*arct
an(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*B*b*x^2 + 3*A*b*x + 2*B*a)*sqrt(b*x^2
+ a))/b]
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \begin{cases} \frac{Aa\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2}+\sqrt{a+bx^2}\left(\frac{Ax}{2}+\frac{Ba}{3b}+\frac{Bx^2}{3}\right) & \text{for}\ b\neq 0 \\ \sqrt{a}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b),
 Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + sqrt(a + b*x**2)*(A*x/2 + B
*a/(3*b) + B*x**2/3), Ne(b, 0)), (sqrt(a)*(A*x + B*x**2/2), True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.67

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{1}{2}\sqrt{bx^2+a}Ax + \frac{Aa\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} + \frac{(bx^2+a)^{\frac{3}{2}}B}{3\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*A*x + 1/2*A*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/3*(b*
x^2 + a)^(3/2)*B/b
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.82

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = -\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\sqrt{b}} + \frac{1}{6}\sqrt{bx^2+a}\left((2\,Bx+3\,A)x+\frac{2\,Ba}{b}\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*A*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/6*sqrt(b*x^2 +
 a)*((2*B*x + 3*A)*x + 2*B*a/b)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int (A+Bx)\sqrt{a+bx^2}\,dx = \frac{B\,(b\,x^2+a)^{3/2}}{3\,b} + \frac{A\,x\,\sqrt{b\,x^2+a}}{2} + \frac{A\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

input
```
int((a + b*x^2)^(1/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(3/2))/(3*b) + (A*x*(a + b*x^2)^(1/2))/2 + (A*a*log(b^(1/2)
*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.5 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

3.5.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx=\frac{1}{2}(2A+Bx)\sqrt{a+bx^2}+\frac{aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}-\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+1/2*a*B*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(1/2)+1/2*(B*x+2*A)*(b*x^2+a)^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx=\frac{1}{2}\left((2A+Bx)\sqrt{a+bx^2}+4\sqrt{a}A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

output
```
((2*A + B*x)*Sqrt[a + b*x^2] + 4*Sqrt[a]*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b
*x^2])/Sqrt[a]] - (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/2
```

3.5.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{x}\,dx$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{2}a\int \frac{2A+Bx}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}a\left(2A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}a\left(2A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}a\left(2A\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\left(A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}a\left(\frac{2A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

$$\downarrow \text{221}$$

$$\frac{1}{2}a\left(\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(2A+Bx)$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/x,x]
```

output
```
((2*A + B*x)*Sqrt[a + b*x^2])/2 + (a*((B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^
2]])/Sqrt[b] - (2*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2
```

3.5.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.5.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00

method	result	size
default	$B\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)+A\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)$	79

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))+A*((
b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 341, normalized size of antiderivative = 4.32

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$$
$$= \left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+\cdots}}{4\,b}\right.$$
$$-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b},\ \frac{4\,A\sqrt{-a}b\,\mathrm{arcta}\cdots}{\cdots}$$
$$\left.-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-2\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx+2\,Ab)\sqrt{bx^2+a}}{2\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*A*sq
rt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(B*b*x + 2
*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) - A*sqrt(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) -
(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, 1/4*(4*A*sqrt(-a)*b*arctan(sqrt(-a)/sq
rt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b, -1/2*(B*a*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - 2*A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) -
 (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b]
```

3.5.6 Sympy [A] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = -A\sqrt{a}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}+\frac{A\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+B\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/x,x)
```

output
```
-A*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1
)) + A*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*Piecewise((a*Piecewise((log(2*sq
rt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2)
, True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \frac{1}{2}\sqrt{bx^2+a}Bx + \frac{Ba\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{b}} - A\sqrt{a}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}A$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x + 1/2*B*a*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*sqrt(
a)*arcsinh(a/(sqrt(a*b)*abs(x))) + sqrt(b*x^2 + a)*A
```

3.5.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.5.9 Mupad [B] (verification not implemented)

Time = 5.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx = A\sqrt{b\,x^2+a} - A\sqrt{a}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + \frac{B\,x\sqrt{b\,x^2+a}}{2} + \frac{B\,a\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/x,x)
```

output
```
A*(a + b*x^2)^(1/2) - A*a^(1/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + (B*x*(a
 + b*x^2)^(1/2))/2 + (B*a*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/(2*b^(1/2))
```

3.5. $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x}\,dx$

3.6 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2} \, dx$

3.6.1 Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2} \, dx = -\frac{(A-Bx)\sqrt{a+bx^2}}{x} + A\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \sqrt{a}B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))*a^(1/2)+A*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))*b^(1/2)-(-B*x+A)*(b*x^2+a)^(1/2)/x
```

3.6.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2} \, dx = \frac{(-A+Bx)\sqrt{a+bx^2}}{x} + 2\sqrt{a}B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

output
```
((-A + B*x)*Sqrt[a + b*x^2])/x + 2*Sqrt[a]*B*ArcTanh[(Sqrt[b]*x - Sqrt[a +
 b*x^2])/Sqrt[a]] - A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]
```

3.6.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{x^2} dx$$

$$\Big\downarrow \ 536$$

$$\int \frac{aB+Abx}{x\sqrt{bx^2+a}}dx - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 538$$

$$Ab\int \frac{1}{\sqrt{bx^2+a}}dx + aB\int \frac{1}{x\sqrt{bx^2+a}}dx - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 224$$

$$Ab\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}} + aB\int \frac{1}{x\sqrt{bx^2+a}}dx - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 219$$

$$aB\int \frac{1}{x\sqrt{bx^2+a}}dx + A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 243$$

$$\frac{1}{2}aB\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2 + A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 73$$

$$\frac{aB\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b} + A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}(A-Bx)}{x}$$

$$\Big\downarrow \ 221$$

$$A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{\sqrt{a+bx^2}(A-Bx)}{x} - \sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/x^2,x]
```

output
```
-(((A - B*x)*Sqrt[a + b*x^2])/x) + A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a +
b*x^2]] - Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]
```

3.6.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.6.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

method	result	size
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{x}+A\sqrt{b}\,\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)+\sqrt{b\,x^{2}+a}\,B-B\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)$	78
default	$B\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)+A\left(-\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{a}\right)$	103

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-A*(b*x^2+a)^(1/2)/x+A*b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+(b*x^2+a)^(1/
2)*B-B*a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.44

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx$$

$$= \left[\frac{A\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\right.$$

$$-\frac{2\,A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{a}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,\sqrt{bx^2+a}(Bx-A)}{2\,x},\ \frac{2\,B\sqrt{-a}x\,\mathrm{arcta}}{}$$

$$\left.-\frac{A\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}(Bx-A)}{x}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + B*sqrt
(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*sqrt(b*x^2 +
 a)*(B*x - A))/x, -1/2*(2*A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
- B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt
(b*x^2 + a)*(B*x - A))/x, 1/2*(2*B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 +
 a)) + A*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqr
t(b*x^2 + a)*(B*x - A))/x, -(A*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a
)) - B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - sqrt(b*x^2 + a)*(B*x
- A))/x]
```

3.6.6 Sympy [A] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = -\frac{A\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}} + A\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) - \frac{Abx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}} - B\sqrt{a}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{B\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/x**2,x)
```

output
```
-A*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + A*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) - A
*b*x/(sqrt(a)*sqrt(1 + b*x**2/a)) - B*sqrt(a)*asinh(sqrt(a)/(sqrt(b)*x)) +
 B*a/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) + B*sqrt(b)*x/sqrt(a/(b*x**2) + 1)
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = A\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - B\sqrt{a}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \sqrt{bx^2+a}B - \frac{\sqrt{bx^2+a}A}{x}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
A*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(x)))
 + sqrt(b*x^2 + a)*B - sqrt(b*x^2 + a)*A/x
```

3.6.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = \frac{2\,Ba\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - A\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \sqrt{bx^2+a}B + \frac{2\,Aa\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*B*a*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - A*sqrt(b)
*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + sqrt(b*x^2 + a)*B + 2*A*a*sqrt(b
)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.6.9 Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^2}\,dx = B\sqrt{b\,x^2+a} - \frac{A\sqrt{b\,x^2+a}}{x} - B\sqrt{a}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) - \frac{A\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/x^2,x)
```

output
```
B*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(1/2))/x - B*a^(1/2)*atanh((a + b*x^2
)^(1/2)/a^(1/2)) - (A*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1/2))*(a + b*x^2)^(1/
2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.7 $\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$

3.7.1 Optimal result

Integrand size = 20, antiderivative size = 80

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2\sqrt{a}}$$

output
```
-1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))*b^(1/2)-1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2
```

3.7.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{(A+2Bx)\sqrt{a+bx^2}}{2x^2}+\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

output
```
-1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + (A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a
+ b*x^2])/Sqrt[a]])/Sqrt[a] - Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
]
```

3.7.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx^2}(A+Bx)}{x^3}\,dx$$

$$\Big\downarrow \text{537}$$

$$-\frac{1}{2}b\int -\frac{A+2Bx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}b\int \frac{A+2Bx}{x\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{2}b\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+2B\int \frac{1}{\sqrt{bx^2+a}}dx\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}b\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+2B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}b\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}b\left(\frac{1}{2}A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}b\left(\frac{A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}b\left(\frac{2B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\frac{\sqrt{a+bx^2}(A+2Bx)}{2x^2}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x^2])/x^3,x]
```

output
```
-1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/x^2 + (b*((2*B*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2
```

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.7.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{(2Bx+A)\sqrt{b\,x^2+a}}{2x^2}+\sqrt{b}\,B\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)-\frac{bA\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{2\sqrt{a}}$
default	$B\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{ax}+\frac{2b\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{a}\right)+A\left(-\frac{(b\,x^2+a)^{\frac{3}{2}}}{2a\,x^2}+\frac{b\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right.}{2a}\right.$

input
```
int((B*x+A)*(b*x^2+a)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*B*x+A)*(b*x^2+a)^(1/2)/x^2+b^(1/2)*B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
-1/2*b*A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.7.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 4.71

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx$$
$$= \left[\frac{2\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{\cdots}}{4\,ax^2}\right.$$
$$-\frac{4\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,ax^2}, \frac{A\sqrt{-a}\cdots}{\cdots}$$
$$\left.-\frac{2\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,ax^2}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(2*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(2
*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), -1/4*(4*B*a*sqrt(-b)*x^2*arctan(sq
rt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a
)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a*x^2), 1/2*(A
*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*x^2*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - (2*B*a*x + A*a)*sqrt(b*x^2 + a)
)/(a*x^2), -1/2*(2*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A
*sqrt(-a)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*
x^2 + a))/(a*x^2)]
```

3.7.6 Sympy [A] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x}-\frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2\sqrt{a}}-\frac{B\sqrt{a}}{x\sqrt{1+\frac{bx^2}{a}}}+B\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\frac{Bbx}{\sqrt{a}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(1/2)/x**3,x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*x) - A*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*
sqrt(a)) - B*sqrt(a)/(x*sqrt(1 + b*x**2/a)) + B*sqrt(b)*asinh(sqrt(b)*x/sq
rt(a)) - B*b*x/(sqrt(a)*sqrt(1 + b*x**2/a))
```

3.7.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = B\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\sqrt{a}} + \frac{\sqrt{bx^2+a}Ab}{2a} - \frac{\sqrt{bx^2+a}B}{x} - \frac{(bx^2+a)^{\frac{3}{2}}A}{2ax^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
B*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/s
qrt(a) + 1/2*sqrt(b*x^2 + a)*A*b/a - sqrt(b*x^2 + a)*B/x - 1/2*(b*x^2 + a)
^(3/2)*A/(a*x^2)
```

3.7.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(62) = 124$.

Time = 0.31 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = \frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - B\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(1/2)/x^3,x, algorithm="giac")
```

output
```
A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - B*sqrt(b)*l
og(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A
*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x
^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

3.7.9 Mupad [B] (verification not implemented)

Time = 6.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\sqrt{a+bx^2}}{x^3}\,dx = -\frac{A\sqrt{b\,x^2+a}}{2\,x^2} - \frac{B\sqrt{b\,x^2+a}}{x} - \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{B\sqrt{b}\,\mathrm{asin}\left(\frac{\sqrt{b}\,x\,1\mathrm{i}}{\sqrt{a}}\right)\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}\sqrt{\frac{b\,x^2}{a}+1}}$$

input
```
int(((a + b*x^2)^(1/2)*(A + B*x))/x^3,x)
```

output
```
- (A*(a + b*x^2)^(1/2))/(2*x^2) - (B*(a + b*x^2)^(1/2))/x - (A*b*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/(2*a^(1/2)) - (B*b^(1/2)*asin((b^(1/2)*x*1i)/a^(1
/2))*(a + b*x^2)^(1/2)*1i)/(a^(1/2)*((b*x^2)/a + 1)^(1/2))
```

3.8 $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.8.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{3a^3Bx\sqrt{a+bx^2}}{128b^2}+\frac{a^2Bx\left(a+bx^2\right)^{3/2}}{64b^2}+\frac{Ax^2\left(a+bx^2\right)^{5/2}}{7b}+\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{a(32A+35Bx)\left(a+bx^2\right)^{5/2}}{560b^2}+\frac{3a^4B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{5/2}}$$

output
```
1/64*a^2*B*x*(b*x^2+a)^(3/2)/b^2+1/7*A*x^2*(b*x^2+a)^(5/2)/b+1/8*B*x^3*(b*
x^2+a)^(5/2)/b-1/560*a*(35*B*x+32*A)*(b*x^2+a)^(5/2)/b^2+3/128*a^4*B*arcta
nh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+3/128*a^3*B*x*(b*x^2+a)^(1/2)/b^2
```

3.8.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}\left(80b^3x^6(8A+7Bx)+2a^2bx^2(64A+35Bx)+8ab^2x^4(128A+105Bx)-a^3(256A\right.}{4480b^{5/2}}$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(80*b^3*x^6*(8*A + 7*B*x) + 2*a^2*b*x^2*(64*A + 3
5*B*x) + 8*a*b^2*x^4*(128*A + 105*B*x) - a^3*(256*A + 105*B*x)) - 105*a^4*
B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(4480*b^(5/2))
```

3.8.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {533, 533, 25, 27, 533, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\int x^2(3aB-8Abx)\left(bx^2+a\right)^{3/2}dx}{8b}$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{-\frac{\int -abx(16A+21Bx)\left(bx^2+a\right)^{3/2}dx}{7b}-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 25$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{\int abx(16A+21Bx)\left(bx^2+a\right)^{3/2}dx}{7b}-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\int x(16A+21Bx)\left(bx^2+a\right)^{3/2}dx-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 533$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{\int 3(7aB-32Abx)\left(bx^2+a\right)^{3/2}dx}{6b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \; 27$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{\int (7aB-32Abx)\left(bx^2+a\right)^{3/2}dx}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{7aB\int\left(bx^2+a\right)^{3/2}dx-\frac{32}{5}A\left(a+bx^2\right)^{5/2}}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{7aB\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{32}{5}A\left(a+bx^2\right)^{5/2}}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{7aB\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{32}{5}A\left(a+bx^2\right)^{5/2}}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{224}$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{7aB\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{32}{5}A\left(a+bx^2\right)^{5/2}}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^3\left(a+bx^2\right)^{5/2}}{8b}-\frac{\frac{1}{7}a\left(\frac{7Bx\left(a+bx^2\right)^{5/2}}{2b}-\frac{7aB\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{32}{5}A\left(a+bx^2\right)^{5/2}}{2b}\right)-\frac{8}{7}Ax^2\left(a+bx^2\right)^{5/2}}{8b}$$

input
```
Int[x^3*(A + B*x)*(a + b*x^2)^(3/2),x]
```

3.8. $\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

output
```
(B*x^3*(a + b*x^2)^(5/2))/(8*b) - ((-8*A*x^2*(a + b*x^2)^(5/2))/7 + (a*((7
*B*x*(a + b*x^2)^(5/2))/(2*b) - ((-32*A*(a + b*x^2)^(5/2))/5 + 7*a*B*((x*(
a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)
/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/(2*b)))/7)/(8*b)
```

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.8.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{\left(-560 b^{3} B \,x^{7}-640 x^{6} b^{3} A -840 B a \,b^{2} x^{5}-1024 a A \,b^{2} x^{4}-70 B \,a^{2} b \,x^{3}-128 a^{2} A b \,x^{2}+105 a^{3} B x +256 a^{3} A \right) \sqrt{b \,x^{2}+a}}{4480 b^{2}}+\frac{3 B \,a^{4} \ln \left(x \sqrt{b}\right.}{128 b}$
default	$B \left(\frac{x^{3} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{8 b}-\frac{3 a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6 b}\right)}{8 b}\right)+A \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{7 b}-\right.$

input
```
int(x^3*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4480*(-560*B*b^3*x^7-640*A*b^3*x^6-840*B*a*b^2*x^5-1024*A*a*b^2*x^4-70*
B*a^2*b*x^3-128*A*a^2*b*x^2+105*B*a^3*x+256*A*a^3)/b^2*(b*x^2+a)^(1/2)+3/1
28*B*a^4/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.69

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5}{8960\,b^3}\right.$$

$$-\frac{105\,Ba^4\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(560\,Bb^4x^7+640\,Ab^4x^6+840\,Bab^3x^5+1024\,Aab^3x^4+70\,Ba^2b^2x^3+12}{4480\,b^3}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/8960*(105*B*a^4*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^4 +
 70*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sqrt(
b*x^2 + a))/b^3, -1/4480*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
+ a)) - (560*B*b^4*x^7 + 640*A*b^4*x^6 + 840*B*a*b^3*x^5 + 1024*A*a*b^3*x^
4 + 70*B*a^2*b^2*x^3 + 128*A*a^2*b^2*x^2 - 105*B*a^3*b*x - 256*A*a^3*b)*sq
rt(b*x^2 + a))/b^3]
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.12

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}\frac{3Ba^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b^2}+\sqrt{a+bx^2}\left(-\frac{2Aa^3}{35b^2}+\frac{Aa^2x^2}{35b}+\frac{8Aax^4}{35}+\frac{Abx^6}{7}-\frac{3Ba^3x}{128b^2}\right. \\ a^{\frac{3}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right)\end{cases}$$

input
```
integrate(x**3*(B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((3*B*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b**2) + sqrt(a + b*x*
*2)*(-2*A*a**3/(35*b**2) + A*a**2*x**2/(35*b) + 8*A*a*x**4/35 + A*b*x**6/7
 - 3*B*a**3*x/(128*b**2) + B*a**2*x**3/(64*b) + 3*B*a*x**5/16 + B*b*x**7/8
), Ne(b, 0)), (a**(3/2)*(A*x**4/4 + B*x**5/5), True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{(bx^2+a)^{\frac{5}{2}}Bx^3}{8\,b}+\frac{(bx^2+a)^{\frac{5}{2}}Ax^2}{7\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Bax}{16\,b^2}+\frac{(bx^2+a)^{\frac{3}{2}}Ba^2x}{64\,b^2}+\frac{3\sqrt{bx^2+a}Ba^3x}{128\,b^2}+\frac{3\,Ba^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{5}{2}}}-\frac{2\,(bx^2+a)^{\frac{5}{2}}Aa}{35\,b^2}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(5/2)*B*x^3/b + 1/7*(b*x^2 + a)^(5/2)*A*x^2/b - 1/16*(b*x^
2 + a)^(5/2)*B*a*x/b^2 + 1/64*(b*x^2 + a)^(3/2)*B*a^2*x/b^2 + 3/128*sqrt(b
*x^2 + a)*B*a^3*x/b^2 + 3/128*B*a^4*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/35*
(b*x^2 + a)^(5/2)*A*a/b^2
```

3.8.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = -\frac{3\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{5}{2}}} - \frac{1}{4480}\sqrt{bx^2+a}\left(\frac{256\,Aa^3}{b^2}+\left(\frac{105\,Ba^3}{b^2}-2\left(\frac{64\,Aa^2}{b}+\left(\frac{35\,Ba^2}{b}+4\left(128\,Aa+5\left(21\,Ba+2\left(7\,Bbx+8\,A\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/4480*sqrt(
b*x^2 + a)*(256*A*a^3/b^2 + (105*B*a^3/b^2 - 2*(64*A*a^2/b + (35*B*a^2/b +
 4*(128*A*a + 5*(21*B*a + 2*(7*B*b*x + 8*A*b)*x)*x)*x)*x)*x)*x)
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \int x^3\,\left(b\,x^2+a\right)^{3/2}\,(A+B\,x)\;dx$$

input
```
int(x^3*(a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^3*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.9 $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.9.1 Optimal result

Integrand size = 20, antiderivative size = 127

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^2Ax\sqrt{a+bx^2}}{16b}-\frac{aAx(a+bx^2)^{3/2}}{24b}+\frac{Bx^2(a+bx^2)^{5/2}}{7b}-\frac{(12aB-35Abx)\left(a+bx^2\right)^{5/2}}{210b^2}-\frac{a^3A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
-1/24*a*A*x*(b*x^2+a)^(3/2)/b+1/7*B*x^2*(b*x^2+a)^(5/2)/b-1/210*(-35*A*b*x
+12*B*a)*(b*x^2+a)^(5/2)/b^2-1/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))
/b^(3/2)-1/16*a^2*A*x*(b*x^2+a)^(1/2)/b
```

3.9.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx)\,(a+bx^2)^{3/2}\,dx=\frac{\sqrt{a+bx^2}\left(-96a^3B+40b^3x^5(7A+6Bx)+3a^2bx(35A+16Bx)+2ab^2x^3(245A+192Bx)\right)+}{1680b^2}$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-96*a^3*B + 40*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(35*A +
 16*B*x) + 2*a*b^2*x^3*(245*A + 192*B*x)) + 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b
]*x) + Sqrt[a + b*x^2]])/(1680*b^2)
```

3.9.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {533, 533, 25, 27, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\int x(2aB-7Abx)\left(bx^2+a\right)^{3/2}dx}{7b}$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{-\frac{\int -ab(7A+12Bx)\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{25}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{\int ab(7A+12Bx)\left(bx^2+a\right)^{3/2}dx}{6b}-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\int(7A+12Bx)\left(bx^2+a\right)^{3/2}dx-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\left(7A\int\left(bx^2+a\right)^{3/2}dx+\frac{12B\left(a+bx^2\right)^{5/2}}{5b}\right)-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\left(7A\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{12B\left(a+bx^2\right)^{5/2}}{5b}\right)-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow 211$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\left(7A\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{12B\left(a+bx^2\right)^{5/2}}{5b}\right)-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow 224$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\left(7A\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{12B\left(a+bx^2\right)^{5/2}}{5b}\right)-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

$$\Big\downarrow 219$$

$$\frac{Bx^2\left(a+bx^2\right)^{5/2}}{7b}-\frac{\frac{1}{6}a\left(7A\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{12B\left(a+bx^2\right)^{5/2}}{5b}\right)-\frac{7}{6}Ax\left(a+bx^2\right)^{5/2}}{7b}$$

input
```
Int[x^2*(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(B*x^2*(a + b*x^2)^(5/2))/(7*b) - ((-7*A*x*(a + b*x^2)^(5/2))/6 + (a*((12*
B*(a + b*x^2)^(5/2))/(5*b) + 7*A*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[
a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))
)/6)/(7*b)
```

3.9. $\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.9.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.82

method	result	si
risch	$\frac{\left(240 b^{3} B \,x^{6}+280 A \,b^{3} x^{5}+384 B a \,b^{2} x^{4}+490 a A \,b^{2} x^{3}+48 B \,a^{2} b \,x^{2}+105 a^{2} A b x -96 a^{3} B \right) \sqrt{b \,x^{2}+a}}{1680 b^{2}}-\frac{A \,a^{3} \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{16 b^{\frac{3}{2}}}$	10
default	$B \left(\frac{x^{2} \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{7 b}-\frac{2 a \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{35 b^{2}}\right)+A \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{5}{2}}}{6 b}-\frac{a \left(\frac{x \left(b \,x^{2}+a \right)^{\frac{3}{2}}}{4}+\frac{3 a \left(\frac{x \sqrt{b \,x^{2}+a}}{2}+\frac{a \ln \left(x \sqrt{b}+\sqrt{b \,x^{2}+a}\right)}{2 \sqrt{b}}\right)}{4}\right)}{6 b}\right)$	11

input
```
int(x^2*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1680*(240*B*b^3*x^6+280*A*b^3*x^5+384*B*a*b^2*x^4+490*A*a*b^2*x^3+48*B*a
^2*b*x^2+105*A*a^2*b*x-96*B*a^3)/b^2*(b*x^2+a)^(1/2)-1/16*A*a^3/b^(3/2)*ln
(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.76

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(240\,Bb^3x^6+280\,Ab^3x^5+384\,Bab^2x^4+}{3360\,b^2}\right.$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/3360*(105*A*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(240*B*b^3*x^6 + 280*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 +
48*B*a^2*b*x^2 + 105*A*a^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2, 1/1680*(1
05*A*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (240*B*b^3*x^6 + 28
0*A*b^3*x^5 + 384*B*a*b^2*x^4 + 490*A*a*b^2*x^3 + 48*B*a^2*b*x^2 + 105*A*a
^2*b*x - 96*B*a^3)*sqrt(b*x^2 + a))/b^2]
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.18

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \begin{cases} -\frac{Aa^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b} + \sqrt{a+bx^2}\left(\frac{Aa^2x}{16b}+\frac{7Aax^3}{24}+\frac{Abx^5}{6}-\frac{2Ba^3}{35b^2}+\frac{Ba^2x^2}{35b} - \right. \\ a^{\frac{3}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right) \end{cases}$$

input
```
integrate(x**2*(B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-A*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b) + sqrt(a + b*x**2)*(
A*a**2*x/(16*b) + 7*A*a*x**3/24 + A*b*x**5/6 - 2*B*a**3/(35*b**2) + B*a**2
*x**2/(35*b) + 8*B*a*x**4/35 + B*b*x**6/7), Ne(b, 0)), (a**(3/2)*(A*x**3/3
 + B*x**4/4), True))
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx^2}{7\,b} + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ax}{6\,b} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}Aax}{24\,b} - \frac{\sqrt{bx^2+a}\,Aa^2x}{16\,b} - \frac{Aa^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}} - \frac{2\left(bx^2+a\right)^{\frac{5}{2}}Ba}{35\,b^2}$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(b*x^2 + a)^(5/2)*B*x^2/b + 1/6*(b*x^2 + a)^(5/2)*A*x/b - 1/24*(b*x^2
+ a)^(3/2)*A*a*x/b - 1/16*sqrt(b*x^2 + a)*A*a^2*x/b - 1/16*A*a^3*arcsinh(b
*x/sqrt(a*b))/b^(3/2) - 2/35*(b*x^2 + a)^(5/2)*B*a/b^2
```

3.9.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.81

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}$$
$$-\frac{1}{1680}\sqrt{bx^2+a}\left(\frac{96\,Ba^3}{b^2}-\left(\frac{105\,Aa^2}{b}+2\left(\frac{24\,Ba^2}{b}+(245\,Aa+4\,(48\,Ba+5\,(6\,Bbx+7\,Ab)x)x)x\right)x\right)x\right)$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/1680*sqrt(b*
x^2 + a)*(96*B*a^3/b^2 - (105*A*a^2/b + 2*(24*B*a^2/b + (245*A*a + 4*(48*B
*a + 5*(6*B*b*x + 7*A*b)*x)*x)*x)*x)*x)
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\int x^2\,\left(b\,x^2+a\right)^{3/2}\,(A+B\,x)\;dx$$

input
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^2*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.10 $\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.10.1 Optimal result

Integrand size = 18, antiderivative size = 103

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{a^2Bx\sqrt{a+bx^2}}{16b}-\frac{aBx\left(a+bx^2\right)^{3/2}}{24b}+\frac{(6A+5Bx)\left(a+bx^2\right)^{5/2}}{30b}-\frac{a^3B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{3/2}}$$

output
```
-1/24*a*B*x*(b*x^2+a)^(3/2)/b+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)/b-1/16*a^3*
B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/16*a^2*B*x*(b*x^2+a)^(1/2)/
b
```

3.10.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.98

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}(48a^2A+15a^2Bx+96aAbx^2+70abBx^3+48Ab^2x^4+40b^2Bx^5)}{240b}+\frac{a^3B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{16b^{3/2}}$$

input
```
Integrate[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(48*a^2*A + 15*a^2*B*x + 96*a*A*b*x^2 + 70*a*b*B*x^3 + 48
*A*b^2*x^4 + 40*b^2*B*x^5))/(240*b) + (a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b
*x^2]])/(16*b^(3/2))
```

3.10.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {533, 455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{3/2}(A+Bx)\,dx$$

$$\Big\downarrow \; 533$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{\int (aB-6Abx)\left(bx^2+a\right)^{3/2}dx}{6b}$$

$$\Big\downarrow \; 455$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{aB\int \left(bx^2+a\right)^{3/2}dx-\frac{6}{5}A\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \; 211$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{aB\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{6}{5}A\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \; 211$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{aB\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{6}{5}A\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \; 224$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{aB\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{6}{5}A\left(a+bx^2\right)^{5/2}}{6b}$$

$$\Big\downarrow \; 219$$

$$\frac{Bx\left(a+bx^2\right)^{5/2}}{6b}-\frac{aB\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)-\frac{6}{5}A\left(a+bx^2\right)^{5/2}}{6b}$$

input
```
Int[x*(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(B*x*(a + b*x^2)^(5/2))/(6*b) - ((-6*A*(a + b*x^2)^(5/2))/5 + a*B*((x*(a +
 b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sq
rt[a + b*x^2]])/(2*Sqrt[b])))/4))/(6*b)
```

3.10.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.10.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{(40b^2Bx^5+48Ab^2x^4+70Babx^3+96aAbx^2+15a^2Bx+48a^2A)\sqrt{bx^2+a}}{240b}-\frac{Ba^3\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{16b^{\frac{3}{2}}}$	89
default	$B\left(\frac{x(bx^2+a)^{\frac{5}{2}}}{6b}-\frac{a\left(\frac{x(bx^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{bx^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6b}\right)+\frac{A(bx^2+a)^{\frac{5}{2}}}{5b}$	92

input
```
int(x*(B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/240*(40*B*b^2*x^5+48*A*b^2*x^4+70*B*a*b*x^3+96*A*a*b*x^2+15*B*a^2*x+48*A
*a^2)/b*(b*x^2+a)^(1/2)-1/16*B*a^3/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\left[\frac{15\,Ba^3\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(40\,Bb^3x^5+48\,Ab^3x^4+70\,Bab^2x^3+96\,\ldots}{480\,b^2}\right.$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/480*(15*B*a^3*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 2*(40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x^3 + 96*A*a*b^2*x^2 + 15*B*a
^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2, 1/240*(15*B*a^3*sqrt(-b)*arctan
(sqrt(-b)*x/sqrt(b*x^2 + a)) + (40*B*b^3*x^5 + 48*A*b^3*x^4 + 70*B*a*b^2*x
^3 + 96*A*a*b^2*x^2 + 15*B*a^2*b*x + 48*A*a^2*b)*sqrt(b*x^2 + a))/b^2]
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\begin{cases}-\frac{Ba^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b}+\sqrt{a+bx^2}\left(\frac{Aa^2}{5b}+\frac{2Aax^2}{5}+\frac{Abx^4}{5}+\frac{Ba^2x}{16b}+\frac{7Bax^3}{24}+\right.\\ a^{\frac{3}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right)\end{cases}$$

input
```
integrate(x*(B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((-B*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(16*b) + sqrt(a + b*x**2)*(
A*a**2/(5*b) + 2*A*a*x**2/5 + A*b*x**4/5 + B*a**2*x/(16*b) + 7*B*a*x**3/24
 + B*b*x**5/6), Ne(b, 0)), (a**(3/2)*(A*x**2/2 + B*x**3/3), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{5}{2}}Bx}{6\,b}-\frac{\left(bx^2+a\right)^{\frac{3}{2}}Bax}{24\,b}-\frac{\sqrt{bx^2+a}Ba^2x}{16\,b}-\frac{Ba^3\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{3}{2}}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{5\,b}$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*B*x/b - 1/24*(b*x^2 + a)^(3/2)*B*a*x/b - 1/16*sqrt(b
*x^2 + a)*B*a^2*x/b - 1/16*B*a^3*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 1/5*(b*x
^2 + a)^(5/2)*A/b
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{Ba^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{3}{2}}}+\frac{1}{240}\sqrt{bx^2+a}\left(\frac{48\,Aa^2}{b}+\left(\frac{15\,Ba^2}{b}+2\,(48\,Aa+(35\,Ba+4\,(5\,Bbx+6\,Ab)x)x)x\right)x\right)$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/16*B*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/240*sqrt(b*x
^2 + a)*(48*A*a^2/b + (15*B*a^2/b + 2*(48*A*a + (35*B*a + 4*(5*B*b*x + 6*A
*b)*x)*x)*x)*x)
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\int x\,\left(b\,x^2+a\right)^{3/2}\,(A+B\,x)\,dx$$

input
```
int(x*(a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x*(a + b*x^2)^(3/2)*(A + B*x), x)
```

3.11 $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.11.1 Optimal result

Integrand size = 17, antiderivative size = 87

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{3}{8}aAx\sqrt{a+bx^2}+\frac{1}{4}Ax\left(a+bx^2\right)^{3/2}+\frac{B\left(a+bx^2\right)^{5/2}}{5b}+\frac{3a^2A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}}$$

output
```
1/4*A*x*(b*x^2+a)^(3/2)+1/5*B*(b*x^2+a)^(5/2)/b+3/8*a^2*A*arctanh(x*b^(1/2
)/(b*x^2+a)^(1/2))/b^(1/2)+3/8*a*A*x*(b*x^2+a)^(1/2)
```

3.11.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{\sqrt{a+bx^2}\left(8a^2B+2b^2x^3(5A+4Bx)+abx(25A+16Bx)\right)-15a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx\right.}{40b}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*B + 2*b^2*x^3*(5*A + 4*B*x) + a*b*x*(25*A + 16*B*x
)) - 15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(40*b)
```

3.11.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {455, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{3/2} (A+Bx) \, dx$$

$$\Big\downarrow \text{455}$$

$$A \int \left(bx^2+a\right)^{3/2}dx+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A \left(\frac{3}{4} a \int \sqrt{bx^2+a}dx+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{211}$$

$$A \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2} x \sqrt{a+bx^2}\right)+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{224}$$

$$A \left(\frac{3}{4} a \left(\frac{1}{2} a \int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2} x \sqrt{a+bx^2}\right)+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$A \left(\frac{3}{4} a \left(\frac{a \text{arctanh}\left(\frac{\sqrt{b} x}{\sqrt{a+bx^2}}\right)}{2 \sqrt{b}}+\frac{1}{2} x \sqrt{a+bx^2}\right)+\frac{1}{4} x \left(a+bx^2\right)^{3/2}\right)+\frac{B\left(a+bx^2\right)^{5/2}}{5b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(3/2),x]
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + A*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4)
```

3.11.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.11.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)+\frac{B\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5b}$	70
risch	$\frac{\left(8b^{2}B\,x^{4}+10A\,b^{2}x^{3}+16Bab\,x^{2}+25aAbx+8a^{2}B\right)\sqrt{b\,x^{2}+a}}{40b}+\frac{3a^{2}A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8\sqrt{b}}$	80

input
```
int((B*x+A)*(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))))+1/5*B*(b*x^2+a)^(5/2)/b
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.02

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \left[\frac{15\,Aa^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aa\right.}{80\,b}$$

$$\left.-\frac{15\,Aa^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,Bb^2x^4+10\,Ab^2x^3+16\,Babx^2+25\,Aabx+8\,Ba^2)\sqrt{bx^2+a}}{40\,b}\right]$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/80*(15*A*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
2*(8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt(
b*x^2 + a))/b, -1/40*(15*A*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a))
 - (8*B*b^2*x^4 + 10*A*b^2*x^3 + 16*B*a*b*x^2 + 25*A*a*b*x + 8*B*a^2)*sqrt
(b*x^2 + a))/b]
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx = \begin{cases} \frac{3Aa^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for}\ a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8} + \sqrt{a+bx^2}\cdot\left(\frac{5Aax}{8}+\frac{Abx^3}{4}+\frac{Ba^2}{5b}+\frac{2Bax^2}{5}+\frac{Bbx^4}{5}\right) \\ a^{\frac{3}{2}}\left(Ax+\frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2),x)
```

output
```
Piecewise((3*A*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/8 + sqrt(a + b*x**2)*(5*A*
a*x/8 + A*b*x**3/4 + B*a**2/(5*b) + 2*B*a*x**2/5 + B*b*x**4/5), Ne(b, 0)),
 (a**(3/2)*(A*x + B*x**2/2), True))
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.70

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Ax+\frac{3}{8}\sqrt{bx^2+a}Aax+\frac{3\,Aa^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{b}}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{5\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*A*x + 3/8*sqrt(b*x^2 + a)*A*a*x + 3/8*A*a^2*arcsinh(
b*x/sqrt(a*b))/sqrt(b) + 1/5*(b*x^2 + a)^(5/2)*B/b
```

3.11.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.87

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=-\frac{3\,Aa^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,\sqrt{b}}+\frac{1}{40}\sqrt{bx^2+a}\left(\frac{8\,Ba^2}{b}+(25\,Aa+2\,(8\,Ba+(4\,Bbx+5\,Ab)x)x)x\right)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-3/8*A*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/40*sqrt(b*x^
2 + a)*(8*B*a^2/b + (25*A*a + 2*(8*B*a + (4*B*b*x + 5*A*b)*x)*x)*x)
```

3.11.9 Mupad [B] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.62

$$\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx=\frac{B\,\left(b\,x^2+a\right)^{5/2}}{5\,b}+\frac{A\,x\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int((a + b*x^2)^(3/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(5/2))/(5*b) + (A*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.11. $\int (A+Bx)\left(a+bx^2\right)^{3/2}\,dx$

3.12 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx$

3.12.1 Optimal result

Integrand size = 20, antiderivative size = 106

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx = \frac{1}{8}a(8A+3Bx)\sqrt{a+bx^2} + \frac{1}{12}(4A+3Bx)\left(a+bx^2\right)^{3/2} + \frac{3a^2 B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8\sqrt{b}} - a^{3/2}A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/12*(3*B*x+4*A)*(b*x^2+a)^(3/2)-a^(3/2)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)+3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+1/8*a*(3*B*x+8*A)*(
b*x^2+a)^(1/2)
```

3.12.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x} \, dx = 2a^{3/2}A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{24}\left(\sqrt{a+bx^2}\left(32aA+15aBx+8Abx^2+6bBx^3\right) - \frac{9a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

output
```
2*a^(3/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x
^2]*(32*a*A + 15*a*B*x + 8*A*b*x^2 + 6*b*B*x^3) - (9*a^2*B*Log[-(Sqrt[b]*x
) + Sqrt[a + b*x^2]])/Sqrt[b])/24
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {535, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{4}a\int \frac{(4A+3Bx)\sqrt{bx^2+a}}{x}dx+\frac{1}{12}(a+bx^2)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\int \frac{8A+3Bx}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}(a+bx^2)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(8A\int \frac{1}{x\sqrt{bx^2+a}}dx+3B\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}(a+bx^2)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(8A\int \frac{1}{x\sqrt{bx^2+a}}dx+3B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}(a+bx^2)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(8A\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}(a+bx^2)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(4A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{8A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{8A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(8A+3Bx)\right)+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4A+3Bx)$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/x,x]
```

output
```
((4*A + 3*B*x)*(a + b*x^2)^(3/2))/12 + (a*(((8*A + 3*B*x)*Sqrt[a + b*x^2])
/2 + (a*((3*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (8*A*ArcTanh
[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]))/2))/4
```

3.12.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.12.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

method	result
default	$B\left(\frac{x(b\,x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)+A\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b}}{x}\right.\right.\right.$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b
^(1/2)+(b*x^2+a)^(1/2))))+A*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2
)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.14

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+24\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}}{x^2}\right.}{48\,b}\right.$$

$$-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-12\,Aa^{\frac{3}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,A\right.}{24\,b}$$

$$-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-24\,A\sqrt{-a}ab\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2x^3+8\,Ab^2x^2+15\,Babx+32\,Aab\right)\sqrt{}}{24\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
4*A*a^(3/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(6*B
*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, -1/24*
(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 12*A*a^(3/2)*b*log(
-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^3 + 8*A*b^2*x
^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b, 1/48*(48*A*sqrt(-a)*a*b*ar
ctan(sqrt(-a)/sqrt(b*x^2 + a)) + 9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*
a*b)*sqrt(b*x^2 + a))/b, -1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*
x^2 + a)) - 24*A*sqrt(-a)*a*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*
x^3 + 8*A*b^2*x^2 + 15*B*a*b*x + 32*A*a*b)*sqrt(b*x^2 + a))/b]
```

3.12.6 Sympy [A] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.58

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = -Aa^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Aa^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Aa\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Ab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) + Ba\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/x,x)
```

output
```
-A*a**(3/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**2/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + A*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + A*b*Piecewise((a*sqrt(a + b*
x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
+ B*a*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt
(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, N
e(b, 0)), (sqrt(a)*x, True)) + B*b*Piecewise((-a**2*Piecewise((log(2*sqrt(
b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), T
rue))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b,
0)), (sqrt(a)*x**3/3, True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{1}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bx + \frac{3}{8}\sqrt{bx^2+a}Bax + \frac{3\,Ba^2\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{b}} - Aa^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}A + \sqrt{bx^2+a}Aa$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^(3/2)*B*x + 3/8*sqrt(b*x^2 + a)*B*a*x + 3/8*B*a^2*arcsinh(
b*x/sqrt(a*b))/sqrt(b) - A*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*
x^2 + a)^(3/2)*A + sqrt(b*x^2 + a)*A*a
```

3.12.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.12.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx = \frac{A\left(b\,x^2+a\right)^{3/2}}{3} - A\,a^{3/2}\operatorname{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + A\,a\,\sqrt{b\,x^2+a} + \frac{B\,x\left(b\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/x,x)
```

output
```
(A*(a + b*x^2)^(3/2))/3 - A*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + A*a
*(a + b*x^2)^(1/2) + (B*x*(a + b*x^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(
b*x^2)/a))/((b*x^2)/a + 1)^(3/2)
```

3.12. $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x}\,dx$

3.13 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^2}\,dx$

3.13.1 Optimal result

Integrand size = 20, antiderivative size = 108

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{2}(2aB+3Abx)\sqrt{a+bx^2}-\frac{(3A-Bx)\left(a+bx^2\right)^{3/2}}{3x} + \frac{3}{2}aA\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
-1/3*(-B*x+3*A)*(b*x^2+a)^(3/2)/x-a^(3/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2
))+3/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+1/2*(3*A*b*x+2*B*a)*
(b*x^2+a)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{a+bx^2}\left(bx^2(3A+2Bx)+a(-6A+8Bx)\right)}{6x} + 2a^{3/2}B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{3}{2}aA\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(b*x^2*(3*A + 2*B*x) + a*(-6*A + 8*B*x)))/(6*x) + 2*a^(3/
2)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - (3*a*A*Sqrt[b]*Log[-
(Sqrt[b]*x) + Sqrt[a + b*x^2]])/2
```

3.13.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {536, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{3/2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{ 536}$$

$$\int \frac{(aB+3Abx)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}$$

$$\Big\downarrow \text{ 535}$$

$$\frac{1}{2}a\int \frac{2aB+3Abx}{x\sqrt{bx^2+a}}dx-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{ 538}$$

$$\frac{1}{2}a\left(3Ab\int \frac{1}{\sqrt{bx^2+a}}dx+2aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}a\left(3Ab\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+2aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}a\left(2aB\int \frac{1}{x\sqrt{bx^2+a}}dx+3A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{\left(a+bx^2\right)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}a\left(aB\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+3A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{(a+bx^2)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}a\left(\frac{2aB\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+3A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)-\frac{(a+bx^2)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}a\left(3A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-2\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)-\frac{(a+bx^2)^{3/2}(3A-Bx)}{3x}+\frac{1}{2}\sqrt{a+bx^2}(2aB+3Abx)$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/x^2,x]
```

output
```
((2*a*B + 3*A*b*x)*Sqrt[a + b*x^2])/2 - ((3*A - B*x)*(a + b*x^2)^(3/2))/(3
*x) + (a*(3*A*Sqrt[b]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - 2*Sqrt[a]*B*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/2
```

3.13.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.13.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05

method	result
risch	$-\frac{aA\sqrt{b\,x^{2}+a}}{x}+\frac{Bb\,x^{2}\sqrt{b\,x^{2}+a}}{3}+\frac{4aB\sqrt{b\,x^{2}+a}}{3}+\frac{bAx\sqrt{b\,x^{2}+a}}{2}+\frac{3a\sqrt{b}\,A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2}-a^{\frac{3}{2}}B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x}}{x}\right.$
default	$B\left(\frac{(b\,x^{2}+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^{2}+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)\right)\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(x\sqrt{b}\right.}{}\right.}{}\right.$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*A*(b*x^2+a)^(1/2)/x+1/3*B*b*x^2*(b*x^2+a)^(1/2)+4/3*a*B*(b*x^2+a)^(1/2)
+1/2*b*A*x*(b*x^2+a)^(1/2)+3/2*a*b^(1/2)*A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-a
^(3/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 411, normalized size of antiderivative = 3.81

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \left[\frac{9\,Aa\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+6\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{}}{x^2}\right.}{12\,x}\right.$$

$$-\frac{9\,Aa\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,Ba^{\frac{3}{2}}x\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa\right)\sqrt{bx}}{6\,x}$$

$$-\frac{9\,Aa\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-6\,B\sqrt{-a}ax\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(2\,Bbx^3+3\,Abx^2+8\,Bax-6\,Aa\right)\sqrt{bx^2+a}}{6\,x}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[1/12*(9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 6
*B*a^(3/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*
b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*A*a*sqrt(
-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*B*a^(3/2)*x*log(-(b*x^2 - 2*s
qrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*
A*a)*sqrt(b*x^2 + a))/x, 1/12*(12*B*sqrt(-a)*a*x*arctan(sqrt(-a)/sqrt(b*x^
2 + a)) + 9*A*a*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sqrt(b*x^2 + a))/x, -1/6*(9*
A*a*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 6*B*sqrt(-a)*a*x*arcta
n(sqrt(-a)/sqrt(b*x^2 + a)) - (2*B*b*x^3 + 3*A*b*x^2 + 8*B*a*x - 6*A*a)*sq
rt(b*x^2 + a))/x]
```

3.13.6 Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)\,(a+bx^2)^{3/2}}{x^2}\,dx = -\frac{Aa^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{A\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Ab\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right) - Ba^{\frac{3}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba^2}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}} + Bb\left(\begin{cases} \frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0 \\ \frac{\sqrt{a}x^2}{2} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/x**2,x)
```

output
```
-A*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - A*sqrt(a)*b*x/sqrt(1 + b*x**2/a) + A*
a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + A*b*Piecewise((a*Piecewise((log(2*sqr
t(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2),
 True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) - B*a**(3/
2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**2/(sqrt(b)*x*sqrt(a/(b*x**2) + 1)) +
B*a*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + B*b*Piecewise((a*sqrt(a + b*x**2)/(3*
b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\,(a+bx^2)^{3/2}}{x^2}\,dx = \frac{3}{2}\sqrt{bx^2+a}Abx + \frac{3}{2}Aa\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}B + \sqrt{bx^2+a}Ba - \frac{(bx^2+a)^{\frac{3}{2}}A}{x}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*A*b*x + 3/2*A*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(
3/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1/3*(b*x^2 + a)^(3/2)*B + sqrt(b*x^2
+ a)*B*a - (b*x^2 + a)^(3/2)*A/x
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{2\,Ba^2\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3}{2}\,Aa\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^2\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{6}\sqrt{bx^2+a}(8\,Ba+(2\,Bbx+3\,Ab)x)$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2*B*a^2*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*A*a
*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^2*sqrt(b)/((sqrt(b
)*x - sqrt(b*x^2 + a))^2 - a) + 1/6*sqrt(b*x^2 + a)*(8*B*a + (2*B*b*x + 3*
A*b)*x)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 6.73 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx = \frac{B\left(b\,x^2+a\right)^{3/2}}{3} - B\,a^{3/2}\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right) + B\,a\,\sqrt{b\,x^2+a} - \frac{A\left(b\,x^2+a\right)^{3/2}{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/x^2,x)
```

output

```
(B*(a + b*x^2)^(3/2))/3 - B*a^(3/2)*atanh((a + b*x^2)^(1/2)/a^(1/2)) + B*a
*(a + b*x^2)^(1/2) - (A*(a + b*x^2)^(3/2)*hypergeom([-3/2, -1/2], 1/2, -(b
*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.13. $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^2}\,dx$

3.14 $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^3}\,dx$

3.14.1 Optimal result

Integrand size = 20, antiderivative size = 111

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{3(aB-Abx)\sqrt{a+bx^2}}{2x} - \frac{(A-Bx)\left(a+bx^2\right)^{3/2}}{2x^2} + \frac{3}{2}a\sqrt{b}B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{3}{2}\sqrt{a}Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
-1/2*(-B*x+A)*(b*x^2+a)^(3/2)/x^2-3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))
*a^(1/2)+3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))*b^(1/2)-3/2*(-A*b*x+B*
a)*(b*x^2+a)^(1/2)/x
```

3.14.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \frac{1}{2}\left(\frac{\sqrt{a+bx^2}(bx^2(2A+Bx)-a(A+2Bx))}{x^2} + 6\sqrt{a}Ab\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) - 3a\sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)\right)$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

output
```
((Sqrt[a + b*x^2]*(b*x^2*(2*A + B*x) - a*(A + 2*B*x)))/x^2 + 6*Sqrt[a]*A*b
*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] - 3*a*Sqrt[b]*B*Log[-(Sqrt
[b]*x) + Sqrt[a + b*x^2]])/2
```

3.14.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {537, 25, 535, 27, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{3/2}(A+Bx)}{x^3}dx$$

$$\Big\downarrow \; 537$$

$$-\frac{3}{2}b\int -\frac{(A+2Bx)\sqrt{bx^2+a}}{x}dx-\frac{(a+bx^2)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 25$$

$$\frac{3}{2}b\int \frac{(A+2Bx)\sqrt{bx^2+a}}{x}dx-\frac{(a+bx^2)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 535$$

$$\frac{3}{2}b\left(\frac{1}{2}a\int \frac{2(A+Bx)}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(A+Bx)\right)-\frac{(a+bx^2)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}b\left(a\int \frac{A+Bx}{x\sqrt{bx^2+a}}dx+\sqrt{a+bx^2}(A+Bx)\right)-\frac{(a+bx^2)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 538$$

$$\frac{3}{2}b\left(a\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{(a+bx^2)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 224$$

$$\frac{3}{2}b\left(a\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{\left(a+bx^2\right)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 219$$

$$\frac{3}{2}b\left(a\left(A\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{\left(a+bx^2\right)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 243$$

$$\frac{3}{2}b\left(a\left(\frac{1}{2}A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{\left(a+bx^2\right)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 73$$

$$\frac{3}{2}b\left(a\left(\frac{A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{\left(a+bx^2\right)^{3/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \; 221$$

$$\frac{3}{2}b\left(a\left(\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\sqrt{a+bx^2}(A+Bx)\right)-\frac{\left(a+bx^2\right)^{3/2}(A+2Bx)}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(3/2))/x^3,x]
```

output
```
-1/2*((A + 2*B*x)*(a + b*x^2)^(3/2))/x^2 + (3*b*((A + B*x)*Sqrt[a + b*x^2]
 + a*((B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a
 + b*x^2]/Sqrt[a]])/Sqrt[a])))/2
```

3.14. $\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx$

3.14.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.14.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.92

method	result
risch	$-\frac{a\sqrt{b x^2+a}\,(2Bx+A)}{2x^2}+\frac{3\sqrt{b}\,Ba\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{2}+\frac{bBx\sqrt{b x^2+a}}{2}+bA\sqrt{b x^2+a}-\frac{3bA\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b x^2+a}}{x}\right)}{2}$
default	$B\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{ax}+\frac{4b\left(\frac{x(b x^2+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{a}\right)+A\left(-\frac{(b x^2+a)^{\frac{5}{2}}}{2a x^2}+\frac{3b\left(\frac{(b x^2+a)^{\frac{3}{2}}}{3}+a\left(\cdots\right.\right.}{\cdots}\right.$

input
```
int((B*x+A)*(b*x^2+a)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(b*x^2+a)^(1/2)*(2*B*x+A)/x^2+3/2*b^(1/2)*B*a*ln(x*b^(1/2)+(b*x^2+a
)^(1/2))+1/2*b*B*x*(b*x^2+a)^(1/2)+b*A*(b*x^2+a)^(1/2)-3/2*b*A*a^(1/2)*ln(
(2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 425, normalized size of antiderivative = 3.83

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \left[\frac{3\,Ba\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+3\,A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2}}{x}\right.}{4\,x^2}\right.$$

$$-\frac{6\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{a}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(Bbx^3+2\,Abx^2-2\,Bax-Aa)\sqrt{}}{4\,x^2}$$

$$-\frac{3\,Ba\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-3\,A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(Bbx^3+2\,Abx^2-2\,Bax-Aa)\sqrt{bx^2+a}}{2\,x^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
3*A*sqrt(a)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2, -1/4*(6*B*a*sq
rt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(a)*b*x^2*log(-(b*
x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(B*b*x^3 + 2*A*b*x^2 - 2*B
*a*x - A*a)*sqrt(b*x^2 + a))/x^2, 1/4*(6*A*sqrt(-a)*b*x^2*arctan(sqrt(-a)/
sqrt(b*x^2 + a)) + 3*B*a*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt
(b)*x - a) + 2*(B*b*x^3 + 2*A*b*x^2 - 2*B*a*x - A*a)*sqrt(b*x^2 + a))/x^2,
 -1/2*(3*B*a*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 3*A*sqrt(-a
)*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (B*b*x^3 + 2*A*b*x^2 - 2*B*a*x
- A*a)*sqrt(b*x^2 + a))/x^2]
```

3.14.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.02

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = -\frac{3A\sqrt{a}b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{Aa\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} - \frac{Ba^{\frac{3}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{B\sqrt{a}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba\sqrt{b}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + Bb\left(\begin{cases} \frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0 \\ \sqrt{a}x & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(3/2)/x**3,x)
```

output
```
-3*A*sqrt(a)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a*sqrt(b)*sqrt(a/(b*x**2)
+ 1)/(2*x) + A*a*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + A*b**(3/2)*x/sqrt(a/(b
*x**2) + 1) - B*a**(3/2)/(x*sqrt(1 + b*x**2/a)) - B*sqrt(a)*b*x/sqrt(1 + b
*x**2/a) + B*a*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + B*b*Piecewise((a*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True
))
```

3.14.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \frac{3}{2}\sqrt{bx^2+a}Bbx + \frac{3}{2}Ba\sqrt{b}\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{3}{2}A\sqrt{a}b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{3}{2}\sqrt{bx^2+a}Ab + \frac{\left(bx^2+a\right)^{\frac{3}{2}}Ab}{2a} - \frac{\left(bx^2+a\right)^{\frac{3}{2}}B}{x} - \frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{2ax^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/2*sqrt(b*x^2 + a)*B*b*x + 3/2*B*a*sqrt(b)*arcsinh(b*x/sqrt(a*b)) - 3/2*A
*sqrt(a)*b*arcsinh(a/(sqrt(a*b)*abs(x))) + 3/2*sqrt(b*x^2 + a)*A*b + 1/2*(
b*x^2 + a)^(3/2)*A*b/a - (b*x^2 + a)^(3/2)*B/x - 1/2*(b*x^2 + a)^(5/2)*A/(
a*x^2)
```

3.14.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(88) = 176$.

Time = 0.31 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = \frac{3\,Aab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{3}{2}\,Ba\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{1}{2}\,(Bbx+2\,Ab)\sqrt{bx^2+a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^2b-2\,Ba^3\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(3/2)/x^3,x, algorithm="giac")
```

output
```
3*A*a*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 3/2*B*a
*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/2*(B*b*x + 2*A*b)*sqrt
(b*x^2 + a) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*a*b + 2*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*B*a^2*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^2*b - 2*B
*a^3*sqrt(b))/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2
```

3.14.9 Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{3/2}}{x^3}\,dx = A\,b\,\sqrt{b\,x^2+a} - \frac{A\,a\,\sqrt{b\,x^2+a}}{2\,x^2} - \frac{3\,A\,\sqrt{a}\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2} - \frac{B\,\left(b\,x^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,\left(\frac{b\,x^2}{a}+1\right)^{3/2}}$$

input
```
int(((a + b*x^2)^(3/2)*(A + B*x))/x^3,x)
```

output
```
A*b*(a + b*x^2)^(1/2) - (A*a*(a + b*x^2)^(1/2))/(2*x^2) - (3*A*a^(1/2)*b*a
tanh((a + b*x^2)^(1/2)/a^(1/2)))/2 - (B*(a + b*x^2)^(3/2)*hypergeom([-3/2,
 -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(3/2))
```

3.14. $\int \frac{(A+Bx)(a+bx^2)^{3/2}}{x^3}\,dx$

3.15 $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

3.15.1 Optimal result

Integrand size = 20, antiderivative size = 173

$$\begin{aligned}\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=&\frac{3a^4Bx\sqrt{a+bx^2}}{256b^2}+\frac{a^3Bx(a+bx^2)^{3/2}}{128b^2}\\&+\frac{a^2Bx(a+bx^2)^{5/2}}{160b^2}+\frac{Ax^2(a+bx^2)^{7/2}}{9b}+\frac{Bx^3(a+bx^2)^{7/2}}{10b}\\&-\frac{a(160A+189Bx)\left(a+bx^2\right)^{7/2}}{5040b^2}+\frac{3a^5B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{5/2}}\end{aligned}$$

output
```
1/128*a^3*B*x*(b*x^2+a)^(3/2)/b^2+1/160*a^2*B*x*(b*x^2+a)^(5/2)/b^2+1/9*A*
x^2*(b*x^2+a)^(7/2)/b+1/10*B*x^3*(b*x^2+a)^(7/2)/b-1/5040*a*(189*B*x+160*A
)*(b*x^2+a)^(7/2)/b^2+3/256*a^5*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/
2)+3/256*a^4*B*x*(b*x^2+a)^(1/2)/b^2
```

3.15.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.79

$$\int x^3(A+Bx)\,(a+bx^2)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}(896b^4x^8(10A+9Bx)+10a^3bx^2(128A+63Bx)-5a^4(512A+189Bx)+24a^2b}{80640b^5}$$

input
```
Integrate[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(896*b^4*x^8*(10*A + 9*B*x) + 10*a^3*b*x^2*(128*A
 + 63*B*x) - 5*a^4*(512*A + 189*B*x) + 24*a^2*b^2*x^4*(800*A + 651*B*x) +
16*a*b^3*x^6*(1520*A + 1323*B*x)) - 945*a^5*B*Log[-(Sqrt[b]*x) + Sqrt[a +
b*x^2]])/(80640*b^(5/2))
```

3.15.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {533, 533, 25, 27, 533, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b x^2\right)^{5/2} (A+B x) \, dx$$

$$\Big\downarrow \ 533$$

$$\frac{B x^3 \left(a+b x^2\right)^{7/2}}{10 b}-\frac{\int x^2 (3 a B-10 A b x) \left(b x^2+a\right)^{5/2}dx}{10 b}$$

$$\Big\downarrow \ 533$$

$$\frac{B x^3 \left(a+b x^2\right)^{7/2}}{10 b}-\frac{-\frac{\int -a b x (20 A+27 B x) \left(b x^2+a\right)^{5/2}dx}{9 b}-\frac{10}{9} A x^2 \left(a+b x^2\right)^{7/2}}{10 b}$$

$$\Big\downarrow \ 25$$

$$\frac{B x^3 \left(a+b x^2\right)^{7/2}}{10 b}-\frac{\frac{\int a b x (20 A+27 B x) \left(b x^2+a\right)^{5/2}dx}{9 b}-\frac{10}{9} A x^2 \left(a+b x^2\right)^{7/2}}{10 b}$$

$$\Big\downarrow \ 27$$

$$\frac{B x^3 \left(a+b x^2\right)^{7/2}}{10 b}-\frac{\frac{1}{9} a \int x (20 A+27 B x) \left(b x^2+a\right)^{5/2}dx-\frac{10}{9} A x^2 \left(a+b x^2\right)^{7/2}}{10 b}$$

$$\Big\downarrow \ 533$$

$$\frac{B x^3 \left(a+b x^2\right)^{7/2}}{10 b}-\frac{\frac{1}{9} a \left(\frac{27 B x \left(a+b x^2\right)^{7/2}}{8 b}-\frac{\int (27 a B-160 A b x) \left(b x^2+a\right)^{5/2}dx}{8 b}\right)-\frac{10}{9} A x^2 \left(a+b x^2\right)^{7/2}}{10 b}$$

$$\Big\downarrow \ 455$$

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\int\left(bx^2+a\right)^{5/2}dx-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)-\frac{10}{9}Ax^2\left(a+bx^2\right)^{7/2}}{10b}$$

$\big\downarrow$ 211

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)-\frac{10}{9}Ax^2\left(a+bx^2\right)^{7/2}}{10b}$$

$\big\downarrow$ 211

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)-\frac{10}{9}Ax^2\left(a+bx^2\right)^{7/2}}{10b}$$

$\big\downarrow$ 211

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)-\frac{10}{9}Ax}{10b}$$

$\big\downarrow$ 224

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)}{10b}$$

$\big\downarrow$ 219

$$\frac{Bx^3\left(a+bx^2\right)^{7/2}}{10b}-\frac{\frac{1}{9}a\left(\frac{27Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{27aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{160}{7}A\left(a+bx^2\right)^{7/2}}{8b}\right)-}{10b}$$

3.15. $\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

input
```
Int[x^3*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(B*x^3*(a + b*x^2)^(7/2))/(10*b) - ((-10*A*x^2*(a + b*x^2)^(7/2))/9 + (a*(
(27*B*x*(a + b*x^2)^(7/2))/(8*b) - ((-160*A*(a + b*x^2)^(7/2))/7 + 27*a*B*
((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a
 + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/
6))/(8*b)))/9)/(10*b)
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.15.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\left(-8064 B\, b^{4} x^{9}-8960 A\, b^{4} x^{8}-21168 B a\, b^{3} x^{7}-24320 A a\, b^{3} x^{6}-15624 B\, a^{2} b^{2} x^{5}-19200 A\, a^{2} b^{2} x^{4}-630 B\, a^{3} b\, x^{3}-1280 A\, a^{3} b\, x^{2}+945 B}{80640 b^{2}}$
default	$B\left(\frac{x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{10 b}-\frac{3 a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{7}{2}}}{8 b}-\frac{a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5 a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3 a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8 b}\right)}{10 b}\right)+A\Bigg($

input
```
int(x^3*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/80640*(-8064*B*b^4*x^9-8960*A*b^4*x^8-21168*B*a*b^3*x^7-24320*A*a*b^3*x
^6-15624*B*a^2*b^2*x^5-19200*A*a^2*b^2*x^4-630*B*a^3*b*x^3-1280*A*a^3*b*x^
2+945*B*a^4*x+2560*A*a^4)/b^2*(b*x^2+a)^(1/2)+3/256*a^5*B/b^(5/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.75

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{945\,Ba^5\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab\right.$$

$$-\frac{945\,Ba^5\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8064\,Bb^5x^9+8960\,Ab^5x^8+21168\,Bab^4x^7+24320\,Aab^4x^6+15624\,Ba^2}{80640\,b^3}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/161280*(945*B*a^5*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
a) + 2*(8064*B*b^5*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^
4*x^6 + 15624*B*a^2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 12
80*A*a^3*b^2*x^2 - 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3, -1/
80640*(945*B*a^5*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (8064*B*b^5
*x^9 + 8960*A*b^5*x^8 + 21168*B*a*b^4*x^7 + 24320*A*a*b^4*x^6 + 15624*B*a^
2*b^3*x^5 + 19200*A*a^2*b^3*x^4 + 630*B*a^3*b^2*x^3 + 1280*A*a^3*b^2*x^2 -
 945*B*a^4*b*x - 2560*A*a^4*b)*sqrt(b*x^2 + a))/b^3]
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.15

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}\frac{3Ba^5\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{256b^2}+\sqrt{a+bx^2}\left(-\frac{2Aa^4}{63b^2}+\frac{Aa^3x^2}{63b}+\frac{5Aa^2x^4}{21}+\frac{19Aabx^6}{63}+A\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right)\end{cases}$$

input
```
integrate(x**3*(B*x+A)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((3*B*a**5*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(256*b**2) + sqrt(a + b*x*
*2)*(-2*A*a**4/(63*b**2) + A*a**3*x**2/(63*b) + 5*A*a**2*x**4/21 + 19*A*a*
b*x**6/63 + A*b**2*x**8/9 - 3*B*a**4*x/(256*b**2) + B*a**3*x**3/(128*b) +
31*B*a**2*x**5/160 + 21*B*a*b*x**7/80 + B*b**2*x**9/10), Ne(b, 0)), (a**(5
/2)*(A*x**4/4 + B*x**5/5), True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.84

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\left(bx^2+a\right)^{\frac{7}{2}}Bx^3}{10\,b}+\frac{\left(bx^2+a\right)^{\frac{7}{2}}Ax^2}{9\,b}-\frac{3\left(bx^2+a\right)^{\frac{7}{2}}Bax}{80\,b^2}+\frac{\left(bx^2+a\right)^{\frac{5}{2}}Ba^2x}{160\,b^2}+\frac{\left(bx^2+a\right)^{\frac{3}{2}}Ba^3x}{128\,b^2}+\frac{3\sqrt{bx^2+a}Ba^4x}{256\,b^2}+\frac{3\,Ba^5\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{5}{2}}}-\frac{2\left(bx^2+a\right)^{\frac{7}{2}}Aa}{63\,b^2}$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/10*(b*x^2 + a)^(7/2)*B*x^3/b + 1/9*(b*x^2 + a)^(7/2)*A*x^2/b - 3/80*(b*x
^2 + a)^(7/2)*B*a*x/b^2 + 1/160*(b*x^2 + a)^(5/2)*B*a^2*x/b^2 + 1/128*(b*x
^2 + a)^(3/2)*B*a^3*x/b^2 + 3/256*sqrt(b*x^2 + a)*B*a^4*x/b^2 + 3/256*B*a^
5*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/63*(b*x^2 + a)^(7/2)*A*a/b^2
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{3\,Ba^5\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{5}{2}}}$$
$$-\frac{1}{80640}\left(\frac{2560\,Aa^4}{b^2}+\left(\frac{945\,Ba^4}{b^2}-2\left(\frac{640\,Aa^3}{b}+\left(\frac{315\,Ba^3}{b}+4\left(2400\,Aa^2+\left(1953\,Ba^2+2\left(1520\,Aab+7\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^3*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-3/256*B*a^5*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2) - 1/80640*(256
0*A*a^4/b^2 + (945*B*a^4/b^2 - 2*(640*A*a^3/b + (315*B*a^3/b + 4*(2400*A*a
^2 + (1953*B*a^2 + 2*(1520*A*a*b + 7*(189*B*a*b + 8*(9*B*b^2*x + 10*A*b^2)
*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int x^3(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x^3\,\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^3*(a + b*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x^3*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.16 $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

3.16.1 Optimal result

Integrand size = 20, antiderivative size = 150

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^3Ax\sqrt{a+bx^2}}{128b}-\frac{5a^2Ax\left(a+bx^2\right)^{3/2}}{192b}-\frac{aAx\left(a+bx^2\right)^{5/2}}{48b}+\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{(16aB-63Abx)\left(a+bx^2\right)^{7/2}}{504b^2}-\frac{5a^4A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
-5/192*a^2*A*x*(b*x^2+a)^(3/2)/b-1/48*a*A*x*(b*x^2+a)^(5/2)/b+1/9*B*x^2*(b
*x^2+a)^(7/2)/b-1/504*(-63*A*b*x+16*B*a)*(b*x^2+a)^(7/2)/b^2-5/128*a^4*A*a
rctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-5/128*a^3*A*x*(b*x^2+a)^(1/2)/b
```

3.16.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\sqrt{a+bx^2}\left(-256a^4B+112b^4x^7(9A+8Bx)+a^3bx(315A+128Bx)+8ab^3x^5(357A+304Bx\right.}{8064b^2}$$

input
```
Integrate[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-256*a^4*B + 112*b^4*x^7*(9*A + 8*B*x) + a^3*b*x*(315*A
+ 128*B*x) + 8*a*b^3*x^5*(357*A + 304*B*x) + 6*a^2*b^2*x^3*(413*A + 320*B*
x)) + 315*a^4*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8064*b^2)
```

3.16.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {533, 533, 25, 27, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\int x(2aB-9Abx)\left(bx^2+a\right)^{5/2}dx}{9b}$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{-\frac{\int -ab(9A+16Bx)\left(bx^2+a\right)^{5/2}dx}{8b}-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{25}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{\int ab(9A+16Bx)\left(bx^2+a\right)^{5/2}dx}{8b}-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\int (9A+16Bx)\left(bx^2+a\right)^{5/2}dx-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\int\left(bx^2+a\right)^{5/2}dx+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\left(\frac{5}{6}a\int\left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$\Big\downarrow$ 211

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\left(\frac{5}{6}a\left(\frac{3}{4}a\int\sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)-\frac{9}{8}Ax\left(a+bx^2\right)^{7/2}}{9b}$$

$\Big\downarrow$ 211

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)-\frac{9}{8}Ax(}{9b}$$

$\Big\downarrow$ 224

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)}{9b}$$

$\Big\downarrow$ 219

$$\frac{Bx^2\left(a+bx^2\right)^{7/2}}{9b}-\frac{\frac{1}{8}a\left(9A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{16B\left(a+bx^2\right)^{7/2}}{7b}\right)}{9b}$$

input
```
Int[x^2*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(B*x^2*(a + b*x^2)^(7/2))/(9*b) - ((-9*A*x*(a + b*x^2)^(7/2))/8 + (a*((16*
B*(a + b*x^2)^(7/2))/(7*b) + 9*A*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a +
b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqr
t[a + b*x^2]])/(2*Sqrt[b])))/4))/6)))/8)/(9*b)
```

3.16. $\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.16.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

method	result
default	$B\left(\frac{x^{2}(b\,x^{2}+a)^{\frac{7}{2}}}{9b}-\frac{2a(b\,x^{2}+a)^{\frac{7}{2}}}{63b^{2}}\right)+A\left(\frac{x(b\,x^{2}+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}}\right.}{2\sqrt{b}}\right.}{4}\right.}{6}\right.}{8b}\right.$
risch	$\frac{(896B\,x^{8}b^{4}+1008A\,b^{4}x^{7}+2432B\,x^{6}a\,b^{3}+2856Aa\,b^{3}x^{5}+1920a^{2}B\,b^{2}x^{4}+2478A\,a^{2}b^{2}x^{3}+128B\,a^{3}b\,x^{2}+315A\,a^{3}bx-256B\,a^{4})\sqrt{b\,x^{2}+a}}{8064b^{2}}$

input
```
int(x^2*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/9*x^2*(b*x^2+a)^(7/2)/b-2/63*a/b^2*(b*x^2+a)^(7/2))+A*(1/8*x*(b*x^2+a
)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*
a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))))))
```

3.16.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.81

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \left[\frac{315\,Aa^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(896\,Bb^4x^8+1008\,Ab^4x^7+2432\,Bab^3x}{1}\right.$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/16128*(315*A*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 2*(896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^3*x^6 + 2856*A*a*b^3*x^
5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B*a^3*b*x^2 + 315*A*a^3*
b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2, 1/8064*(315*A*a^4*sqrt(-b)*arctan(s
qrt(-b)*x/sqrt(b*x^2 + a)) + (896*B*b^4*x^8 + 1008*A*b^4*x^7 + 2432*B*a*b^
3*x^6 + 2856*A*a*b^3*x^5 + 1920*B*a^2*b^2*x^4 + 2478*A*a^2*b^2*x^3 + 128*B
*a^3*b*x^2 + 315*A*a^3*b*x - 256*B*a^4)*sqrt(b*x^2 + a))/b^2]
```

3.16.6 Sympy [A] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\begin{cases}-\frac{5Aa^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b}+\sqrt{a+bx^2}\cdot\left(\frac{5Aa^3x}{128b}+\frac{59Aa^2x^3}{192}+\frac{17Aabx^5}{48}+\frac{Ab^2x^7}{8}\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right)\end{cases}$$

input
```
integrate(x**2*(B*x+A)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-5*A*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b) + sqrt(a + b*x**2
)*(5*A*a**3*x/(128*b) + 59*A*a**2*x**3/192 + 17*A*a*b*x**5/48 + A*b**2*x**
7/8 - 2*B*a**4/(63*b**2) + B*a**3*x**2/(63*b) + 5*B*a**2*x**4/21 + 19*B*a*
b*x**6/63 + B*b**2*x**8/9), Ne(b, 0)), (a**(5/2)*(A*x**3/3 + B*x**4/4), Tr
ue))
```

3.16.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{(bx^2+a)^{\frac{7}{2}}Bx^2}{9\,b}+\frac{(bx^2+a)^{\frac{7}{2}}Ax}{8\,b}-\frac{(bx^2+a)^{\frac{5}{2}}Aax}{48\,b}-\frac{5\,(bx^2+a)^{\frac{3}{2}}Aa^2x}{192\,b}-\frac{5\,\sqrt{bx^2+a}Aa^3x}{128\,b}-\frac{5\,Aa^4\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}}-\frac{2\,(bx^2+a)^{\frac{7}{2}}Ba}{63\,b^2}$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(b*x^2 + a)^(7/2)*B*x^2/b + 1/8*(b*x^2 + a)^(7/2)*A*x/b - 1/48*(b*x^2
+ a)^(5/2)*A*a*x/b - 5/192*(b*x^2 + a)^(3/2)*A*a^2*x/b - 5/128*sqrt(b*x^2
+ a)*A*a^3*x/b - 5/128*A*a^4*arcsinh(b*x/sqrt(a*b))/b^(3/2) - 2/63*(b*x^2
+ a)^(7/2)*B*a/b^2
```

3.16.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5\,Aa^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}}-\frac{1}{8064}\left(\frac{256\,Ba^4}{b^2}-\left(\frac{315\,Aa^3}{b}+2\left(\frac{64\,Ba^3}{b}+\left(1239\,Aa^2+4\left(240\,Ba^2+\left(357\,Aab+2\left(152\,Bab+7\left(8\,Bb^2\right.\right.\right.\right.\right.\right.\right.\right.$$

input
```
integrate(x^2*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
5/128*A*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) - 1/8064*(256*B
*a^4/b^2 - (315*A*a^3/b + 2*(64*B*a^3/b + (1239*A*a^2 + 4*(240*B*a^2 + (35
7*A*a*b + 2*(152*B*a*b + 7*(8*B*b^2*x + 9*A*b^2)*x)*x)*x)*x)*x)*x)*x)*sqrt
(b*x^2 + a)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x^2\,\left(b\,x^2+a\right)^{5/2}\,(A+B\,x)\;dx$$

input
```
int(x^2*(a + b*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x^2*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.17 $\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

3.17.1 Optimal result

Integrand size = 18, antiderivative size = 126

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5a^3Bx\sqrt{a+bx^2}}{128b}-\frac{5a^2Bx\left(a+bx^2\right)^{3/2}}{192b}-\frac{aBx\left(a+bx^2\right)^{5/2}}{48b}+\frac{(8A+7Bx)\left(a+bx^2\right)^{7/2}}{56b}-\frac{5a^4B\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{3/2}}$$

output
```
-5/192*a^2*B*x*(b*x^2+a)^(3/2)/b-1/48*a*B*x*(b*x^2+a)^(5/2)/b+1/56*(7*B*x+
8*A)*(b*x^2+a)^(7/2)/b-5/128*a^4*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3
/2)-5/128*a^3*B*x*(b*x^2+a)^(1/2)/b
```

3.17.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.94

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\sqrt{b}\sqrt{a+bx^2}\left(48b^3x^6(8A+7Bx)+3a^3(128A+35Bx)+8ab^2x^4(144A+119Bx)+2a^2bx^2(57\right.}{2688b^{3/2}}$$

input
```
Integrate[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[b]*Sqrt[a + b*x^2]*(48*b^3*x^6*(8*A + 7*B*x) + 3*a^3*(128*A + 35*B*x
) + 8*a*b^2*x^4*(144*A + 119*B*x) + 2*a^2*b*x^2*(576*A + 413*B*x)) + 105*a
^4*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2688*b^(3/2))
```

3.17.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {533, 455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^{5/2}(A+Bx)\,dx$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{\int (aB-8Abx)\left(bx^2+a\right)^{5/2}dx}{8b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{aB\int \left(bx^2+a\right)^{5/2}dx-\frac{8}{7}A\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{aB\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{8}{7}A\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{aB\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{8}{7}A\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{211}$$

$$\frac{Bx\left(a+bx^2\right)^{7/2}}{8b}-\frac{aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)-\frac{8}{7}A\left(a+bx^2\right)^{7/2}}{8b}$$

$$\Big\downarrow \text{224}$$

$$\frac{Bx(a+bx^2)^{7/2}}{8b}-\frac{aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)-\frac{8}{7}A(a+bx^2)^{7/2}}{8b}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{Bx(a+bx^2)^{7/2}}{8b}-\frac{aB\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x(a+bx^2)^{3/2}\right)+\frac{1}{6}x(a+bx^2)^{5/2}\right)-\frac{8}{7}A(a+bx^2)^{7/2}}{8b}$$

input
```
Int[x*(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(B*x*(a + b*x^2)^(7/2))/(8*b) - ((-8*A*(a + b*x^2)^(7/2))/7 + a*B*((x*(a +
 b*x^2)^(5/2))/6 + (5*a*((x*(a + b*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2
])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/4))/6))/(8*b
)
```

3.17.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.17.4 Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.86

method	result
default	$B\left(\frac{x(b\,x^2+a)^{\frac{7}{2}}}{8b}-\frac{a\left(\frac{x\left(b\,x^2+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^2+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{8b}\right)+\frac{A(b\,x^2+a)^{\frac{7}{2}}}{7b}$
risch	$\frac{(336b^3B\,x^7+384x^6b^3A+952Ba\,b^2x^5+1152aA\,b^2x^4+826B\,a^2b\,x^3+1152a^2Ab\,x^2+105a^3Bx+384a^3A)\sqrt{b\,x^2+a}}{2688b}-\frac{5B\,a^4\ln\left(x\sqrt{b}+\sqrt{\right.}}{128b^{\frac{3}{2}}}$

input
```
int(x*(B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/8*x*(b*x^2+a)^(7/2)/b-1/8*a/b*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*
x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))))))+1/7*A*(b*x^2+a)^(7/2)/b
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.01

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \left[\frac{105\,Ba^4\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(336\,Bb^4x^7+384\,Ab^4x^6+952\,Bab^3x^5+}{5376\,b^2}\right.$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/5376*(105*B*a^4*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
 + 2*(336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x^4 +
 826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)*sqr
t(b*x^2 + a))/b^2, 1/2688*(105*B*a^4*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) + (336*B*b^4*x^7 + 384*A*b^4*x^6 + 952*B*a*b^3*x^5 + 1152*A*a*b^3*x
^4 + 826*B*a^2*b^2*x^3 + 1152*A*a^2*b^2*x^2 + 105*B*a^3*b*x + 384*A*a^3*b)
*sqrt(b*x^2 + a))/b^2]
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.33

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \begin{cases} -\frac{5Ba^4\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{128b}+\sqrt{a+bx^2}\left(\frac{Aa^3}{7b}+\frac{3Aa^2x^2}{7}+\frac{3Aabx^4}{7}+\frac{Ab^2x^6}{7}+\frac{5Ba}{128}\right. \\ a^{\frac{5}{2}}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) \end{cases}$$

input
```
integrate(x*(B*x+A)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((-5*B*a**4*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sq
rt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(128*b) + sqrt(a + b*x**2
)*(A*a**3/(7*b) + 3*A*a**2*x**2/7 + 3*A*a*b*x**4/7 + A*b**2*x**6/7 + 5*B*a
**3*x/(128*b) + 59*B*a**2*x**3/192 + 17*B*a*b*x**5/48 + B*b**2*x**7/8), Ne
(b, 0)), (a**(5/2)*(A*x**2/2 + B*x**3/3), True))
```

3.17.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.83

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{(bx^2+a)^{\frac{7}{2}}Bx}{8\,b} - \frac{(bx^2+a)^{\frac{5}{2}}Bax}{48\,b} - \frac{5\,(bx^2+a)^{\frac{3}{2}}Ba^2x}{192\,b} - \frac{5\,\sqrt{bx^2+a}Ba^3x}{128\,b} - \frac{5\,Ba^4\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{3}{2}}} + \frac{(bx^2+a)^{\frac{7}{2}}A}{7\,b}$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/8*(b*x^2 + a)^(7/2)*B*x/b - 1/48*(b*x^2 + a)^(5/2)*B*a*x/b - 5/192*(b*x^
2 + a)^(3/2)*B*a^2*x/b - 5/128*sqrt(b*x^2 + a)*B*a^3*x/b - 5/128*B*a^4*arc
sinh(b*x/sqrt(a*b))/b^(3/2) + 1/7*(b*x^2 + a)^(7/2)*A/b
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.90

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{5\,Ba^4\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{3}{2}}} + \frac{1}{2688}\left(\frac{384\,Aa^3}{b} + \left(\frac{105\,Ba^3}{b} + 2\left(576\,Aa^2 + \left(413\,Ba^2 + 4\left(144\,Aab + \left(119\,Bab + 6\left(7\,Bb^2x + 8\,Ab^2\right)x\right)x\right.\right.\right.\right.\right.$$

input
```
integrate(x*(B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
5/128*B*a^4*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2) + 1/2688*(384*A
*a^3/b + (105*B*a^3/b + 2*(576*A*a^2 + (413*B*a^2 + 4*(144*A*a*b + (119*B*
a*b + 6*(7*B*b^2*x + 8*A*b^2)*x)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.17.9 Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\int x\,\left(b\,x^2+a\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x*(a + b*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x*(a + b*x^2)^(5/2)*(A + B*x), x)
```

3.18 $\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx$

3.18.1 Optimal result

Integrand size = 17, antiderivative size = 107

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{5}{16}a^2Ax\sqrt{a+bx^2}+\frac{5}{24}aAx\left(a+bx^2\right)^{3/2}+\frac{1}{6}Ax\left(a+bx^2\right)^{5/2}+\frac{B\left(a+bx^2\right)^{7/2}}{7b}+\frac{5a^3A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}$$

output
```
5/24*a*A*x*(b*x^2+a)^(3/2)+1/6*A*x*(b*x^2+a)^(5/2)+1/7*B*(b*x^2+a)^(7/2)/b
+5/16*a^3*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+5/16*a^2*A*x*(b*x^2
+a)^(1/2)
```

3.18.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{\sqrt{a+bx^2}(48a^3B+8b^3x^5(7A+6Bx)+3a^2bx(77A+48Bx)+2ab^2x^3(91A+72Bx))-105a}{336b}$$

input
```
Integrate[(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(Sqrt[a + b*x^2]*(48*a^3*B + 8*b^3*x^5*(7*A + 6*B*x) + 3*a^2*b*x*(77*A + 4
8*B*x) + 2*a*b^2*x^3*(91*A + 72*B*x)) - 105*a^3*A*Sqrt[b]*Log[-(Sqrt[b]*x)
 + Sqrt[a + b*x^2]])/(336*b)
```

3.18.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {455, 211, 211, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^{5/2} (A+Bx) \, dx$$

$$\Big\downarrow \; 455$$

$$A\int \left(bx^2+a\right)^{5/2}dx+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$A\left(\frac{5}{6}a\int \left(bx^2+a\right)^{3/2}dx+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\int \sqrt{bx^2+a}dx+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \; 211$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \; 224$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

$$\Big\downarrow \; 219$$

$$A\left(\frac{5}{6}a\left(\frac{3}{4}a\left(\frac{a\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2\sqrt{b}}+\frac{1}{2}x\sqrt{a+bx^2}\right)+\frac{1}{4}x\left(a+bx^2\right)^{3/2}\right)+\frac{1}{6}x\left(a+bx^2\right)^{5/2}\right)+\frac{B\left(a+bx^2\right)^{7/2}}{7b}$$

input
```
Int[(A + B*x)*(a + b*x^2)^(5/2),x]
```

output
```
(B*(a + b*x^2)^(7/2))/(7*b) + A*((x*(a + b*x^2)^(5/2))/6 + (5*a*((x*(a + b
*x^2)^(3/2))/4 + (3*a*((x*Sqrt[a + b*x^2])/2 + (a*ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]])/(2*Sqrt[b])))/4))/6)
```

3.18.3.1 Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.18.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.80

method	result	size
default	$A\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+\frac{B(b\,x^{2}+a)^{\frac{7}{2}}}{7b}$	86
risch	$\frac{(48b^{3}B\,x^{6}+56A\,b^{3}x^{5}+144Ba\,b^{2}x^{4}+182aA\,b^{2}x^{3}+144B\,a^{2}b\,x^{2}+231a^{2}Abx+48a^{3}B)\sqrt{b\,x^{2}+a}}{336b}+\frac{5a^{3}A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16\sqrt{b}}$	104

input
```
int((B*x+A)*(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
A*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+1/7*B*(b*x^2+a)^(7
/2)/b
```

3.18.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.09

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\left[\frac{105\,Aa^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+1}{672\,b}\right.$$

$$-\frac{105\,Aa^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(48\,Bb^3x^6+56\,Ab^3x^5+144\,Bab^2x^4+182\,Aab^2x^3+144\,Ba^2bx^2+231\,A}{336\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/672*(105*A*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a)
+ 2*(48*B*b^3*x^6 + 56*A*b^3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144
*B*a^2*b*x^2 + 231*A*a^2*b*x + 48*B*a^3)*sqrt(b*x^2 + a))/b, -1/336*(105*A
*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (48*B*b^3*x^6 + 56*A*b^
3*x^5 + 144*B*a*b^2*x^4 + 182*A*a*b^2*x^3 + 144*B*a^2*b*x^2 + 231*A*a^2*b*
x + 48*B*a^3)*sqrt(b*x^2 + a))/b]
```

3.18.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.40

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \begin{cases} \frac{5Aa^3\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a \neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{16} + \sqrt{a+bx^2}\cdot\left(\frac{11Aa^2x}{16} + \frac{13Aabx^3}{24} + \frac{Ab^2x^5}{6} + \frac{Ba^3}{7b} + \frac{3B\ldots \\ a^{\frac{5}{2}}\left(Ax+\frac{Bx^2}{2}\right) \end{cases}$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2),x)
```

output
```
Piecewise((5*A*a**3*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/16 + sqrt(a + b*x**2)*(11*
A*a**2*x/16 + 13*A*a*b*x**3/24 + A*b**2*x**5/6 + B*a**3/(7*b) + 3*B*a**2*x
**2/7 + 3*B*a*b*x**4/7 + B*b**2*x**6/7), Ne(b, 0)), (a**(5/2)*(A*x + B*x**
2/2), True))
```

3.18.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.72

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Ax + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Aax + \frac{5}{16}\sqrt{bx^2+a}Aa^2x + \frac{5\,Aa^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} + \frac{\left(bx^2+a\right)^{\frac{7}{2}}B}{7\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*A*x + 5/24*(b*x^2 + a)^(3/2)*A*a*x + 5/16*sqrt(b*x^2
 + a)*A*a^2*x + 5/16*A*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) + 1/7*(b*x^2 + a
)^(7/2)*B/b
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=-\frac{5\,Aa^3\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,\sqrt{b}}+\frac{1}{336}\left(\frac{48\,Ba^3}{b}+\left(231\,Aa^2+2\left(72\,Ba^2+\left(91\,Aab+4\left(18\,Bab+\left(6\,Bb^2x+7\,Ab^2\right)x\right)x\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-5/16*A*a^3*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + 1/336*(48*B*a
^3/b + (231*A*a^2 + 2*(72*B*a^2 + (91*A*a*b + 4*(18*B*a*b + (6*B*b^2*x + 7
*A*b^2)*x)*x)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.50

$$\int (A+Bx)\left(a+bx^2\right)^{5/2}\,dx=\frac{B\left(b\,x^2+a\right)^{7/2}}{7\,b}+\frac{A\,x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}}$$

input
```
int((a + b*x^2)^(5/2)*(A + B*x),x)
```

output
```
(B*(a + b*x^2)^(7/2))/(7*b) + (A*x*(a + b*x^2)^(5/2)*hypergeom([-5/2, 1/2]
, 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.19 $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$

3.19.1 Optimal result

Integrand size = 20, antiderivative size = 132

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx=\frac{1}{16}a^2(16A+5Bx)\sqrt{a+bx^2}+\frac{1}{24}a(8A+5Bx)\left(a+bx^2\right)^{3/2}+\frac{1}{30}(6A+5Bx)\left(a+bx^2\right)^{5/2}+\frac{5a^3B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16\sqrt{b}}-a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/24*a*(5*B*x+8*A)*(b*x^2+a)^(3/2)+1/30*(5*B*x+6*A)*(b*x^2+a)^(5/2)-a^(5/2
)*A*arctanh((b*x^2+a)^(1/2)/a^(1/2))+5/16*a^3*B*arctanh(x*b^(1/2)/(b*x^2+a
)^(1/2))/b^(1/2)+1/16*a^2*(5*B*x+16*A)*(b*x^2+a)^(1/2)
```

3.19.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx=2a^{5/2}A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)+\frac{1}{240}\left(\sqrt{a+bx^2}\left(8b^2x^4(6A+5Bx)+2abx^2(88A+65Bx)+a^2(368A+165Bx)\right)-\frac{75a^3B\log\left(-\sqrt{b}x+\sqrt{a+b\right.}{\sqrt{b}}\right.$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

output
```
2*a^(5/2)*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + (Sqrt[a + b*x
^2]*(8*b^2*x^4*(6*A + 5*B*x) + 2*a*b*x^2*(88*A + 65*B*x) + a^2*(368*A + 16
5*B*x)) - (75*a^3*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b])/240
```

3.19.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {535, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx)}{x}dx$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{6}a\int \frac{(6A+5Bx)(bx^2+a)^{3/2}}{x}dx+\frac{1}{30}(a+bx^2)^{5/2}(6A+5Bx)$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{6}a\left(\frac{1}{4}a\int \frac{3(8A+5Bx)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}(a+bx^2)^{3/2}(8A+5Bx)\right)+\frac{1}{30}(a+bx^2)^{5/2}(6A+5Bx)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}a\left(\frac{3}{4}a\int \frac{(8A+5Bx)\sqrt{bx^2+a}}{x}dx+\frac{1}{4}(a+bx^2)^{3/2}(8A+5Bx)\right)+\frac{1}{30}(a+bx^2)^{5/2}(6A+5Bx)$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\int \frac{16A+5Bx}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}(a+bx^2)^{3/2}(8A+5Bx)\right)+\frac{1}{30}(a+bx^2)^{5/2}(6A+5Bx)$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(16A\int \frac{1}{x\sqrt{bx^2+a}}dx+5B\int \frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}(a+bx^2)^{3/2}(8A+5\right.$$
$$\frac{1}{30}(a+bx^2)^{5/2}(6A+5Bx)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(16A\int\frac{1}{x\sqrt{bx^2+a}}dx+5B\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}\right.$$
$$\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

$\big\downarrow$ 219

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(16A\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{5B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8A\right.$$
$$\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

$\big\downarrow$ 243

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(8A\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{5B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8A\right.$$
$$\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

$\big\downarrow$ 73

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(\frac{16A\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{5B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8\right.$$
$$\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

$\big\downarrow$ 221

$$\frac{1}{6}a\left(\frac{3}{4}a\left(\frac{1}{2}a\left(\frac{5B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{16A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(16A+5Bx)\right)+\frac{1}{4}\left(a+bx^2\right)^{3/2}(8A\right.$$
$$\frac{1}{30}\left(a+bx^2\right)^{5/2}(6A+5Bx)$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/x,x]
```

output
```
((6*A + 5*B*x)*(a + b*x^2)^(5/2))/30 + (a*(((8*A + 5*B*x)*(a + b*x^2)^(3/2
))/4 + (3*a*((((16*A + 5*B*x)*Sqrt[a + b*x^2])/2 + (a*((5*B*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (16*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/S
qrt[a]))/2))/4))/6
```

3.19. $\int\frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.19.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

method	result
default	$B\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)+A\left(\frac{\left(b\,x^{2}+a\right)^{\frac{5}{2}}}{5}+a\left(\frac{\left(b\,x^{2}+a\right)^{\frac{3}{2}}}{3}+a\left(\right.\right.\right.$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
B*(1/6*x*(b*x^2+a)^(5/2)+5/6*a*(1/4*x*(b*x^2+a)^(3/2)+3/4*a*(1/2*x*(b*x^2+
a)^(1/2)+1/2*a/b^(1/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))))+A*(1/5*(b*x^2+a)^(
5/2)+a*(1/3*(b*x^2+a)^(3/2)+a*((b*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x))))
```

3.19.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 539, normalized size of antiderivative = 4.08

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \left[\frac{75\,Ba^3\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+240\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2\cdots}}{x^2\cdots}\right.}{\cdots}\right.$$

$$-\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,Aa^{\frac{5}{2}}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3\cdots\right.}{240\,b}$$

$$-\frac{75\,Ba^3\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-240\,A\sqrt{-a}a^2b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(40\,Bb^3x^5+48\,Ab^3x^4+130\,Bab^2x^3+1\cdots\right.}{240\,b}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="fricas")
```

output
```
[1/480*(75*B*a^3*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) +
 240*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(
40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^
2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(
sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*A*a^(5/2)*b*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) - (40*B*b^3*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 +
 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*sqrt(b*x^2 + a))/b, 1/480*
(480*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 75*B*a^3*sqrt(b)*
log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(40*B*b^3*x^5 + 48*A*b
^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x + 368*A*a^2*b)*
sqrt(b*x^2 + a))/b, -1/240*(75*B*a^3*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2
 + a)) - 240*A*sqrt(-a)*a^2*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (40*B*b^3
*x^5 + 48*A*b^3*x^4 + 130*B*a*b^2*x^3 + 176*A*a*b^2*x^2 + 165*B*a^2*b*x +
368*A*a^2*b)*sqrt(b*x^2 + a))/b]
```

3.19. $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x}\,dx$

3.19.6 Sympy [A] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.59

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = -Aa^{\frac{5}{2}}\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)+\frac{Aa^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}}$$
$$+\frac{Aa^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}+2Aab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+Ab^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2}+\frac{ax^2\sqrt{a+bx^2}}{15b}+\frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$
$$+Ba^2\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$
$$+2Bab\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$
$$+Bb^2\left(\begin{cases}\frac{a^3\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{16b^2}-\frac{a^2x\sqrt{a+bx^2}}{16b^2}+\frac{ax^3\sqrt{a+bx^2}}{24b}+\frac{x^5\sqrt{a+bx^2}}{6} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^5}{5} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/x,x)
```

output
```
-A*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + A*a**3/(sqrt(b)*x*sqrt(a/(b*x**2)
 + 1)) + A*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*A*a*b*Piecewise((a*sqrt
(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2,
True)) + A*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*sqr
t(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4/4
, True)) + B*a**2*Piecewise((a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) +
 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/2 + x*sqrt(a +
b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + 2*B*a*b*Piecewise((-a**2*Piecew
ise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)
/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) + x**3*sqrt(a + b
*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True)) + B*b**2*Piecewise((a**3*Piec
ewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(
x)/sqrt(b*x**2), True))/(16*b**2) - a**2*x*sqrt(a + b*x**2)/(16*b**2) + a*
x**3*sqrt(a + b*x**2)/(24*b) + x**5*sqrt(a + b*x**2)/6, Ne(b, 0)), (sqrt(a
)*x**5/5, True))
```

3.19.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{1}{6}\left(bx^2+a\right)^{\frac{5}{2}}Bx + \frac{5}{24}\left(bx^2+a\right)^{\frac{3}{2}}Bax + \frac{5}{16}\sqrt{bx^2+a}Ba^2x + \frac{5\,Ba^3\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,\sqrt{b}} - Aa^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}A + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Aa + \sqrt{bx^2+a}Aa^2$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^(5/2)*B*x + 5/24*(b*x^2 + a)^(3/2)*B*a*x + 5/16*sqrt(b*x^2
 + a)*B*a^2*x + 5/16*B*a^3*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*a^(5/2)*arcs
inh(a/(sqrt(a*b)*abs(x))) + 1/5*(b*x^2 + a)^(5/2)*A + 1/3*(b*x^2 + a)^(3/2
)*A*a + sqrt(b*x^2 + a)*A*a^2
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.19.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x}\,dx = \frac{A\left(b\,x^2+a\right)^{5/2}}{5} + A\,a^2\,\sqrt{b\,x^2+a} + \frac{A\,a\left(b\,x^2+a\right)^{3/2}}{3} + \frac{B\,x\left(b\,x^2+a\right)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{\left(\frac{b\,x^2}{a}+1\right)^{5/2}} + A\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/x,x)
```

output
```
(A*(a + b*x^2)^(5/2))/5 + A*a^2*(a + b*x^2)^(1/2) + A*a^(5/2)*atan(((a + b
*x^2)^(1/2)*1i)/a^(1/2))*1i + (A*a*(a + b*x^2)^(3/2))/3 + (B*x*(a + b*x^2)
^(5/2)*hypergeom([-5/2, 1/2], 3/2, -(b*x^2)/a))/((b*x^2)/a + 1)^(5/2)
```

3.20 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2}\,dx$

3.20.1 Optimal result

Integrand size = 20, antiderivative size = 136

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx=\frac{1}{8}a(8aB+15Abx)\sqrt{a+bx^2}+\frac{1}{12}(4aB+15Abx)\left(a+bx^2\right)^{3/2}-\frac{(5A-Bx)\left(a+bx^2\right)^{5/2}}{5x}+\frac{15}{8}a^2A\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-a^{5/2}B\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
1/12*(15*A*b*x+4*B*a)*(b*x^2+a)^(3/2)-1/5*(-B*x+5*A)*(b*x^2+a)^(5/2)/x-a^(
5/2)*B*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*A*arctanh(x*b^(1/2)/(b*x^
2+a)^(1/2))*b^(1/2)+1/8*a*(15*A*b*x+8*B*a)*(b*x^2+a)^(1/2)
```

3.20.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx=\frac{\sqrt{a+bx^2}(-8a^2(15A-23Bx)+6b^2x^4(5A+4Bx)+abx^2(135A+88Bx))}{120x}+2a^{5/2}B\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)-\frac{15}{8}a^2A\sqrt{b}\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*(15*A - 23*B*x) + 6*b^2*x^4*(5*A + 4*B*x) + a*b*x
^2*(135*A + 88*B*x)))/(120*x) + 2*a^(5/2)*B*ArcTanh[(Sqrt[b]*x - Sqrt[a +
b*x^2])/Sqrt[a]] - (15*a^2*A*Sqrt[b]*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/
8
```

3.20.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {536, 535, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx^2\right)^{5/2}(A+Bx)}{x^2}dx$$

$$\Big\downarrow \text{536}$$

$$\int \frac{(aB+5Abx)\left(bx^2+a\right)^{3/2}}{x}dx-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}a\int \frac{(4aB+15Abx)\sqrt{bx^2+a}}{x}dx-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$$\Big\downarrow \text{535}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\int \frac{8aB+15Abx}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$$\Big\downarrow \text{538}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(15Ab\int \frac{1}{\sqrt{bx^2+a}}dx+8aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}a\left(\frac{1}{2}a\left(15Ab\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+8aB\int \frac{1}{x\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$\Big\downarrow$ 219

$$\frac{1}{4}a\left(\frac{1}{2}a\left(8aB\int \frac{1}{x\sqrt{bx^2+a}}dx+15A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$\Big\downarrow$ 243

$$\frac{1}{4}a\left(\frac{1}{2}a\left(4aB\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+15A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$\Big\downarrow$ 73

$$\frac{1}{4}a\left(\frac{1}{2}a\left(\frac{8aB\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+15A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

$\Big\downarrow$ 221

$$\frac{1}{4}a\left(\frac{1}{2}a\left(15A\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)-8\sqrt{a}B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)+\frac{1}{2}\sqrt{a+bx^2}(8aB+15Abx)\right)-\frac{\left(a+bx^2\right)^{5/2}(5A-Bx)}{5x}+\frac{1}{12}\left(a+bx^2\right)^{3/2}(4aB+15Abx)$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/x^2,x]
```

output
```
((4*a*B + 15*A*b*x)*(a + b*x^2)^(3/2))/12 - ((5*A - B*x)*(a + b*x^2)^(5/2)
)/(5*x) + (a*(((8*a*B + 15*A*b*x)*Sqrt[a + b*x^2])/2 + (a*(15*A*Sqrt[b]*Ar
cTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]] - 8*Sqrt[a]*B*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]]))/2))/4
```

3.20. $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx$

3.20.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.20.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{a^2A\sqrt{b\,x^2+a}}{x}+\frac{B\,b^2x^4\sqrt{b\,x^2+a}}{5}+\frac{11Bba\,x^2\sqrt{b\,x^2+a}}{15}+\frac{23a^2B\sqrt{b\,x^2+a}}{15}+\frac{b^2A\,x^3\sqrt{b\,x^2+a}}{4}+\frac{9bAax\sqrt{b\,x^2+a}}{8}+\frac{15a^2\sqrt{b}}{}$
default	$B\left(\frac{(b\,x^2+a)^{\frac{5}{2}}}{5}+a\left(\frac{(b\,x^2+a)^{\frac{3}{2}}}{3}+a\left(\sqrt{b\,x^2+a}-\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)\right)\right)\right)+A\left(-\frac{(b\,x^2+a)^{\frac{7}{2}}}{ax}+\frac{6b}{}\right.$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2*A*(b*x^2+a)^(1/2)/x+1/5*B*b^2*x^4*(b*x^2+a)^(1/2)+11/15*B*b*a*x^2*(b*
x^2+a)^(1/2)+23/15*a^2*B*(b*x^2+a)^(1/2)+1/4*b^2*A*x^3*(b*x^2+a)^(1/2)+9/8
*b*A*a*x*(b*x^2+a)^(1/2)+15/8*a^2*b^(1/2)*A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))-
a^(5/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.20.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 519, normalized size of antiderivative = 3.82

$$\int \frac{(A+Bx)\,(a+bx^2)^{5/2}}{x^2}\,dx = \left[\frac{225\,Aa^2\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+120\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\sqrt{}}{}\right.}{}\right.$$

$$-\frac{225\,Aa^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,Ba^{\frac{5}{2}}x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3}{120\,x}$$

$$-\frac{225\,Aa^2\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-120\,B\sqrt{-a}a^2x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(24\,Bb^2x^5+30\,Ab^2x^4+88\,Babx^3+}{120\,x}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[1/240*(225*A*a^2*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 120*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*
x - 120*A*a^2)*sqrt(b*x^2 + a))/x, -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - 60*B*a^(5/2)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) - (24*B*b^2*x^5 + 30*A*b^2*x^4 + 88*B*a*b*x^3 + 135*A*
a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x, 1/240*(240*B*sqrt(-
a)*a^2*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 225*A*a^2*sqrt(b)*x*log(-2*b*x
^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(24*B*b^2*x^5 + 30*A*b^2*x^4 + 8
8*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 + a))/x,
 -1/120*(225*A*a^2*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 120*B*s
qrt(-a)*a^2*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (24*B*b^2*x^5 + 30*A*b^2*
x^4 + 88*B*a*b*x^3 + 135*A*a*b*x^2 + 184*B*a^2*x - 120*A*a^2)*sqrt(b*x^2 +
 a))/x]
```

3.20. $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2}\,dx$

3.20.6 Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 420, normalized size of antiderivative = 3.09

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = -\frac{Aa^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Aa^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Aa^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)$$
$$+ 2Aab\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2} + \frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right)$$
$$+ Ab^2\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b} + \frac{ax\sqrt{a+bx^2}}{8b} + \frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$
$$- Ba^{\frac{5}{2}}\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right) + \frac{Ba^3}{\sqrt{b}x\sqrt{\frac{a}{bx^2}+1}} + \frac{Ba^2\sqrt{b}x}{\sqrt{\frac{a}{bx^2}+1}}$$
$$+ 2Bab\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b} + \frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right)$$
$$+ Bb^2\left(\begin{cases}-\frac{2a^2\sqrt{a+bx^2}}{15b^2} + \frac{ax^2\sqrt{a+bx^2}}{15b} + \frac{x^4\sqrt{a+bx^2}}{5} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^4}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/x**2,x)
```

output
```
-A*a**(5/2)/(x*sqrt(1 + b*x**2/a)) - A*a**(3/2)*b*x/sqrt(1 + b*x**2/a) + A
*a**2*sqrt(b)*asinh(sqrt(b)*x/sqrt(a)) + 2*A*a*b*Piecewise((a*Piecewise((l
og(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(
b*x**2), True))/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) +
A*b**2*Piecewise((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)
/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b
*x**2)/(8*b) + x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
 - B*a**(5/2)*asinh(sqrt(a)/(sqrt(b)*x)) + B*a**3/(sqrt(b)*x*sqrt(a/(b*x**
2) + 1)) + B*a**2*sqrt(b)*x/sqrt(a/(b*x**2) + 1) + 2*B*a*b*Piecewise((a*sq
rt(a + b*x**2)/(3*b) + x**2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2
, True)) + B*b**2*Piecewise((-2*a**2*sqrt(a + b*x**2)/(15*b**2) + a*x**2*s
qrt(a + b*x**2)/(15*b) + x**4*sqrt(a + b*x**2)/5, Ne(b, 0)), (sqrt(a)*x**4
/4, True))
```

3.20.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^2}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Abx + \frac{15}{8}\sqrt{bx^2+a}Aabx + \frac{15}{8}Aa^2\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - Ba^{\frac{5}{2}}\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{1}{5}\left(bx^2+a\right)^{\frac{5}{2}}B + \frac{1}{3}\left(bx^2+a\right)^{\frac{3}{2}}Ba + \sqrt{bx^2+a}Ba^2 - \frac{\left(bx^2+a\right)^{\frac{5}{2}}A}{x}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
5/4*(b*x^2 + a)^(3/2)*A*b*x + 15/8*sqrt(b*x^2 + a)*A*a*b*x + 15/8*A*a^2*sq
rt(b)*arcsinh(b*x/sqrt(a*b)) - B*a^(5/2)*arcsinh(a/(sqrt(a*b)*abs(x))) + 1
/5*(b*x^2 + a)^(5/2)*B + 1/3*(b*x^2 + a)^(3/2)*B*a + sqrt(b*x^2 + a)*B*a^2
 - (b*x^2 + a)^(5/2)*A/x
```

3.20.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2}\,dx = \frac{2\,Ba^3\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}\,Aa^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{2\,Aa^3\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a} + \frac{1}{120}\left(184\,Ba^2+\left(135\,Aab+2\left(44\,Bab+3\left(4\,Bb^2x+5\,Ab^2\right)x\right)x\right)x\right)\sqrt{bx^2+a}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2*B*a^3*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*A*
a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 2*A*a^3*sqrt(b)/((sqr
t(b)*x - sqrt(b*x^2 + a))^2 - a) + 1/120*(184*B*a^2 + (135*A*a*b + 2*(44*B
*a*b + 3*(4*B*b^2*x + 5*A*b^2)*x)*x)*x)*sqrt(b*x^2 + a)
```

3.20.9 Mupad [B] (verification not implemented)

Time = 6.72 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^2}\,dx = \frac{B\,(b\,x^2+a)^{5/2}}{5} + B\,a^2\sqrt{b\,x^2+a} + \frac{B\,a\,(b\,x^2+a)^{3/2}}{3} - \frac{A\,(b\,x^2+a)^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\left(\frac{b\,x^2}{a}+1\right)^{5/2}} + B\,a^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,1\mathrm{i}$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/x^2,x)
```

output
```
(B*(a + b*x^2)^(5/2))/5 + B*a^2*(a + b*x^2)^(1/2) + B*a^(5/2)*atan(((a + b
*x^2)^(1/2)*1i)/a^(1/2))*1i + (B*a*(a + b*x^2)^(3/2))/3 - (A*(a + b*x^2)^(
5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^2)/a + 1)^(5/2))
```

3.21 $\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^3} dx$

3.21.1 Optimal result

Integrand size = 20, antiderivative size = 141

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^3} dx = \frac{5}{8}ab(4A+3Bx)\sqrt{a+bx^2} - \frac{5(3aB-2Abx)(a+bx^2)^{3/2}}{12x} - \frac{(2A-Bx)(a+bx^2)^{5/2}}{4x^2} + \frac{15}{8}a^2\sqrt{b}B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right) - \frac{5}{2}a^{3/2}Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)$$

output
```
-5/12*(-2*A*b*x+3*B*a)*(b*x^2+a)^(3/2)/x-1/4*(-B*x+2*A)*(b*x^2+a)^(5/2)/x^
2-5/2*a^(3/2)*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))+15/8*a^2*B*arctanh(x*b^
(1/2)/(b*x^2+a)^(1/2))*b^(1/2)+5/8*a*b*(3*B*x+4*A)*(b*x^2+a)^(1/2)
```

3.21.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(a+bx^2)^{5/2}}{x^3} dx = 5a^{3/2}Ab\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right) + \frac{1}{24}\left(\frac{\sqrt{a+bx^2}(-12a^2(A+2Bx)+2b^2x^4(4A+3Bx)+abx^2(56A+27Bx))}{x^2} - 45a^2\sqrt{b}B\log\left(-\sqrt{b}x+\sqrt{a-}\right.\right.$$

input
```
Integrate[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

output
```
5*a^(3/2)*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]] + ((Sqrt[a +
b*x^2]*(-12*a^2*(A + 2*B*x) + 2*b^2*x^4*(4*A + 3*B*x) + a*b*x^2*(56*A + 27
*B*x)))/x^2 - 45*a^2*Sqrt[b]*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/24
```

3.21.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {537, 25, 535, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^{5/2}(A+Bx)}{x^3}dx$$

$$\Big\downarrow \text{537}$$

$$-\frac{5}{2}b\int -\frac{(A+2Bx)(bx^2+a)^{3/2}}{x}dx-\frac{(a+bx^2)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5}{2}b\int \frac{(A+2Bx)(bx^2+a)^{3/2}}{x}dx-\frac{(a+bx^2)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{5}{2}b\left(\frac{1}{4}a\int \frac{2(2A+3Bx)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}(a+bx^2)^{3/2}(2A+3Bx)\right)-\frac{(a+bx^2)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{2}b\left(\frac{1}{2}a\int \frac{(2A+3Bx)\sqrt{bx^2+a}}{x}dx+\frac{1}{6}(a+bx^2)^{3/2}(2A+3Bx)\right)-\frac{(a+bx^2)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \text{535}$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\int \frac{4A+3Bx}{x\sqrt{bx^2+a}}dx+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}(a+bx^2)^{3/2}(2A+3Bx)\right)-\frac{(a+bx^2)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 538$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4A\int\frac{1}{x\sqrt{bx^2+a}}dx+3B\int\frac{1}{\sqrt{bx^2+a}}dx\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2A+3Bx$$

$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 224$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4A\int\frac{1}{x\sqrt{bx^2+a}}dx+3B\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2$$

$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 219$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(4A\int\frac{1}{x\sqrt{bx^2+a}}dx+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2A+$$

$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 243$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(2A\int\frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2A$$

$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 73$$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{4A\int\frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2A$$

$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

$$\Big\downarrow \ 221$$

3.21. $\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx$

$$\frac{5}{2}b\left(\frac{1}{2}a\left(\frac{1}{2}a\left(\frac{3B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{4A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)+\frac{1}{2}\sqrt{a+bx^2}(4A+3Bx)\right)+\frac{1}{6}\left(a+bx^2\right)^{3/2}(2A+$$
$$\frac{\left(a+bx^2\right)^{5/2}(A+2Bx)}{2x^2}$$

input
```
Int[((A + B*x)*(a + b*x^2)^(5/2))/x^3,x]
```

output
```
-1/2*((A + 2*B*x)*(a + b*x^2)^(5/2))/x^2 + (5*b*(((2*A + 3*B*x)*(a + b*x^2
)^(3/2))/6 + (a*(((4*A + 3*B*x)*Sqrt[a + b*x^2])/2 + (a*((3*B*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (4*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])
/Sqrt[a]))/2))/2))/2
```

3.21.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.21.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{a^{2}\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2x^{2}}+\frac{15\sqrt{b}\,a^{2}B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8}+\frac{B\,b^{2}x^{3}\sqrt{b\,x^{2}+a}}{4}+\frac{9Bbax\sqrt{b\,x^{2}+a}}{8}+\frac{b^{2}A\,x^{2}\sqrt{b\,x^{2}+a}}{3}+\frac{7bAa\sqrt{}}{3}$
default	$B\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{ax}+\frac{6b\left(\frac{x(b\,x^{2}+a)^{\frac{5}{2}}}{6}+\frac{5a\left(\frac{x(b\,x^{2}+a)^{\frac{3}{2}}}{4}+\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2}+\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2\sqrt{b}}\right)}{4}\right)}{6}\right)}{a}\right)+A\left(-\frac{(b\,x^{2}+a)^{\frac{7}{2}}}{2a\,x^{2}}+\right.$

input
```
int((B*x+A)*(b*x^2+a)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*(b*x^2+a)^(1/2)*(2*B*x+A)/x^2+15/8*b^(1/2)*a^2*B*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))+1/4*B*b^2*x^3*(b*x^2+a)^(1/2)+9/8*B*b*a*x*(b*x^2+a)^(1/2)+1/
3*b^2*A*x^2*(b*x^2+a)^(1/2)+7/3*b*A*a*(b*x^2+a)^(1/2)-5/2*b*a^(3/2)*A*ln((
2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 535, normalized size of antiderivative = 3.79

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = \left[\frac{45\,Ba^2\sqrt{b}x^2\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+60\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\sqrt{\cdots}}{\cdots}\right.\cdots}{\cdots}\right.$$

$$-\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-30\,Aa^{\frac{3}{2}}bx^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3\cdots\right.}{24\,x^2}$$

$$-\frac{45\,Ba^2\sqrt{-b}x^2\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-60\,A\sqrt{-a}abx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\left(6\,Bb^2x^5+8\,Ab^2x^4+27\,Babx^3+56\,\cdots\right.}{24\,x^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[1/48*(45*B*a^2*sqrt(b)*x^2*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) + 60*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
 + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x
 - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqr
t(-b)*x/sqrt(b*x^2 + a)) - 30*A*a^(3/2)*b*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 +
 a)*sqrt(a) + 2*a)/x^2) - (6*B*b^2*x^5 + 8*A*b^2*x^4 + 27*B*a*b*x^3 + 56*A
*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2, 1/48*(120*A*sqrt(-
a)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + 45*B*a^2*sqrt(b)*x^2*log(-2*
b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(6*B*b^2*x^5 + 8*A*b^2*x^4 +
27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))/x^2,
 -1/24*(45*B*a^2*sqrt(-b)*x^2*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - 60*A*sq
rt(-a)*a*b*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (6*B*b^2*x^5 + 8*A*b^2*x
^4 + 27*B*a*b*x^3 + 56*A*a*b*x^2 - 24*B*a^2*x - 12*A*a^2)*sqrt(b*x^2 + a))
/x^2]
```

3.21.6 Sympy [A] (verification not implemented)

Time = 3.21 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.70

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = -\frac{5Aa^{\frac{3}{2}}b\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2} - \frac{Aa^2\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2x} + \frac{2Aa^2\sqrt{b}}{x\sqrt{\frac{a}{bx^2}+1}} + \frac{2Aab^{\frac{3}{2}}x}{\sqrt{\frac{a}{bx^2}+1}} + Ab^2\left(\begin{cases}\frac{a\sqrt{a+bx^2}}{3b}+\frac{x^2\sqrt{a+bx^2}}{3} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^2}{2} & \text{otherwise}\end{cases}\right) - \frac{Ba^{\frac{5}{2}}}{x\sqrt{1+\frac{bx^2}{a}}} - \frac{Ba^{\frac{3}{2}}bx}{\sqrt{1+\frac{bx^2}{a}}} + Ba^2\sqrt{b}\,\mathrm{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right) + 2Bab\left(\begin{cases}\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2}+\frac{x\sqrt{a+bx^2}}{2} & \text{for } b\neq 0\\ \sqrt{a}x & \text{otherwise}\end{cases}\right) + Bb^2\left(\begin{cases}-\frac{a^2\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b}+\frac{ax\sqrt{a+bx^2}}{8b}+\frac{x^3\sqrt{a+bx^2}}{4} & \text{for } b\neq 0\\ \frac{\sqrt{a}x^3}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)*(b*x**2+a)**(5/2)/x**3,x)
```

output
```
-5*A*a**(3/2)*b*asinh(sqrt(a)/(sqrt(b)*x))/2 - A*a**2*sqrt(b)*sqrt(a/(b*x*
*2) + 1)/(2*x) + 2*A*a**2*sqrt(b)/(x*sqrt(a/(b*x**2) + 1)) + 2*A*a*b**(3/2
)*x/sqrt(a/(b*x**2) + 1) + A*b**2*Piecewise((a*sqrt(a + b*x**2)/(3*b) + x*
*2*sqrt(a + b*x**2)/3, Ne(b, 0)), (sqrt(a)*x**2/2, True)) - B*a**(5/2)/(x*
sqrt(1 + b*x**2/a)) - B*a**(3/2)*b*x/sqrt(1 + b*x**2/a) + B*a**2*sqrt(b)*a
sinh(sqrt(b)*x/sqrt(a)) + 2*B*a*b*Piecewise((a*Piecewise((log(2*sqrt(b)*sq
rt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))
/2 + x*sqrt(a + b*x**2)/2, Ne(b, 0)), (sqrt(a)*x, True)) + B*b**2*Piecewis
e((-a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b) + a*x*sqrt(a + b*x**2)/(8*b) +
x**3*sqrt(a + b*x**2)/4, Ne(b, 0)), (sqrt(a)*x**3/3, True))
```

3.21.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = \frac{5}{4}\left(bx^2+a\right)^{\frac{3}{2}}Bbx + \frac{15}{8}\sqrt{bx^2+a}Babx + \frac{15}{8}Ba^2\sqrt{b}\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right) - \frac{5}{2}Aa^{\frac{3}{2}}b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right) + \frac{5}{6}\left(bx^2+a\right)^{\frac{3}{2}}Ab + \frac{\left(bx^2+a\right)^{\frac{5}{2}}Ab}{2\,a} + \frac{5}{2}\sqrt{bx^2+a}Aab - \frac{\left(bx^2+a\right)^{\frac{5}{2}}B}{x} - \frac{\left(bx^2+a\right)^{\frac{7}{2}}A}{2\,ax^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
5/4*(b*x^2 + a)^(3/2)*B*b*x + 15/8*sqrt(b*x^2 + a)*B*a*b*x + 15/8*B*a^2*sq
rt(b)*arcsinh(b*x/sqrt(a*b)) - 5/2*A*a^(3/2)*b*arcsinh(a/(sqrt(a*b)*abs(x)
)) + 5/6*(b*x^2 + a)^(3/2)*A*b + 1/2*(b*x^2 + a)^(5/2)*A*b/a + 5/2*sqrt(b*
x^2 + a)*A*a*b - (b*x^2 + a)^(5/2)*B/x - 1/2*(b*x^2 + a)^(7/2)*A/(a*x^2)
```

3.21.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx = \frac{5\,Aa^2b\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{15}{8}\,Ba^2\sqrt{b}\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right) + \frac{1}{24}\left(56\,Aab+\left(27\,Bab+2\left(3\,Bb^2x+4\,Ab^2\right)x\right)x\right)\sqrt{bx^2+a} + \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Aa^2b+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^3\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aa^3b-2\,Ba^4\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2}$$

input
```
integrate((B*x+A)*(b*x^2+a)^(5/2)/x^3,x, algorithm="giac")
```

output
```
5*A*a^2*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) - 15/8*
B*a^2*sqrt(b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a))) + 1/24*(56*A*a*b + (2
7*B*a*b + 2*(3*B*b^2*x + 4*A*b^2)*x)*x)*sqrt(b*x^2 + a) + ((sqrt(b)*x - sq
rt(b*x^2 + a))^3*A*a^2*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a^3*sqrt(b)
 + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a^3*b - 2*B*a^4*sqrt(b))/((sqrt(b)*x -
sqrt(b*x^2 + a))^2 - a)^2
```

3.21.9 Mupad [B] (verification not implemented)

Time = 7.04 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(a+bx^2\right)^{5/2}}{x^3}\,dx=\frac{A\,b\,{\left(b\,x^2+a\right)}^{3/2}}{3}+2\,A\,a\,b\,\sqrt{b\,x^2+a}-\frac{A\,a^2\,\sqrt{b\,x^2+a}}{2\,x^2}-\frac{B\,{\left(b\,x^2+a\right)}^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},-\frac{1}{2};\ \frac{1}{2};\ -\frac{b\,x^2}{a}\right)}{x\,{\left(\frac{b\,x^2}{a}+1\right)}^{5/2}}+\frac{A\,a^{3/2}\,b\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{2}$$

input
```
int(((a + b*x^2)^(5/2)*(A + B*x))/x^3,x)
```

output
```
(A*b*(a + b*x^2)^(3/2))/3 + 2*A*a*b*(a + b*x^2)^(1/2) - (A*a^2*(a + b*x^2)
^(1/2))/(2*x^2) + (A*a^(3/2)*b*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/2
- (B*(a + b*x^2)^(5/2)*hypergeom([-5/2, -1/2], 1/2, -(b*x^2)/a))/(x*((b*x^
2)/a + 1)^(5/2))
```

3.22 $\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$

3.22.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{Ax^2\sqrt{a+bx^2}}{3b}+\frac{Bx^3\sqrt{a+bx^2}}{4b} - \frac{a(16A+9Bx)\sqrt{a+bx^2}}{24b^2}+\frac{3a^2B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
3/8*a^2*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)+1/3*A*x^2*(b*x^2+a)^(
1/2)/b+1/4*B*x^3*(b*x^2+a)^(1/2)/b-1/24*a*(9*B*x+16*A)*(b*x^2+a)^(1/2)/b^2
```

3.22.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.74

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-16aA-9aBx+8Abx^2+6bBx^3)}{24b^2} - \frac{3a^2B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-16*a*A - 9*a*B*x + 8*A*b*x^2 + 6*b*B*x^3))/(24*b^2) - (
3*a^2*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(8*b^(5/2))
```

3.22.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {533, 533, 25, 27, 533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\int \frac{x^2(3aB-4Abx)}{\sqrt{bx^2+a}}dx}{4b}$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{-\frac{\int -\frac{abx(8A+9Bx)}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{25}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{\int \frac{abx(8A+9Bx)}{\sqrt{bx^2+a}}dx}{3b}-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{1}{3}a\int \frac{x(8A+9Bx)}{\sqrt{bx^2+a}}dx-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{533}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{1}{3}a\left(\frac{9Bx\sqrt{a+bx^2}}{2b}-\frac{\int \frac{9aB-16Abx}{\sqrt{bx^2+a}}dx}{2b}\right)-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{455}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{1}{3}a\left(\frac{9Bx\sqrt{a+bx^2}}{2b}-\frac{9aB\int \frac{1}{\sqrt{bx^2+a}}dx-16A\sqrt{a+bx^2}}{2b}\right)-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{224}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{1}{3}a\left(\frac{9Bx\sqrt{a+bx^2}}{2b}-\frac{9aB\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-16A\sqrt{a+bx^2}}{2b}\right)-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^3\sqrt{a+bx^2}}{4b}-\frac{\frac{1}{3}a\left(\frac{9Bx\sqrt{a+bx^2}}{2b}-\frac{\frac{9aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-16A\sqrt{a+bx^2}}{2b}\right)-\frac{4}{3}Ax^2\sqrt{a+bx^2}}{4b}$$

input
```
Int[(x^3*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
(B*x^3*Sqrt[a + b*x^2])/(4*b) - ((-4*A*x^2*Sqrt[a + b*x^2])/3 + (a*((9*B*x
*Sqrt[a + b*x^2])/(2*b) - (-16*A*Sqrt[a + b*x^2] + (9*a*B*ArcTanh[(Sqrt[b]
*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b)))/3)/(4*b)
```

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.22.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.62

method	result	size
risch	$-\frac{\left(-6bB\,x^{3}-8Ab\,x^{2}+9Bax+16Aa\right)\sqrt{b\,x^{2}+a}}{24b^{2}}+\frac{3a^{2}B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{8b^{\frac{5}{2}}}$	65
default	$B\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+A\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)$	101

input
```
int(x^3*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*B*b*x^3-8*A*b*x^2+9*B*a*x+16*A*a)/b^2*(b*x^2+a)^(1/2)+3/8*a^2*B/
b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.22.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.52

$$\int\frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{9\,Ba^2\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab)\sqrt{bx^2+a}}{48\,b^3},\right.$$
$$\left.-\frac{9\,Ba^2\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(6\,Bb^2x^3+8\,Ab^2x^2-9\,Babx-16\,Aab)\sqrt{bx^2+a}}{24\,b^3}\right]$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(9*B*a^2*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2
*(6*B*b^2*x^3 + 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3,
-1/24*(9*B*a^2*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (6*B*b^2*x^3
+ 8*A*b^2*x^2 - 9*B*a*b*x - 16*A*a*b)*sqrt(b*x^2 + a))/b^3]
```

3.22.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3Ba^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b^2} + \sqrt{a+bx^2}\left(-\frac{2Aa}{3b^2}+\frac{Ax^2}{3b}-\frac{3Bax}{8b^2}+\frac{Bx^3}{4b}\right) & \text{for } b\neq 0 \\ \frac{\frac{Ax^4}{4}+\frac{Bx^5}{5}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*B*a**2*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqr
t(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2
)*(-2*A*a/(3*b**2) + A*x**2/(3*b) - 3*B*a*x/(8*b**2) + B*x**3/(4*b)), Ne(b
, 0)), ((A*x**4/4 + B*x**5/5)/sqrt(a), True))
```

3.22.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^3}{4\,b} + \frac{\sqrt{bx^2+a}Ax^2}{3\,b} - \frac{3\,\sqrt{bx^2+a}Bax}{8\,b^2} + \frac{3\,Ba^2\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{2\,\sqrt{bx^2+a}Aa}{3\,b^2}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*B*x^3/b + 1/3*sqrt(b*x^2 + a)*A*x^2/b - 3/8*sqrt(b*x^2
 + a)*B*a*x/b^2 + 3/8*B*a^2*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 2/3*sqrt(b*x^
2 + a)*A*a/b^2
```

3.22.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.71

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{24}\sqrt{bx^2+a}\left(\left(2\left(\frac{3\,Bx}{b}+\frac{4\,A}{b}\right)x-\frac{9\,Ba}{b^2}\right)x-\frac{16\,Aa}{b^2}\right) - \frac{3\,Ba^2\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(b*x^2 + a)*((2*(3*B*x/b + 4*A/b)*x - 9*B*a/b^2)*x - 16*A*a/b^2)
- 3/8*B*a^2*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^3\,(A+B\,x)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x^2)^(1/2), x)
```

3.23 $\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$

3.23.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{Bx^2\sqrt{a+bx^2}}{3b} - \frac{(4aB-3Abx)\sqrt{a+bx^2}}{6b^2} - \frac{aA\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
-1/2*a*A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/3*B*x^2*(b*x^2+a)^(1
/2)/b-1/6*(-3*A*b*x+4*B*a)*(b*x^2+a)^(1/2)/b^2
```

3.23.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-4aB+3Abx+2bBx^2)}{6b^2} - \frac{aA\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-4*a*B + 3*A*b*x + 2*b*B*x^2))/(6*b^2) - (a*A*ArcTanh[(S
qrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2)
```

3.23.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {533, 533, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 533$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\int \frac{x(2aB-3Abx)}{\sqrt{bx^2+a}}dx}{3b}$$

$$\Big\downarrow \ 533$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{-\frac{\int -\frac{ab(3A+4Bx)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \ 25$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\frac{\int \frac{ab(3A+4Bx)}{\sqrt{bx^2+a}}dx}{2b}-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \ 27$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\frac{1}{2}a\int \frac{3A+4Bx}{\sqrt{bx^2+a}}dx-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \ 455$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\frac{1}{2}a\left(3A\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{4B\sqrt{a+bx^2}}{b}\right)-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \ 224$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\frac{1}{2}a\left(3A\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{4B\sqrt{a+bx^2}}{b}\right)-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

$$\Big\downarrow \ 219$$

$$\frac{Bx^2\sqrt{a+bx^2}}{3b}-\frac{\frac{1}{2}a\left(\frac{3A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{4B\sqrt{a+bx^2}}{b}\right)-\frac{3}{2}Ax\sqrt{a+bx^2}}{3b}$$

input
```
Int[(x^2*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
(B*x^2*Sqrt[a + b*x^2])/(3*b) - ((-3*A*x*Sqrt[a + b*x^2])/2 + (a*((4*B*Sqr
t[a + b*x^2])/b + (3*A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/2)/
(3*b)
```

3.23.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.23.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{(2bB\,x^2+3Abx-4Ba)\sqrt{b\,x^2+a}}{6b^2}-\frac{aA\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}$	56
default	$B\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+A\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)$	77

input
```
int(x^2*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*B*b*x^2+3*A*b*x-4*B*a)/b^2*(b*x^2+a)^(1/2)-1/2*a*A/b^(3/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{3\,Aa\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,Bbx^2+3\,Abx-4\,Ba)\sqrt{bx^2+a}}{12\,b^2},\frac{3\,Aa\sqrt{-b}\arctan\left(\right.}{}\right.$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*A*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(
2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2 + a))/b^2, 1/6*(3*A*a*sqrt(-b)*arc
tan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (2*B*b*x^2 + 3*A*b*x - 4*B*a)*sqrt(b*x^2
 + a))/b^2]
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.26

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{Aa\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{Ax}{2b}-\frac{2Ba}{3b^2}+\frac{Bx^2}{3b}\right) & \text{for } b\neq 0\\ \frac{\frac{Ax^3}{3}+\frac{Bx^4}{4}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-A*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(A*x/
(2*b) - 2*B*a/(3*b**2) + B*x**2/(3*b)), Ne(b, 0)), ((A*x**3/3 + B*x**4/4)/
sqrt(a), True))
```

3.23.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx^2}{3\,b} + \frac{\sqrt{bx^2+a}Ax}{2\,b} - \frac{Aa\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} - \frac{2\,\sqrt{bx^2+a}Ba}{3\,b^2}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*B*x^2/b + 1/2*sqrt(b*x^2 + a)*A*x/b - 1/2*A*a*arcsinh(
b*x/sqrt(a*b))/b^(3/2) - 2/3*sqrt(b*x^2 + a)*B*a/b^2
```

3.23.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{6}\sqrt{bx^2+a}\left(\left(\frac{2\,Bx}{b}+\frac{3\,A}{b}\right)x-\frac{4\,Ba}{b^2}\right)+\frac{Aa\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(b*x^2 + a)*((2*B*x/b + 3*A/b)*x - 4*B*a/b^2) + 1/2*A*a*log(abs(-s
qrt(b)*x + sqrt(b*x^2 + a)))/b^(3/2)
```

3.23.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.15

$$\int \frac{x^2(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{3\,B\,x^4+4\,A\,x^3}{12\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{A\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}-\frac{B\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2} & \text{if } b\neq 0 \end{cases}$$

input
```
int((x^2*(A + B*x))/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (4*A*x^3 + 3*B*x^4)/(12*a^(1/2)), b ~= 0, - (A*a*log(2*b
^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (A*x*(a + b*x^2)^(1/2))/(2*
b) - (B*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2))
```

3.24 $\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx$

3.24.1 Optimal result

Integrand size = 18, antiderivative size = 56

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} - \frac{aB\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
-1/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+1/2*(B*x+2*A)*(b*x^2+a
)^(1/2)/b
```

3.24.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.04

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{(2A+Bx)\sqrt{a+bx^2}}{2b} + \frac{aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
((2*A + B*x)*Sqrt[a + b*x^2])/(2*b) + (a*B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(2*b^(3/2))
```

3.24.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {533, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \ 533$$

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{\int \frac{aB-2Abx}{\sqrt{bx^2+a}}dx}{2b}$$

$$\Big\downarrow \ 455$$

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{aB\int \frac{1}{\sqrt{bx^2+a}}dx-2A\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 224$$

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{aB\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2A\sqrt{a+bx^2}}{2b}$$

$$\Big\downarrow \ 219$$

$$\frac{Bx\sqrt{a+bx^2}}{2b}-\frac{\frac{aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-2A\sqrt{a+bx^2}}{2b}$$

input
```
Int[(x*(A + B*x))/Sqrt[a + b*x^2],x]
```

output
```
(B*x*Sqrt[a + b*x^2])/(2*b) - (-2*A*Sqrt[a + b*x^2] + (a*B*ArcTanh[(Sqrt[b
]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(2*b)
```

3.24.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] && !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.24.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{(Bx+2A)\sqrt{b\,x^{2}+a}}{2b}-\frac{aB\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}$	46
default	$B\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)+\frac{A\sqrt{b\,x^{2}+a}}{b}$	56

input
```
int(x*(B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x+2*A)*(b*x^2+a)^(1/2)/b-1/2*a/b^(3/2)*B*ln(x*b^(1/2)+(b*x^2+a)^(1/
2))
```

3.24.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bbx+2\,Ab)\sqrt{bx^2+a}}{4\,b^2}, \frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bbx\right.$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(B*a*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(B*b
*x + 2*A*b)*sqrt(b*x^2 + a))/b^2, 1/2*(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt
(b*x^2 + a)) + (B*b*x + 2*A*b)*sqrt(b*x^2 + a))/b^2]
```

3.24.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.55

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{Ba\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\sqrt{a+bx^2}\left(\frac{A}{b}+\frac{Bx}{2b}\right) & \text{for } b\neq 0\\ \frac{\frac{Ax^2}{2}+\frac{Bx^3}{3}}{\sqrt{a}} & \text{otherwise}\end{cases}$$

input
```
integrate(x*(B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-B*a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b)
, Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(A/b
+ B*x/(2*b)), Ne(b, 0)), ((A*x**2/2 + B*x**3/3)/sqrt(a), True))
```

3.24.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}Bx}{2b} - \frac{Ba\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2b^{\frac{3}{2}}} + \frac{\sqrt{bx^2+a}A}{b}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*B*x/b - 1/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(3/2) + sqrt(
b*x^2 + a)*A/b
```

3.24.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \frac{1}{2}\sqrt{bx^2+a}\left(\frac{Bx}{b}+\frac{2A}{b}\right) + \frac{Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*(B*x/b + 2*A/b) + 1/2*B*a*log(abs(-sqrt(b)*x + sqrt(b*
x^2 + a)))/b^(3/2)
```

3.24.9 Mupad [B] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{x(A+Bx)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{2\,B\,x^3+3\,A\,x^2}{6\,\sqrt{a}} & \text{if } b=0 \\ \frac{A\,\sqrt{b\,x^2+a}}{b} - \frac{B\,a\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}} + \frac{B\,x\,\sqrt{b\,x^2+a}}{2\,b} & \text{if } b\neq 0 \end{cases}$$

input
```
int((x*(A + B*x))/(a + b*x^2)^(1/2),x)
```

output
```
piecewise(b == 0, (3*A*x^2 + 2*B*x^3)/(6*a^(1/2)), b ~= 0, (A*(a + b*x^2)^
(1/2))/b - (B*a*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (B*x
*(a + b*x^2)^(1/2))/(2*b))
```

3.25 $\int \frac{A+Bx}{\sqrt{a+bx^2}} dx$

3.25.1 Optimal result

Integrand size = 17, antiderivative size = 43

$$\int \frac{A+Bx}{\sqrt{a+bx^2}} dx = \frac{B\sqrt{a+bx^2}}{b} + \frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

3.25.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{A+Bx}{\sqrt{a+bx^2}} dx = \frac{B\sqrt{a+bx^2}}{b} - \frac{A\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b - (A*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

3.25.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{455}$$

$$A\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \text{224}$$

$$A\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{B\sqrt{a+bx^2}}{b}$$

$$\Big\downarrow \text{219}$$

$$\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}$$

input
```
Int[(A + B*x)/Sqrt[a + b*x^2],x]
```

output
```
(B*Sqrt[a + b*x^2])/b + (A*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]
```

3.25.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

3.25.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}$	37
risch	$\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+\frac{B\sqrt{b\,x^{2}+a}}{b}$	37

input
```
int((B*x+A)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
A*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)+B*(b*x^2+a)^(1/2)/b
```

3.25.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.14

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \left[\frac{A\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,\sqrt{bx^2+a}B}{2\,b}, -\frac{A\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}B}{b}\right]$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(A*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*sqrt(b
*x^2 + a)*B)/b, -(A*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - sqrt(b*x
^2 + a)*B)/b]
```

3.25.6 Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.65

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \begin{cases} A\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right) + \frac{B\sqrt{a+bx^2}}{b} & \text{for } b\neq 0 \\ \frac{Ax+\frac{Bx^2}{2}}{\sqrt{a}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((A*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), N
e(a, 0)), (x*log(x)/sqrt(b*x**2), True)) + B*sqrt(a + b*x**2)/b, Ne(b, 0))
, ((A*x + B*x**2/2)/sqrt(a), True))
```

3.25.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{A\,\text{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
A*arcsinh(b*x/sqrt(a*b))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

3.25.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = -\frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{\sqrt{b}} + \frac{\sqrt{bx^2+a}B}{b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-A*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/sqrt(b) + sqrt(b*x^2 + a)*B/b
```

3.25.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{a+bx^2}}\,dx = \frac{B\sqrt{b\,x^2+a}}{b} + \frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}$$

input
```
int((A + B*x)/(a + b*x^2)^(1/2),x)
```

output
```
(B*(a + b*x^2)^(1/2))/b + (A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2)
```

3.26 $\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$

3.26.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx=\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^
(1/2))/b^(1/2)
```

3.26.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx=\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/Sqrt[a] - (B*Log[-(Sq
rt[b]*x) + Sqrt[a + b*x^2]])/Sqrt[b]
```

3.26.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{ 538}$$

$$A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{\sqrt{bx^2+a}}dx$$

$$\Big\downarrow \text{ 224}$$

$$A\int \frac{1}{x\sqrt{bx^2+a}}dx+B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}$$

$$\Big\downarrow \text{ 219}$$

$$A\int \frac{1}{x\sqrt{bx^2+a}}dx+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}+\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*x)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b] - (A*ArcTanh[Sqrt[a + b*x
^2]/Sqrt[a]])/Sqrt[a]
```

3.26.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

3.26.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{B\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}-\frac{A\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}$	52

input
```
int((B*x+A)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)-A/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2
+a)^(1/2))/x)
```

3.26.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.15

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \left[\frac{Ba\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,ab},\right.$$
$$-\frac{2\,Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{a}b\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)}{2\,ab}, \frac{2\,A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+Ba\sqrt{b}\log}{2\,ab}$$
$$\left.-\frac{Ba\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-A\sqrt{-a}b\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)}{ab}\right]$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + A*sqrt
(a)*b*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), -1/2*(2*
B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - A*sqrt(a)*b*log(-(b*x^2
- 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2))/(a*b), 1/2*(2*A*sqrt(-a)*b*arctan
(sqrt(-a)/sqrt(b*x^2 + a)) + B*a*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(b)*x - a))/(a*b), -(B*a*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) -
 A*sqrt(-a)*b*arctan(sqrt(-a)/sqrt(b*x^2 + a)))/(a*b)]
```

3.26.6 Sympy [A] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = -\frac{A\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}+B\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/x/(b*x**2+a)**(1/2),x)
```

output
```
-A*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + B*Piecewise((log(2*sqrt(b)*sqrt(a
+ b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2),
Ne(b, 0)), (x/sqrt(a), True))
```

3.26.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{A\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
B*arcsinh(b*x/sqrt(a*b))/sqrt(b) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a)
```

3.26.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.26.9 Mupad [B] (verification not implemented)

Time = 6.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x\sqrt{a+bx^2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
int((A + B*x)/(x*(a + b*x^2)^(1/2)),x)
```

output
```
(B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (A*atanh((a + b*x^2)^(1/2
)/a^(1/2)))/a^(1/2)
```

3.27 $\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx$

3.27.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)-A*(b*x^2+a)^(1/2)/a/x
```

3.27.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{ax}+\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/
Sqrt[a]])/Sqrt[a]
```

3.27.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 534}$$

$$B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*x)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a
]
```

3.27.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.27.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

method	result	size
default	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^{2}+a}}{a x}$	49
risch	$-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}-\frac{A\sqrt{b\,x^{2}+a}}{a x}$	49

input
```
int((B*x+A)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-B/a^(1/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)-A*(b*x^2+a)^(1/2)/a/x
```

3.27.5 Fricas [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = \left[\frac{B\sqrt{a}x\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\sqrt{bx^2+a}A}{2ax}, \frac{B\sqrt{-a}x\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-\sqrt{bx^2+a}A}{ax}\right]$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(B*sqrt(a)*x*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*
sqrt(b*x^2 + a)*A)/(a*x), (B*sqrt(-a)*x*arctan(sqrt(-a)/sqrt(b*x^2 + a)) -
 sqrt(b*x^2 + a)*A)/(a*x)]
```

3.27.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}-\frac{B\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}}$$

input
```
integrate((B*x+A)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/a - B*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a)
```

3.27.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{B\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}}-\frac{\sqrt{bx^2+a}A}{ax}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-B*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - sqrt(b*x^2 + a)*A/(a*x)
```

3.27.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2\,A\sqrt{b}}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/sqrt(-a) + 2*A*sqrt(b)
/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)
```

3.27.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx}{x^2\sqrt{a+bx^2}}\,dx = -\frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{A\,\sqrt{b\,x^2+a}}{a\,x}$$

input
```
int((A + B*x)/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
- (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (A*(a + b*x^2)^(1/2))/(a*
x)
```

3.28 $\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx$

3.28.1 Optimal result

Integrand size = 20, antiderivative size = 72

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=-\frac{A\sqrt{a+bx^2}}{2ax^2}-\frac{B\sqrt{a+bx^2}}{ax}+\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)-1/2*A*(b*x^2+a)^(1/2)/a/x
^2-B*(b*x^2+a)^(1/2)/a/x
```

3.28.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx=-\frac{(A+2Bx)\sqrt{a+bx^2}}{2ax^2}-\frac{Ab\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*((A + 2*B*x)*Sqrt[a + b*x^2])/(a*x^2) - (A*b*ArcTanh[(Sqrt[b]*x - Sqr
t[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

3.28.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {539, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{539}$$

$$-\frac{\int -\frac{2aB-Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2aB-Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{534}$$

$$\frac{-Ab\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{243}$$

$$\frac{-\frac{1}{2}Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{-A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{Ab\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}$$

input
```
Int[(A + B*x)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*x^2])/x + (A*b*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)
```

3.28.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 539
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[c*x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))
   Int[x^(m + 1)*(a + b*x^2)^p*(a*d*(m + 1) - b*c*(m + 2*p + 3)*x), x], x]
/; FreeQ[{a, b, c, d, p}, x] && ILtQ[m, -1] && GtQ[p, -1] && IntegerQ[2*p]
```

3.28.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2a\,x^{2}}+\frac{Ab\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}$	55
default	$-\frac{B\sqrt{b\,x^{2}+a}}{ax}+A\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)$	69

input
```
int((B*x+A)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(2*B*x+A)/a/x^2+1/2*A/a^(3/2)*b*ln((2*a+2*a^(1/2)*(b*
x^2+a)^(1/2))/x)
```

3.28.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.71

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = \left[\frac{A\sqrt{a}bx^2\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,Bax+Aa)\sqrt{bx^2+a}}{4\,a^2x^2},\ -\frac{A\sqrt{-a}bx^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(2\,Bax+Aa)\sqrt{bx^2+a}}{2\,a^2x^2}\right]$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(A*sqrt(a)*b*x^2*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2)
- 2*(2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2), -1/2*(A*sqrt(-a)*b*x^2*arc
tan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*x^2)
]
```

3.28.6 Sympy [A] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = -\frac{A\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{Ab\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{B\sqrt{b}\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate((B*x+A)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
-A*sqrt(b)*sqrt(a/(b*x**2) + 1)/(2*a*x) + A*b*asinh(sqrt(a)/(sqrt(b)*x))/(
2*a**(3/2)) - B*sqrt(b)*sqrt(a/(b*x**2) + 1)/a
```

3.28.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = \frac{Ab\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{\sqrt{bx^2+a}B}{ax} - \frac{\sqrt{bx^2+a}A}{2\,ax^2}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) - sqrt(b*x^2 + a)*B/(a*x) -
1/2*sqrt(b*x^2 + a)*A/(a*x^2)
```

3.28.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(58) = 116$.

Time = 0.30 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.03

$$\begin{aligned}\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx &= -\frac{Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}\\ &+ \frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3 Ab + 2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2 Ba\sqrt{b} + \left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab - 2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2 a}\end{aligned}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a) + ((sqrt
(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqr
t(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x
 - sqrt(b*x^2 + a))^2 - a)^2*a)
```

3.28.9 Mupad [B] (verification not implemented)

Time = 5.81 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^3\sqrt{a+bx^2}}\,dx = \frac{A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{B\,\sqrt{b\,x^2+a}}{a\,x} - \frac{A\,\sqrt{b\,x^2+a}}{2\,a\,x^2}$$

input
```
int((A + B*x)/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) - (B*(a + b*x^2)^(1/2))
/(a*x) - (A*(a + b*x^2)^(1/2))/(2*a*x^2)
```

3.29 $\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

3.29.1 Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx=-\frac{x^2(A+Bx)}{b\sqrt{a+bx^2}}+\frac{(4A+3Bx)\sqrt{a+bx^2}}{2b^2}-\frac{3aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{5/2}}$$

output
```
-3/2*a*B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-x^2*(B*x+A)/b/(b*x^2+a
)^(1/2)+1/2*(3*B*x+4*A)*(b*x^2+a)^(1/2)/b^2
```

3.29.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx=\frac{4aA+3aBx+2Abx^2+bBx^3}{2b^2\sqrt{a+bx^2}}+\frac{3aB\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{5/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
(4*a*A + 3*a*B*x + 2*A*b*x^2 + b*B*x^3)/(2*b^2*Sqrt[a + b*x^2]) + (3*a*B*L
og[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(5/2))
```

3.29.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {530, 2346, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{530}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\int \frac{\frac{Ba^2}{b^2}-\frac{Bx^2a}{b}-\frac{Axa}{b}}{\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{2346}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\frac{\int \frac{a(3aB-2Abx)}{b\sqrt{bx^2+a}}dx}{2b}-\frac{aBx\sqrt{a+bx^2}}{2b^2}}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\frac{a\int \frac{3aB-2Abx}{\sqrt{bx^2+a}}dx}{2b^2}-\frac{aBx\sqrt{a+bx^2}}{2b^2}}{a}$$

$$\Big\downarrow \text{455}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\frac{a\left(3aB\int \frac{1}{\sqrt{bx^2+a}}dx-2A\sqrt{a+bx^2}\right)}{2b^2}-\frac{aBx\sqrt{a+bx^2}}{2b^2}}{a}$$

$$\Big\downarrow \text{224}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\frac{a\left(3aB\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-2A\sqrt{a+bx^2}\right)}{2b^2}-\frac{aBx\sqrt{a+bx^2}}{2b^2}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{a(A+Bx)}{b^2\sqrt{a+bx^2}}-\frac{\frac{a\left(\frac{3aB\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-2A\sqrt{a+bx^2}\right)}{2b^2}-\frac{aBx\sqrt{a+bx^2}}{2b^2}}{a}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
(a*(A + B*x))/(b^2*Sqrt[a + b*x^2]) - (-1/2*(a*B*x*Sqrt[a + b*x^2])/b^2 +
(a*(-2*A*Sqrt[a + b*x^2] + (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sq
rt[b]))/(2*b^2))/a
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.29.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

method	result	size
risch	$\frac{(Bx+2A)\sqrt{b\,x^{2}+a}}{2b^{2}}+\frac{aBx}{b^{2}\sqrt{b\,x^{2}+a}}-\frac{3aB\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{5}{2}}}+\frac{aA}{b^{2}\sqrt{b\,x^{2}+a}}$	77
default	$B\left(\frac{x^{3}}{2b\sqrt{b\,x^{2}+a}}-\frac{3a\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)}{2b}\right)+A\left(\frac{x^{2}}{\sqrt{b\,x^{2}+a}\,b}+\frac{2a}{b^{2}\sqrt{b\,x^{2}+a}}\right)$	98

input
```
int(x^3*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x+2*A)/b^2*(b*x^2+a)^(1/2)+a/b^2*B*x/(b*x^2+a)^(1/2)-3/2*a/b^(5/2)*
B*ln(x*b^(1/2)+(b*x^2+a)^(1/2))+a/b^2*A/(b*x^2+a)^(1/2)
```

3.29.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.43

$$\int\frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx=\left[\frac{3\left(Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2+2\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\left(Bb^2x^3+2\,Ab^2x^2+3\,B\right.}{4\left(b^4x^2+ab^3\right)}\right.$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(
b)*x - a) + 2*(B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqrt(b*x^2 +
 a))/(b^4*x^2 + a*b^3), 1/2*(3*(B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b
)*x/sqrt(b*x^2 + a)) + (B*b^2*x^3 + 2*A*b^2*x^2 + 3*B*a*b*x + 4*A*a*b)*sqr
t(b*x^2 + a))/(b^4*x^2 + a*b^3)]
```

3.29.6 Sympy [A] (verification not implemented)

Time = 3.63 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.44

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\begin{cases} \frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{3\sqrt{a}x}{2b^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{\frac{5}{2}}} + \frac{x^3}{2\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**3*(B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(b
, 0)), (x**4/(4*a**(3/2)), True)) + B*(3*sqrt(a)*x/(2*b**2*sqrt(1 + b*x**2
/a)) - 3*a*asinh(sqrt(b)*x/sqrt(a))/(2*b**(5/2)) + x**3/(2*sqrt(a)*b*sqrt(
1 + b*x**2/a)))
```

3.29.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.05

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^3}{2\sqrt{bx^2+a}b} + \frac{Ax^2}{\sqrt{bx^2+a}b} + \frac{3\,Bax}{2\sqrt{bx^2+a}b^2} - \frac{3\,Ba\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{5}{2}}} + \frac{2\,Aa}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*x^3/(sqrt(b*x^2 + a)*b) + A*x^2/(sqrt(b*x^2 + a)*b) + 3/2*B*a*x/(sqr
t(b*x^2 + a)*b^2) - 3/2*B*a*arcsinh(b*x/sqrt(a*b))/b^(5/2) + 2*A*a/(sqrt(b
*x^2 + a)*b^2)
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\left(\frac{Bx}{b}+\frac{2A}{b}\right)x+\frac{3Ba}{b^2}\right)x+\frac{4Aa}{b^2}}{2\sqrt{bx^2+a}}+\frac{3Ba\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2b^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
1/2*(((B*x/b + 2*A/b)*x + 3*B*a/b^2)*x + 4*A*a/b^2)/sqrt(b*x^2 + a) + 3/2*
B*a*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(b\,x^2+a)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x^2)^(3/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x^2)^(3/2), x)
```

3.30 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

3.30.1 Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx=-\frac{x(A+Bx)}{b\sqrt{a+bx^2}}+\frac{2B\sqrt{a+bx^2}}{b^2}+\frac{A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
A*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-x*(B*x+A)/b/(b*x^2+a)^(1/2)+2
*B*(b*x^2+a)^(1/2)/b^2
```

3.30.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx=\frac{2aB-Abx+bBx^2}{b^2\sqrt{a+bx^2}}+\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
(2*a*B - A*b*x + b*B*x^2)/(b^2*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x)
/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(3/2)
```

3.30.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {530, 25, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{530}$$

$$\frac{aB-Abx}{b^2\sqrt{a+bx^2}}-\frac{\int -\frac{a(A+Bx)}{b\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(A+Bx)}{b\sqrt{bx^2+a}}dx}{a}+\frac{aB-Abx}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+Bx}{\sqrt{bx^2+a}}dx}{b}+\frac{aB-Abx}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{455}$$

$$\frac{A\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{B\sqrt{a+bx^2}}{b}}{b}+\frac{aB-Abx}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{A\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{B\sqrt{a+bx^2}}{b}}{b}+\frac{aB-Abx}{b^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{A\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{B\sqrt{a+bx^2}}{b}}{b}+\frac{aB-Abx}{b^2\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
(a*B - A*b*x)/(b^2*Sqrt[a + b*x^2]) + ((B*Sqrt[a + b*x^2])/b + (A*ArcTanh[
(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/b
```

3.30.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

3.30.4 Maple [A] (verified)

Time = 3.53 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
risch	$\frac{B\sqrt{b\,x^{2}+a}}{b^{2}}-\frac{A x}{b\sqrt{b\,x^{2}+a}}+\frac{A\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}+\frac{a B}{b^{2}\sqrt{b\,x^{2}+a}}$	68
default	$B\left(\frac{x^{2}}{\sqrt{b\,x^{2}+a}\,b}+\frac{2 a}{b^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)$	74

input
```
int(x^2*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*(b*x^2+a)^(1/2)/b^2-1/b*A*x/(b*x^2+a)^(1/2)+1/b^(3/2)*A*ln(x*b^(1/2)+(b*
x^2+a)^(1/2))+a/b^2*B/(b*x^2+a)^(1/2)
```

3.30.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.48

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Abx^2+Aa)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(Bbx^2-Abx+2\,Ba)\sqrt{bx^2+a}}{2\,(b^3x^2+ab^2)}, -\frac{(Abx^2+Aa)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(Bbx^2-Abx+2\,Ba)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((A*b*x^2 + A*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) + 2*(B*b*x^2 - A*b*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((A
*b*x^2 + A*a)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (B*b*x^2 - A*b
*x + 2*B*a)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2)]
```

3.30.6 Sympy [A] (verification not implemented)

Time = 2.71 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.26

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}\frac{2a}{b^2\sqrt{a+bx^2}} + \frac{x^2}{b\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^4}{4a^{\frac{3}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate(x**2*(B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a))) +
 B*Piecewise((2*a/(b**2*sqrt(a + b*x**2)) + x**2/(b*sqrt(a + b*x**2)), Ne(
b, 0)), (x**4/(4*a**(3/2)), True))
```

3.30.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{Bx^2}{\sqrt{bx^2+a}b} - \frac{Ax}{\sqrt{bx^2+a}b} + \frac{A\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} + \frac{2\,Ba}{\sqrt{bx^2+a}b^2}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*x^2/(sqrt(b*x^2 + a)*b) - A*x/(sqrt(b*x^2 + a)*b) + A*arcsinh(b*x/sqrt(a
*b))/b^(3/2) + 2*B*a/(sqrt(b*x^2 + a)*b^2)
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{\left(\frac{Bx}{b} - \frac{A}{b}\right)x + \frac{2\,Ba}{b^2}}{\sqrt{bx^2+a}} - \frac{A\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
((B*x/b - A/b)*x + 2*B*a/b^2)/sqrt(b*x^2 + a) - A*log(abs(-sqrt(b)*x + sqr
t(b*x^2 + a)))/b^(3/2)
```

3.30.9 Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{A\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{A\,x}{b\,\sqrt{b\,x^2+a}} + \frac{B\,(b\,x^2+2\,a)}{b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((x^2*(A + B*x))/(a + b*x^2)^(3/2),x)
```

output
```
(A*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - (A*x)/(b*(a + b*x^2)^(1/2
)) + (B*(2*a + b*x^2))/(b^2*(a + b*x^2)^(1/2))
```

3.31 $\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx$

3.31.1 Optimal result

Integrand size = 18, antiderivative size = 48

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{A+Bx}{b\sqrt{a+bx^2}} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}$$

output
```
B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)+(-B*x-A)/b/(b*x^2+a)^(1/2)
```

3.31.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{-A-Bx}{b\sqrt{a+bx^2}} - \frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
(-A - B*x)/(b*Sqrt[a + b*x^2]) - (B*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b
^(3/2)
```

3.31.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {530, 25, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}dx$$

$$\Big\downarrow \text{530}$$

$$-\frac{\int -\frac{aB}{b\sqrt{bx^2+a}}dx}{a}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aB}{b\sqrt{bx^2+a}}dx}{a}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{224}$$

$$\frac{B\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{A+Bx}{b\sqrt{a+bx^2}}$$

input
```
Int[(x*(A + B*x))/(a + b*x^2)^(3/2),x]
```

output
```
-((A + B*x)/(b*Sqrt[a + b*x^2])) + (B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]]
)/b^(3/2)
```

3.31.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

3.31.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

method	result	size
default	$B\left(-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}\right)-\frac{A}{b\sqrt{b\,x^{2}+a}}$	55

input
```
int(x*(B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2)))-1/b*A/(b*
x^2+a)^(1/2)
```

3.31.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \left[\frac{(Bbx^2+Ba)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(Bbx+Ab)\sqrt{bx^2+a}}{2\,(b^3x^2+ab^2)},\right.$$
$$\left.-\frac{(Bbx^2+Ba)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(Bbx+Ab)\sqrt{bx^2+a}}{b^3x^2+ab^2}\right]$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*b*x^2 + B*a)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x -
 a) - 2*(B*b*x + A*b)*sqrt(b*x^2 + a))/(b^3*x^2 + a*b^2), -((B*b*x^2 + B*a
)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (B*b*x + A*b)*sqrt(b*x^2 +
 a))/(b^3*x^2 + a*b^2)]
```

3.31.6 Sympy [A] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.38

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = A\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{\frac{3}{2}}} - \frac{x}{\sqrt{a}b\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x*(B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)
) + B*(asinh(sqrt(b)*x/sqrt(a))/b**(3/2) - x/(sqrt(a)*b*sqrt(1 + b*x**2/a)
))
```

3.31.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{Bx}{\sqrt{bx^2+ab}} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}} - \frac{A}{\sqrt{bx^2+ab}}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-B*x/(sqrt(b*x^2 + a)*b) + B*arcsinh(b*x/sqrt(a*b))/b^(3/2) - A/(sqrt(b*x^
2 + a)*b)
```

3.31.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{Bx}{b}+\frac{A}{b}}{\sqrt{bx^2+a}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B*x/b + A/b)/sqrt(b*x^2 + a) - B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/
b^(3/2)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{x(A+Bx)}{(a+bx^2)^{3/2}}\,dx = \frac{B\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{b^{3/2}} - \frac{A}{b\,\sqrt{b\,x^2+a}} - \frac{B\,x}{b\,\sqrt{b\,x^2+a}}$$

input
```
int((x*(A + B*x))/(a + b*x^2)^(3/2),x)
```

output
```
(B*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(3/2) - A/(b*(a + b*x^2)^(1/2)) -
 (B*x)/(b*(a + b*x^2)^(1/2))
```

3.32 $\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx$

3.32.1 Optimal result

Integrand size = 17, antiderivative size = 28

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

output
```
(A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{-aB+Abx}{ab\sqrt{a+bx^2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
(-(a*B) + A*b*x)/(a*b*Sqrt[a + b*x^2])
```

3.32.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{453}$$

$$-\frac{aB-Abx}{ab\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(3/2),x]
```

output
```
-((a*B - A*b*x)/(a*b*Sqrt[a + b*x^2]))
```

3.32.3.1 Defintions of rubi rules used

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

3.32.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
gosper	$\frac{Abx-Ba}{ab\sqrt{b\,x^2+a}}$	26
trager	$\frac{Abx-Ba}{ab\sqrt{b\,x^2+a}}$	26
default	$\frac{Ax}{a\sqrt{b\,x^2+a}}-\frac{B}{b\sqrt{b\,x^2+a}}$	32

input
```
int((B*x+A)/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(A*b*x-B*a)/a/b/(b*x^2+a)^(1/2)
```

3.32.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{(Abx-Ba)\sqrt{bx^2+a}}{ab^2x^2+a^2b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
(A*b*x - B*a)*sqrt(b*x^2 + a)/(a*b^2*x^2 + a^2*b)
```

3.32.6 Sympy [A] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(3/2),x)
```

output
```
A*x/(a**(3/2)*sqrt(1 + b*x**2/a)) + B*Piecewise((-1/(b*sqrt(a + b*x**2)),
Ne(b, 0)), (x**2/(2*a**(3/2)), True))
```

3.32.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{Ax}{\sqrt{bx^2+a}\,a} - \frac{B}{\sqrt{bx^2+a}\,b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
A*x/(sqrt(b*x^2 + a)*a) - B/(sqrt(b*x^2 + a)*b)
```

3.32.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = \frac{\frac{Ax}{a}-\frac{B}{b}}{\sqrt{bx^2+a}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(A*x/a - B/b)/sqrt(b*x^2 + a)
```

3.32.9 Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{(a+bx^2)^{3/2}}\,dx = -\frac{\frac{B}{b}-\frac{A\,x}{a}}{\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x)/(a + b*x^2)^(3/2),x)
```

output
```
-(B/b - (A*x)/a)/(a + b*x^2)^(1/2)
```

3.33 $\int \frac{A+Bx}{x(a+bx^2)^{3/2}}\,dx$

3.33.1 Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/(b*x^2+a)^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx=\frac{A+Bx}{a\sqrt{a+bx^2}}+\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A + B*x)/(a*Sqrt[a + b*x^2]) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])
/Sqrt[a]])/a^(3/2)
```

3.33.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {532, 25, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{\int -\frac{A}{x\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{A}{x\sqrt{bx^2+a}}dx}{a}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{A\int \frac{1}{x\sqrt{bx^2+a}}dx}{a}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{A+Bx}{a\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{A+Bx}{a\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x^2)^(3/2)),x]
```

output
```
(A + B*x)/(a*Sqrt[a + b*x^2]) - (A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/
2)
```

3.33.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.33.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{Bx}{a\sqrt{b\,x^{2}+a}}+A\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)$	61

input
```
int((B*x+A)/x/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
B*x/a/(b*x^2+a)^(1/2)+A*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(
b*x^2+a)^(1/2))/x))
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.11

$$\int \frac{A+Bx}{x\,(a+bx^2)^{3/2}}\,dx = \left[\frac{(Abx^2+Aa)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(Bax+Aa)\sqrt{bx^2+a}}{2\,(a^2bx^2+a^3)}, \frac{(Abx^2+Aa)\cdots}{}\right.$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((A*b*x^2 + A*a)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*
a)/x^2) + 2*(B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^2 + a^3), ((A*b*x^2 +
A*a)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (B*a*x + A*a)*sqrt(b*x^2
+ a))/(a^2*b*x^2 + a^3)]
```

3.33.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(39) = 78$.

Time = 3.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.38

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx = A\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}+\frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}-\frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)+\frac{Bx}{a^{\frac{3}{2}}\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate((B*x+A)/x/(b*x**2+a)**(3/2),x)
```

output
```
A*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**3*log(b
*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1 + b*x**2/a)
+ 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**2/a)/(2*a**(9
/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(2*a*
*(9/2) + 2*a**(7/2)*b*x**2)) + B*x/(a**(3/2)*sqrt(1 + b*x**2/a))
```

3.33.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{3/2}}\,dx = \frac{Bx}{\sqrt{bx^2+a}a}-\frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}}+\frac{A}{\sqrt{bx^2+a}a}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
B*x/(sqrt(b*x^2 + a)*a) - A*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + A/(sqr
t(b*x^2 + a)*a)
```

3.33.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx}{x\,(a+bx^2)^{3/2}}\,dx=\frac{\frac{Bx}{a}+\frac{A}{a}}{\sqrt{bx^2+a}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
(B*x/a + A/a)/sqrt(b*x^2 + a) + 2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/
sqrt(-a))/(sqrt(-a)*a)
```

3.33.9 Mupad [B] (verification not implemented)

Time = 6.49 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x\,(a+bx^2)^{3/2}}\,dx=\frac{A}{a\sqrt{b\,x^2+a}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{B\,x}{a\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x)/(x*(a + b*x^2)^(3/2)),x)
```

output
```
A/(a*(a + b*x^2)^(1/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) + (
B*x)/(a*(a + b*x^2)^(1/2))
```

3.34 $\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 70

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = \frac{A+Bx}{ax\sqrt{a+bx^2}} - \frac{2A\sqrt{a+bx^2}}{a^2x} - \frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

output
```
-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(3/2)+(B*x+A)/a/x/(b*x^2+a)^(1/2)-2*
A*(b*x^2+a)^(1/2)/a^2/x
```

3.34.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = \frac{-aA+aBx-2Abx^2}{a^2x\sqrt{a+bx^2}} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a*A) + a*B*x - 2*A*b*x^2)/(a^2*x*Sqrt[a + b*x^2]) + (2*B*ArcTanh[(Sqrt[
b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(3/2)
```

3.34.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {532, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2 (a+bx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 532}$$

$$\frac{aB-Abx}{a^2\sqrt{a+bx^2}}-\frac{\int -\frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{aB-Abx}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{aB-Abx}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{aB-Abx}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}}{a}+\frac{aB-Abx}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{aB-Abx}{a^2\sqrt{a+bx^2}}+\frac{-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}}{a}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x^2)^(3/2)),x]
```

output
```
(a*B - A*b*x)/(a^2*Sqrt[a + b*x^2]) + (-((A*Sqrt[a + b*x^2])/(a*x)) - (B*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/a
```

3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

3.34.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

method	result	size
risch	$-\frac{A\sqrt{b\,x^{2}+a}}{a^{2}x}-\frac{Abx}{a^{2}\sqrt{b\,x^{2}+a}}+\frac{B}{a\sqrt{b\,x^{2}+a}}-\frac{B\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}$	80
default	$B\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)+A\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)$	82

input
```
int((B*x+A)/x^2/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^2*A*(b*x^2+a)^(1/2)/x-1/a^2*A*b*x/(b*x^2+a)^(1/2)+B/a/(b*x^2+a)^(1/2)
-1/a^(3/2)*B*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.34.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.41

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx=\left[\frac{(Bbx^3+Bax)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(2\,Abx^2-Bax+Aa\right)\sqrt{bx^2+a}}{2\left(a^2bx^3+a^3x\right)},\ldots\right.$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/2*((B*b*x^3 + B*a*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) +
2*a)/x^2) - 2*(2*A*b*x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*
x), ((B*b*x^3 + B*a*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*A*b*
x^2 - B*a*x + A*a)*sqrt(b*x^2 + a))/(a^2*b*x^3 + a^3*x)]
```

3.34.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(60) = 120$.

Time = 4.15 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.36

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = A\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}} - \frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right) + B\left(\frac{2a^3\sqrt{1+\frac{bx^2}{a}}}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^3\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^3\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} + \frac{a^2bx^2\log\left(\frac{bx^2}{a}\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2} - \frac{2a^2bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{2a^{\frac{9}{2}}+2a^{\frac{7}{2}}bx^2}\right)$$

input
```
integrate((B*x+A)/x**2/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(a*sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x*
*2) + 1))) + B*(2*a**3*sqrt(1 + b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2)
 + a**3*log(b*x**2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**3*log(sqrt(1
 + b*x**2/a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) + a**2*b*x**2*log(b*x**
2/a)/(2*a**(9/2) + 2*a**(7/2)*b*x**2) - 2*a**2*b*x**2*log(sqrt(1 + b*x**2/
a) + 1)/(2*a**(9/2) + 2*a**(7/2)*b*x**2))
```

3.34.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^2(a+bx^2)^{3/2}}\,dx = -\frac{2\,Abx}{\sqrt{bx^2+a}a^2} - \frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{3}{2}}} + \frac{B}{\sqrt{bx^2+a}a} - \frac{A}{\sqrt{bx^2+a}ax}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*A*b*x/(sqrt(b*x^2 + a)*a^2) - B*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) +
 B/(sqrt(b*x^2 + a)*a) - A/(sqrt(b*x^2 + a)*a*x)
```

3.34.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{Abx}{a^2}-\frac{B}{a}}{\sqrt{bx^2+a}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(A*b*x/a^2 - B/a)/sqrt(b*x^2 + a) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 +
 a))/sqrt(-a))/(sqrt(-a)*a) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^
2 - a)*a)
```

3.34.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{3/2}}\,dx = \frac{B}{a\sqrt{b\,x^2+a}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{A}{a\,x\sqrt{b\,x^2+a}} - \frac{2\,A\,b\,x}{a^2\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x)/(x^2*(a + b*x^2)^(3/2)),x)
```

output
```
B/(a*(a + b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(3/2) - A
/(a*x*(a + b*x^2)^(1/2)) - (2*A*b*x)/(a^2*(a + b*x^2)^(1/2))
```

3.35 $\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx$

3.35.1 Optimal result

Integrand size = 20, antiderivative size = 95

$$\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx=\frac{A+Bx}{ax^2\sqrt{a+bx^2}}-\frac{3A\sqrt{a+bx^2}}{2a^2x^2}-\frac{2B\sqrt{a+bx^2}}{a^2x}+\frac{3Ab\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}$$

output
```
3/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+(B*x+A)/a/x^2/(b*x^2+a)^(
1/2)-3/2*A*(b*x^2+a)^(1/2)/a^2/x^2-2*B*(b*x^2+a)^(1/2)/a^2/x
```

3.35.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^3(a+bx^2)^{3/2}}\,dx=\frac{-a(A+2Bx)-bx^2(3A+4Bx)}{2a^2x^2\sqrt{a+bx^2}}-\frac{3Ab\,\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
(-(a*(A + 2*B*x)) - b*x^2*(3*A + 4*B*x))/(2*a^2*x^2*Sqrt[a + b*x^2]) - (3*
A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

3.35.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {532, 25, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{-\frac{Abx^2}{a}+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{Abx^2}{a}+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{-\frac{\int -\frac{2aB-3Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2aB-3Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{-3Ab\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{-\frac{3}{2}Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{-3A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

$$\Big\downarrow 221$$

$$\frac{\frac{\frac{3Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}}{a}-\frac{b(A+Bx)}{a^2\sqrt{a+bx^2}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x^2)^(3/2)),x]
```

output
```
-((b*(A + B*x))/(a^2*Sqrt[a + b*x^2])) + (-1/2*(A*Sqrt[a + b*x^2])/(a*x^2)
 + ((-2*B*Sqrt[a + b*x^2])/x + (3*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sq
rt[a])/(2*a))/a
```

3.35.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.35.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(2Bx+A)}{2a^{2}x^{2}}-\frac{bBx}{a^{2}\sqrt{b\,x^{2}+a}}-\frac{Ab}{a^{2}\sqrt{b\,x^{2}+a}}+\frac{3bA\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{5}{2}}}$	88
default	$B\left(-\frac{1}{ax\sqrt{b\,x^{2}+a}}-\frac{2bx}{a^{2}\sqrt{b\,x^{2}+a}}\right)+A\left(-\frac{1}{2a\,x^{2}\sqrt{b\,x^{2}+a}}-\frac{3b\left(\frac{1}{a\sqrt{b\,x^{2}+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{a^{\frac{3}{2}}}\right)}{2a}\right)$	106

input
```
int((B*x+A)/x^3/(b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)^(1/2)*(2*B*x+A)/a^2/x^2-b/a^2*B*x/(b*x^2+a)^(1/2)-1/a^2*A/(
b*x^2+a)^(1/2)*b+3/2*b/a^(5/2)*A*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \left[\frac{3\left(Ab^2x^4+Aabx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)-2\left(4Babx^3+3Aabx^2+2Ba^2x\right.}{4\left(a^3bx^4+a^4x^2\right)}\right.$$
$$\left.-\frac{3\left(Ab^2x^4+Aabx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(4Babx^3+3Aabx^2+2Ba^2x+Aa^2\right)\sqrt{bx^2+a}}{2\left(a^3bx^4+a^4x^2\right)}\right]$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sq
rt(a) + 2*a)/x^2) - 2*(4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2*x + A*a^2)*sqrt
(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2), -1/2*(3*(A*b^2*x^4 + A*a*b*x^2)*sqrt(-
a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (4*B*a*b*x^3 + 3*A*a*b*x^2 + 2*B*a^2
*x + A*a^2)*sqrt(b*x^2 + a))/(a^3*b*x^4 + a^4*x^2)]
```

3.35.6 Sympy [A] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = A\left(-\frac{1}{2a\sqrt{b}x^3\sqrt{\frac{a}{bx^2}+1}}-\frac{3\sqrt{b}}{2a^2x\sqrt{\frac{a}{bx^2}+1}}+\frac{3b\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{5}{2}}}\right)$$
$$+B\left(-\frac{1}{a\sqrt{b}x^2\sqrt{\frac{a}{bx^2}+1}}-\frac{2\sqrt{b}}{a^2\sqrt{\frac{a}{bx^2}+1}}\right)$$

input
```
integrate((B*x+A)/x**3/(b*x**2+a)**(3/2),x)
```

output
```
A*(-1/(2*a*sqrt(b)*x**3*sqrt(a/(b*x**2) + 1)) - 3*sqrt(b)/(2*a**2*x*sqrt(a
/(b*x**2) + 1)) + 3*b*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(5/2))) + B*(-1/(a*
sqrt(b)*x**2*sqrt(a/(b*x**2) + 1)) - 2*sqrt(b)/(a**2*sqrt(a/(b*x**2) + 1))
)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{2\,Bbx}{\sqrt{bx^2+a}a^2}+\frac{3\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{5}{2}}}-\frac{3\,Ab}{2\,\sqrt{bx^2+a}a^2}-\frac{B}{\sqrt{bx^2+a}ax}-\frac{A}{2\,\sqrt{bx^2+a}ax^2}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="maxima")
```

output
```
-2*B*b*x/(sqrt(b*x^2 + a)*a^2) + 3/2*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(
5/2) - 3/2*A*b/(sqrt(b*x^2 + a)*a^2) - B/(sqrt(b*x^2 + a)*a*x) - 1/2*A/(sq
rt(b*x^2 + a)*a*x^2)
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(79) = 158$.

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.80

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = -\frac{\frac{Bbx}{a^2}+\frac{Ab}{a^2}}{\sqrt{bx^2+a}}-\frac{3\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^2}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(3/2),x, algorithm="giac")
```

output
```
-(B*b*x/a^2 + A*b/a^2)/sqrt(b*x^2 + a) - 3*A*b*arctan(-(sqrt(b)*x - sqrt(b
*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b
 + 2*(sqrt(b)*x - sqrt(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2
 + a))*A*a*b - 2*B*a^2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a
^2)
```

3.35.9 Mupad [B] (verification not implemented)

Time = 5.94 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{3/2}}\,dx = \frac{3\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{5/2}} - \frac{3\,A\,b}{2\,a^2\,\sqrt{b\,x^2+a}} - \frac{A}{2\,a\,x^2\,\sqrt{b\,x^2+a}} - \frac{\sqrt{b\,x^2+a}\,\left(\frac{B}{a}+\frac{2\,B\,b\,x^2}{a^2}\right)}{b\,x^3+a\,x}$$

input
```
int((A + B*x)/(x^3*(a + b*x^2)^(3/2)),x)
```

output
```
(3*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(5/2)) - (3*A*b)/(2*a^2*(a +
 b*x^2)^(1/2)) - A/(2*a*x^2*(a + b*x^2)^(1/2)) - ((a + b*x^2)^(1/2)*(B/a +
 (2*B*b*x^2)/a^2))/(a*x + b*x^3)
```

3.36 $\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

3.36.1 Optimal result

Integrand size = 20, antiderivative size = 79

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2(A+Bx)}{3b\,(a+bx^2)^{3/2}} - \frac{2A+3Bx}{3b^2\sqrt{a+bx^2}} + \frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}$$

output
```
-1/3*x^2*(B*x+A)/b/(b*x^2+a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^
(5/2)+1/3*(-3*B*x-2*A)/b^2/(b*x^2+a)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.91

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{-2aA-3aBx-3Abx^2-4bBx^3}{3b^2\,(a+bx^2)^{3/2}} - \frac{B\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{5/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a*A - 3*a*B*x - 3*A*b*x^2 - 4*b*B*x^3)/(3*b^2*(a + b*x^2)^(3/2)) - (B*
Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(5/2)
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {530, 2345, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 530$$

$$\frac{a(A+Bx)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{\frac{Ba^2}{b^2}-\frac{3Bx^2a}{b}-\frac{3Axa}{b}}{\left(bx^2+a\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \; 2345$$

$$\frac{a(A+Bx)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{a(3A+4Bx)}{b^2\sqrt{a+bx^2}}-\frac{\int \frac{3a^2B}{b^2\sqrt{bx^2+a}}dx}{a}}{3a}$$

$$\Big\downarrow \; 27$$

$$\frac{a(A+Bx)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{a(3A+4Bx)}{b^2\sqrt{a+bx^2}}-\frac{3aB\int \frac{1}{\sqrt{bx^2+a}}dx}{b^2}}{3a}$$

$$\Big\downarrow \; 224$$

$$\frac{a(A+Bx)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{a(3A+4Bx)}{b^2\sqrt{a+bx^2}}-\frac{3aB\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b^2}}{3a}$$

$$\Big\downarrow \; 219$$

$$\frac{a(A+Bx)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{a(3A+4Bx)}{b^2\sqrt{a+bx^2}}-\frac{3aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{5/2}}}{3a}$$

input
```
Int[(x^3*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
(a*(A + B*x))/(3*b^2*(a + b*x^2)^(3/2)) - ((a*(3*A + 4*B*x))/(b^2*Sqrt[a +
 b*x^2]) - (3*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(5/2))/(3*a)
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.36.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.23

method	result	size
default	$B\left(-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}\right)+A\left(-\frac{x^{2}}{b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^{2}\left(b\,x^{2}+a\right)^{\frac{3}{2}}}\right)$	97

input
```
int(x^3*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(
1/2)+(b*x^2+a)^(1/2))))+A*(-x^2/b/(b*x^2+a)^(3/2)-2/3*a/b^2/(b*x^2+a)^(3/2
))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.03

$$\int \frac{x^3(A+Bx)}{\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(4\,Bb^2x^3+3\,A\right.}{6\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}\right.$$
$$\left.-\frac{3\left(Bb^2x^4+2\,Babx^2+Ba^2\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+\left(4\,Bb^2x^3+3\,Ab^2x^2+3\,Babx+2\,Aab\right)\sqrt{bx^2+a}}{3\left(b^5x^4+2\,ab^4x^2+a^2b^3\right)}\right]$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(B*b^2*x^4 + 2*B*a*b*x^2 + B*a^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*
x^2 + a)*sqrt(b)*x - a) - 2*(4*B*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a
*b)*sqrt(b*x^2 + a))/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3), -1/3*(3*(B*b^2*x^4
 + 2*B*a*b*x^2 + B*a^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (4*B
*b^2*x^3 + 3*A*b^2*x^2 + 3*B*a*b*x + 2*A*a*b)*sqrt(b*x^2 + a))/(b^5*x^4 +
2*a*b^4*x^2 + a^2*b^3)]
```

3.36.6 Sympy [A] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 400, normalized size of antiderivative = 5.06

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = A\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + B\left(\frac{3a^{\frac{39}{2}}b^{11}\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} + \frac{3a^{\frac{37}{2}}b^{12}x^2\sqrt{1+\frac{bx^2}{a}}\operatorname{asinh}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{3a^{19}b^{\frac{23}{2}}x}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}} - \frac{4a^{18}b^{\frac{25}{2}}x^3}{3a^{\frac{39}{2}}b^{\frac{27}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{37}{2}}b^{\frac{29}{2}}x^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**3*(B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2
)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x**2)),
Ne(b, 0)), (x**4/(4*a**(5/2)), True)) + B*(3*a**(39/2)*b**11*sqrt(1 + b*x*
*2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) +
 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) + 3*a**(37/2)*b**12*x**2*s
qrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(3*a**(39/2)*b**(27/2)*sqrt(1 +
 b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)) - 3*a**19*b**(
23/2)*x/(3*a**(39/2)*b**(27/2)*sqrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*
x**2*sqrt(1 + b*x**2/a)) - 4*a**18*b**(25/2)*x**3/(3*a**(39/2)*b**(27/2)*s
qrt(1 + b*x**2/a) + 3*a**(37/2)*b**(29/2)*x**2*sqrt(1 + b*x**2/a)))
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.29

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{1}{3}\,Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right) - \frac{Ax^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{Bx}{3\,\sqrt{bx^2+a}b^2} + \frac{B\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}} - \frac{2\,Aa}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2)) - A*x
^2/((b*x^2 + a)^(3/2)*b) - 1/3*B*x/(sqrt(b*x^2 + a)*b^2) + B*arcsinh(b*x/s
qrt(a*b))/b^(5/2) - 2/3*A*a/((b*x^2 + a)^(3/2)*b^2)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{\left(\left(\frac{4\,Bx}{b}+\frac{3\,A}{b}\right)x+\frac{3\,Ba}{b^2}\right)x+\frac{2\,Aa}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((4*B*x/b + 3*A/b)*x + 3*B*a/b^2)*x + 2*A*a/b^2)/(b*x^2 + a)^(3/2) -
 B*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(5/2)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(b\,x^2+a)^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(a + b*x^2)^(5/2),x)
```

output
```
int((x^3*(A + B*x))/(a + b*x^2)^(5/2), x)
```

3.37 $\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

3.37.1 Optimal result

Integrand size = 20, antiderivative size = 53

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{x^2(aB-Abx)}{3ab\,(a+bx^2)^{3/2}} - \frac{2B}{3b^2\sqrt{a+bx^2}}$$

output
```
-1/3*x^2*(-A*b*x+B*a)/a/b/(b*x^2+a)^(3/2)-2/3*B/b^2/(b*x^2+a)^(1/2)
```

3.37.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{-2a^2B-3abBx^2+Ab^2x^3}{3ab^2\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
(-2*a^2*B - 3*a*b*B*x^2 + A*b^2*x^3)/(3*a*b^2*(a + b*x^2)^(3/2))
```

3.37.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {530, 25, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{530}$$

$$\frac{aB-Abx}{3b^2(a+bx^2)^{3/2}}-\frac{\int -\frac{a(A+3Bx)}{b(bx^2+a)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a(A+3Bx)}{b(bx^2+a)^{3/2}}dx}{3a}+\frac{aB-Abx}{3b^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{A+3Bx}{(bx^2+a)^{3/2}}dx}{3b}+\frac{aB-Abx}{3b^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{453}$$

$$\frac{aB-Abx}{3b^2(a+bx^2)^{3/2}}-\frac{3aB-Abx}{3ab^2\sqrt{a+bx^2}}$$

input
```
Int[(x^2*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
(a*B - A*b*x)/(3*b^2*(a + b*x^2)^(3/2)) - (3*a*B - A*b*x)/(3*a*b^2*Sqrt[a
+ b*x^2])
```

3.37.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

3.37.4 Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

method	result	size
gosper	$\frac{A\,b^2x^3-3Bab\,x^2-2a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2}$	41
trager	$\frac{A\,b^2x^3-3Bab\,x^2-2a^2B}{3\left(b\,x^2+a\right)^{\frac{3}{2}}a\,b^2}$	41
default	$B\left(-\frac{x^2}{b\left(b\,x^2+a\right)^{\frac{3}{2}}}-\frac{2a}{3b^2\left(b\,x^2+a\right)^{\frac{3}{2}}}\right)+A\left(-\frac{x}{2b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{2b}\right)$	92

input
```
int(x^2*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(A*b^2*x^3-3*B*a*b*x^2-2*B*a^2)/(b*x^2+a)^(3/2)/a/b^2
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{(Ab^2x^3 - 3\,Babx^2 - 2\,Ba^2)\sqrt{bx^2+a}}{3\,(ab^4x^4 + 2\,a^2b^3x^2 + a^3b^2)}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(A*b^2*x^3 - 3*B*a*b*x^2 - 2*B*a^2)*sqrt(b*x^2 + a)/(a*b^4*x^4 + 2*a^2
*b^3*x^2 + a^3*b^2)
```

3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. 2(46) = 92.

Time = 4.25 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.66

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{Ax^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}} + 3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + B\left(\begin{cases} -\frac{2a}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} - \frac{3bx^2}{3ab^2\sqrt{a+bx^2}+3b^3x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^4}{4a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(x**2*(B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/
a)) + B*Piecewise((-2*a/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a +
b*x**2)) - 3*b*x**2/(3*a*b**2*sqrt(a + b*x**2) + 3*b**3*x**2*sqrt(a + b*x*
*2)), Ne(b, 0)), (x**4/(4*a**(5/2)), True))
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.32

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{Bx^2}{(bx^2+a)^{\frac{3}{2}}b} - \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Ax}{3\,\sqrt{bx^2+a}ab} - \frac{2\,Ba}{3\,(bx^2+a)^{\frac{3}{2}}b^2}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-B*x^2/((b*x^2 + a)^(3/2)*b) - 1/3*A*x/((b*x^2 + a)^(3/2)*b) + 1/3*A*x/(sq
rt(b*x^2 + a)*a*b) - 2/3*B*a/((b*x^2 + a)^(3/2)*b^2)
```

3.37.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{Ax}{a} - \frac{3\,B}{b}\right)x^2 - \frac{2\,Ba}{b^2}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((A*x/a - 3*B/b)*x^2 - 2*B*a/b^2)/(b*x^2 + a)^(3/2)
```

3.37.9 Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{B\,a^2 - 3\,B\,a\,(b\,x^2+a) + A\,b\,x\,(b\,x^2+a) - A\,a\,b\,x}{3\,a\,b^2\,(b\,x^2+a)^{3/2}}$$

input
```
int((x^2*(A + B*x))/(a + b*x^2)^(5/2),x)
```

output
```
(B*a^2 - 3*B*a*(a + b*x^2) + A*b*x*(a + b*x^2) - A*a*b*x)/(3*a*b^2*(a + b*
x^2)^(3/2))
```

3.38 $\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx$

3.38.1 Optimal result

Integrand size = 18, antiderivative size = 50

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{-A-Bx}{3b\,(a+bx^2)^{3/2}} + \frac{Bx}{3ab\sqrt{a+bx^2}}$$

output
```
1/3*(-B*x-A)/b/(b*x^2+a)^(3/2)+1/3*B*x/a/b/(b*x^2+a)^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.64

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{-aA+bBx^3}{3ab\,(a+bx^2)^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
(-(a*A) + b*B*x^3)/(3*a*b*(a + b*x^2)^(3/2))
```

3.38.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {530, 25, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 530}$$

$$-\frac{\int -\frac{aB}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{aB}{b(bx^2+a)^{3/2}}dx}{3a}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{B\int \frac{1}{(bx^2+a)^{3/2}}dx}{3b}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 208}$$

$$\frac{Bx}{3ab\sqrt{a+bx^2}}-\frac{A+Bx}{3b\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(x*(A + B*x))/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(A + B*x)/(b*(a + b*x^2)^(3/2)) + (B*x)/(3*a*b*Sqrt[a + b*x^2])
```

3.38.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

3.38.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{-bB\,x^{3}+Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} a b}$	29
trager	$-\frac{-bB\,x^{3}+Aa}{3\left(b\,x^{2}+a\right)^{\frac{3}{2}} a b}$	29
default	$B\left(-\frac{x}{2b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{a\left(\frac{x}{3a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^{2}\sqrt{b\,x^{2}+a}}\right)}{2b}\right)-\frac{A}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}$	72

input
```
int(x*(B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-B*b*x^3+A*a)/(b*x^2+a)^(3/2)/a/b
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.98

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{(Bbx^3-Aa)\sqrt{bx^2+a}}{3\,(ab^3x^4+2\,a^2b^2x^2+a^3b)}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(B*b*x^3 - A*a)*sqrt(b*x^2 + a)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b)
```

3.38.6 Sympy [A] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.90

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = A\left(\begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases}\right) + \frac{Bx^3}{3a^{\frac{5}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{3}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x*(B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(5/2)), True)) + B*x**3/(3*a**(5/2)*sqrt(1 + b*x**2/
a) + 3*a**(3/2)*b*x**2*sqrt(1 + b*x**2/a))
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.02

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = -\frac{Bx}{3\,(bx^2+a)^{\frac{3}{2}}b} + \frac{Bx}{3\,\sqrt{bx^2+a}ab} - \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*x/((b*x^2 + a)^(3/2)*b) + 1/3*B*x/(sqrt(b*x^2 + a)*a*b) - 1/3*A/((b
*x^2 + a)^(3/2)*b)
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.52

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{\frac{Bx^3}{a}-\frac{A}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(B*x^3/a - A/b)/(b*x^2 + a)^(3/2)
```

3.38.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx)}{(a+bx^2)^{5/2}}\,dx = \frac{B\,x^3}{3\,a\,(b\,x^2+a)^{3/2}}-\frac{A}{3\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((x*(A + B*x))/(a + b*x^2)^(5/2),x)
```

output
```
(B*x^3)/(3*a*(a + b*x^2)^(3/2)) - A/(3*b*(a + b*x^2)^(3/2))
```

3.39 $\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx$

3.39.1 Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{-aB+Abx}{3ab\left(a+bx^2\right)^{3/2}}+\frac{2Ax}{3a^2\sqrt{a+bx^2}}$$

output
```
1/3*(A*b*x-B*a)/a/b/(b*x^2+a)^(3/2)+2/3*A*x/a^2/(b*x^2+a)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\left(a+bx^2\right)^{5/2}}\,dx=\frac{-a^2B+3aAbx+2Ab^2x^3}{3a^2b\left(a+bx^2\right)^{3/2}}$$

input
```
Integrate[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
(-(a^2*B) + 3*a*A*b*x + 2*A*b^2*x^3)/(3*a^2*b*(a + b*x^2)^(3/2))
```

3.39.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{454}$$

$$\frac{2A\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{2Ax}{3a^2\sqrt{a+bx^2}}-\frac{aB-Abx}{3ab\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(a + b*x^2)^(5/2),x]
```

output
```
-1/3*(a*B - A*b*x)/(a*b*(a + b*x^2)^(3/2)) + (2*A*x)/(3*a^2*Sqrt[a + b*x^2
])
```

3.39.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

3.39.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

method	result	size
gosper	$\frac{2Ab^2x^3+3aAbx-a^2B}{3(bx^2+a)^{\frac{3}{2}}a^2b}$	40
trager	$\frac{2Ab^2x^3+3aAbx-a^2B}{3(bx^2+a)^{\frac{3}{2}}a^2b}$	40
default	$A\left(\frac{x}{3a(bx^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{bx^2+a}}\right)-\frac{B}{3b(bx^2+a)^{\frac{3}{2}}}$	50

input
```
int((B*x+A)/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*A*b^2*x^3+3*A*a*b*x-B*a^2)/(b*x^2+a)^(3/2)/a^2/b
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{(2\,Ab^2x^3+3\,Aabx-Ba^2)\sqrt{bx^2+a}}{3\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*A*b^2*x^3 + 3*A*a*b*x - B*a^2)*sqrt(b*x^2 + a)/(a^2*b^3*x^4 + 2*a^3
*b^2*x^2 + a^4*b)
```

3.39.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(44) = 88.

Time = 3.59 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.86

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = A\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}} + \frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right) + B\left(\begin{cases}-\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b\neq 0\\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((B*x+A)/(b*x**2+a)**(5/2),x)
```

output
```
A*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 + b*x**
2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1
 + b*x**2/a))) + B*Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqr
t(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True))
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,Ax}{3\,\sqrt{bx^2+a}a^2} + \frac{Ax}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}b}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*A*x/(sqrt(b*x^2 + a)*a^2) + 1/3*A*x/((b*x^2 + a)^(3/2)*a) - 1/3*B/((b*
x^2 + a)^(3/2)*b)
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{\left(\frac{2\,Abx^2}{a^2}+\frac{3\,A}{a}\right)x-\frac{B}{b}}{3\,(bx^2+a)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*((2*A*b*x^2/a^2 + 3*A/a)*x - B/b)/(b*x^2 + a)^(3/2)
```

3.39.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{(a+bx^2)^{5/2}}\,dx = \frac{2\,A\,b\,x\,(b\,x^2+a)-B\,a^2+A\,a\,b\,x}{3\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x)/(a + b*x^2)^(5/2),x)
```

output
```
(2*A*b*x*(a + b*x^2) - B*a^2 + A*a*b*x)/(3*a^2*b*(a + b*x^2)^(3/2))
```

3.40 $\int \frac{A+Bx}{x(a+bx^2)^{5/2}}\,dx$

3.40.1 Optimal result

Integrand size = 20, antiderivative size = 76

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx=\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}+\frac{3A+2Bx}{3a^2\sqrt{a+bx^2}}-\frac{A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
1/3*(B*x+A)/a/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+1
/3*(2*B*x+3*A)/a^2/(b*x^2+a)^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx=\frac{bx^2(3A+2Bx)+a(4A+3Bx)}{3a^2\left(a+bx^2\right)^{3/2}}+\frac{2A\text{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(b*x^2*(3*A + 2*B*x) + a*(4*A + 3*B*x))/(3*a^2*(a + b*x^2)^(3/2)) + (2*A*A
rcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5/2)
```

3.40.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {532, 25, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{532}$$

$$\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3A+2Bx}{x\left(bx^2+a\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{3A+2Bx}{x\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{3A+2Bx}{a\sqrt{a+bx^2}}-\frac{\int -\frac{3A}{x\sqrt{bx^2+a}}dx}{a}}{3a}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3A\int \frac{1}{x\sqrt{bx^2+a}}dx}{a}+\frac{3A+2Bx}{a\sqrt{a+bx^2}}}{3a}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{3A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}+\frac{3A+2Bx}{a\sqrt{a+bx^2}}}{3a}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{3A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{3A+2Bx}{a\sqrt{a+bx^2}}}{3a}+\frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3A+2Bx}{a\sqrt{a+bx^2}} - \frac{3A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{3a} + \frac{A+Bx}{3a\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x*(a + b*x^2)^(5/2)),x]
```

output
```
(A + B*x)/(3*a*(a + b*x^2)^(3/2)) + ((3*A + 2*B*x)/(a*Sqrt[a + b*x^2]) - (
3*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/(3*a)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

3.40.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.29

method	result	size
default	$B\left(\frac{x}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)+A\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)$	98

input
```
int((B*x+A)/x/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))+A*(1/3/a/(b*x^2+a)^(
3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx}{x\,(a+bx^2)^{5/2}}\,dx = \left[\frac{3\,(Ab^2x^4+2\,Aabx^2+Aa^2)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(2\,Babx^3+3\,Aabx^2+}{6\,(a^3b^2x^4+2\,a^4bx^2+a^5)}\right.$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(A*b^2*x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x
^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(2*B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4
*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^4 + 2*a^4*b*x^2 + a^5), 1/3*(3*(A*b^2*
x^4 + 2*A*a*b*x^2 + A*a^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (2*
B*a*b*x^3 + 3*A*a*b*x^2 + 3*B*a^2*x + 4*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x
^4 + 2*a^4*b*x^2 + a^5)]
```

3.40.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(65) = 130$.

Time = 8.43 (sec) , antiderivative size = 840, normalized size of antiderivative = 11.05

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = A\left(\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}\right.$$
$$+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$-\frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$+\frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$+\frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$-\frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$+\frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$+\frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$-\frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$+\frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}$$
$$\left.-\frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}\right)$$
$$+B\left(\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}+\frac{2bx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{bx^2}{a}}+3a^{\frac{5}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate((B*x+A)/x/(b*x**2+a)**(5/2),x)
```

3.40. $\int \frac{A+Bx}{x(a+bx^2)^{5/2}}\,dx$

output
```
A*(8*a**7*sqrt(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(1
5/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2
) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6)
- 6*a**7*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 +
b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*
a**(13/2)*b**3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(1
7/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*
x**2*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a
**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*
x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a*
*(13/2)*b**3*x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(
17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b
**2*x**4*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x
**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**
(13/2)*b**3*x**6) - 6*a**4*b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(6*a**(19
/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6
)) + B*(3*a*x/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**2*sqrt(1 +
b*x**2/a)) + 2*b*x**3/(3*a**(7/2)*sqrt(1 + b*x**2/a) + 3*a**(5/2)*b*x**...
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x\left(a+bx^2\right)^{5/2}}\,dx = \frac{2\,Bx}{3\sqrt{bx^2+a}a^2} + \frac{Bx}{3\left(bx^2+a\right)^{\frac{3}{2}}a} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{A}{\sqrt{bx^2+a}a^2} + \frac{A}{3\left(bx^2+a\right)^{\frac{3}{2}}a}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*x/(sqrt(b*x^2 + a)*a^2) + 1/3*B*x/((b*x^2 + a)^(3/2)*a) - A*arcsinh(
a/(sqrt(a*b)*abs(x)))/a^(5/2) + A/(sqrt(b*x^2 + a)*a^2) + 1/3*A/((b*x^2 +
a)^(3/2)*a)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{x\,(a+bx^2)^{5/2}}\,dx = \frac{\left(\left(\frac{2\,Bbx}{a^2}+\frac{3\,Ab}{a^2}\right)x+\frac{3\,B}{a}\right)x+\frac{4\,A}{a}}{3\,(bx^2+a)^{\frac{3}{2}}}+\frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}$$

input
```
integrate((B*x+A)/x/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
1/3*(((2*B*b*x/a^2 + 3*A*b/a^2)*x + 3*B/a)*x + 4*A/a)/(b*x^2 + a)^(3/2) +
2*A*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
```

3.40.9 Mupad [B] (verification not implemented)

Time = 6.37 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{x\,(a+bx^2)^{5/2}}\,dx = \frac{\frac{A}{3\,a}+\frac{A\,(b\,x^2+a)}{a^2}}{(b\,x^2+a)^{3/2}}+\frac{2\,B\,x\,(b\,x^2+a)+B\,a\,x}{3\,a^2\,(b\,x^2+a)^{3/2}}-\frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
int((A + B*x)/(x*(a + b*x^2)^(5/2)),x)
```

output
```
(A/(3*a) + (A*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) + (2*B*x*(a + b*x^2) + B
*a*x)/(3*a^2*(a + b*x^2)^(3/2)) - (A*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(
5/2)
```

3.41 $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

3.41.1 Optimal result

Integrand size = 20, antiderivative size = 104

$$\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx=\frac{A+Bx}{3ax(a+bx^2)^{3/2}}+\frac{4A+3Bx}{3a^2x\sqrt{a+bx^2}}-\frac{8A\sqrt{a+bx^2}}{3a^3x}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

output
```
1/3*(B*x+A)/a/x/(b*x^2+a)^(3/2)-B*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)
+1/3*(3*B*x+4*A)/a^2/x/(b*x^2+a)^(1/2)-8/3*A*(b*x^2+a)^(1/2)/a^3/x
```

3.41.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx=\frac{-8Ab^2x^4+3abx^2(-4A+Bx)+a^2(-3A+4Bx)}{3a^3x(a+bx^2)^{3/2}}+\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(-8*A*b^2*x^4 + 3*a*b*x^2*(-4*A + B*x) + a^2*(-3*A + 4*B*x))/(3*a^3*x*(a +
 b*x^2)^(3/2)) + (2*B*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(5
/2)
```

3.41.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {532, 25, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 532}$$

$$\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{-\frac{2Abx^2}{a}+3Bx+3A}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-\frac{2Abx^2}{a}+3Bx+3A}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 2336}$$

$$\frac{\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}-\frac{\int -\frac{3(A+Bx)}{x^2\sqrt{bx^2+a}}dx}{a}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{3\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 534}$$

$$\frac{\frac{3\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\frac{3\left(\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\frac{3\left(\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

$\big\downarrow$ 221

$$\frac{\frac{3aB-5Abx}{a^2\sqrt{a+bx^2}}+\frac{3\left(-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{a}}{3a}+\frac{aB-Abx}{3a^2\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^2*(a + b*x^2)^(5/2)),x]
```

output
```
(a*B - A*b*x)/(3*a^2*(a + b*x^2)^(3/2)) + ((3*a*B - 5*A*b*x)/(a^2*Sqrt[a +
 b*x^2]) + (3*(-((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/S
qrt[a]])/Sqrt[a]))/a)/(3*a)
```

3.41.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.41.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.17

method	result
default	$B\left(\frac{1}{3a(b\,x^2+a)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}\right)+A\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)$
risch	$-\frac{A\sqrt{b\,x^2+a}}{a^3x}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)B}{a^{\frac{5}{2}}}-\frac{5\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}\,A}{6a^3\left(x-\frac{\sqrt{-ab}}{b}\right)}+\frac{7\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^2b+2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}}{12a^2\sqrt{-ab}\,\left(x-\frac{\sqrt{-ab}}{b}\right)}$

input
```
int((B*x+A)/x^2/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1
/2)*(b*x^2+a)^(1/2))/x)))+A*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+
a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.54

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{3\left(Bb^2x^5+2\,Babx^3+Ba^2x\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(8\,Ab^2x^4-3\,Babx^3\right.}{6\left(a^3b^2x^5+2\,a^4bx^3+a^5x\right)}\right.$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^
2 - 4*B*a^2*x + 3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5
*x), 1/3*(3*(B*b^2*x^5 + 2*B*a*b*x^3 + B*a^2*x)*sqrt(-a)*arctan(sqrt(-a)/s
qrt(b*x^2 + a)) - (8*A*b^2*x^4 - 3*B*a*b*x^3 + 12*A*a*b*x^2 - 4*B*a^2*x +
3*A*a^2)*sqrt(b*x^2 + a))/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*x)]
```

3.41.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. 2(88) = 176.

Time = 6.91 (sec) , antiderivative size = 910, normalized size of antiderivative = 8.75

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = A\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)+B\left(\frac{8a^7\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^7\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^7\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{14a^6bx^2\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^6bx^2\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^6bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{6a^5b^2x^4\sqrt{1+\frac{bx^2}{a}}}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{9a^5b^2x^4\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{18a^5b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}+\frac{3a^4b^3x^6\log\left(\frac{bx^2}{a}\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}-\frac{6a^4b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{6a^{\frac{19}{2}}+18a^{\frac{17}{2}}bx^2+18a^{\frac{15}{2}}b^2x^4+6a^{\frac{13}{2}}b^3x^6}\right)$$

input
```
integrate((B*x+A)/x**2/(b*x**2+a)**(5/2),x)
```

3.41. $\int \frac{A+Bx}{x^2(a+bx^2)^{5/2}}\,dx$

output
```
A*(-3*a**2*b**(9/2)*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 +
 3*a**3*b**6*x**4) - 12*a*b**(11/2)*x**2*sqrt(a/(b*x**2) + 1)/(3*a**5*b**4
 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4) - 8*b**(13/2)*x**4*sqrt(a/(b*x**2)
 + 1)/(3*a**5*b**4 + 6*a**4*b**5*x**2 + 3*a**3*b**6*x**4)) + B*(8*a**7*sqr
t(1 + b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**
4 + 6*a**(13/2)*b**3*x**6) + 3*a**7*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17
/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 6*a**7*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 14*a**6*b*x**2*sqrt(1 + b*x**2/a)/(6*
a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**
3*x**6) + 9*a**6*b*x**2*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 +
 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**6*b*x**2*log(sqrt
(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2
*x**4 + 6*a**(13/2)*b**3*x**6) + 6*a**5*b**2*x**4*sqrt(1 + b*x**2/a)/(6*a*
*(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*
x**6) + 9*a**5*b**2*x**4*log(b*x**2/a)/(6*a**(19/2) + 18*a**(17/2)*b*x**2
+ 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**3*x**6) - 18*a**5*b**2*x**4*log(
sqrt(1 + b*x**2/a) + 1)/(6*a**(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*
b**2*x**4 + 6*a**(13/2)*b**3*x**6) + 3*a**4*b**3*x**6*log(b*x**2/a)/(6*a**
(19/2) + 18*a**(17/2)*b*x**2 + 18*a**(15/2)*b**2*x**4 + 6*a**(13/2)*b**...
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{8\,Abx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Abx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{B\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{5}{2}}} + \frac{B}{\sqrt{bx^2+a}a^2} + \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a} - \frac{A}{(bx^2+a)^{\frac{3}{2}}ax}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-8/3*A*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*A*b*x/((b*x^2 + a)^(3/2)*a^2) - B*a
rcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + B/(sqrt(b*x^2 + a)*a^2) + 1/3*B/((b
*x^2 + a)^(3/2)*a) - A/((b*x^2 + a)^(3/2)*a*x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = -\frac{\left(\left(\frac{5\,Ab^2x}{a^3}-\frac{3\,Bb}{a^2}\right)x+\frac{6\,Ab}{a^2}\right)x-\frac{4\,B}{a}}{3\left(bx^2+a\right)^{\frac{3}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^2}$$

input
```
integrate((B*x+A)/x^2/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((5*A*b^2*x/a^3 - 3*B*b/a^2)*x + 6*A*b/a^2)*x - 4*B/a)/(b*x^2 + a)^(
3/2) + 2*B*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2)
+ 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^2)
```

3.41.9 Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^2\left(a+bx^2\right)^{5/2}}\,dx = \frac{\frac{B}{3\,a}+\frac{B\,\left(b\,x^2+a\right)}{a^2}}{\left(b\,x^2+a\right)^{3/2}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{A\,a^2-8\,A\,\left(b\,x^2+a\right)^2+4\,A\,a\,\left(b\,x^2+a\right)}{3\,a^3\,x\,\left(b\,x^2+a\right)^{3/2}}$$

input
```
int((A + B*x)/(x^2*(a + b*x^2)^(5/2)),x)
```

output
```
(B/(3*a) + (B*(a + b*x^2))/a^2)/(a + b*x^2)^(3/2) - (B*atanh((a + b*x^2)^(
1/2)/a^(1/2)))/a^(5/2) + (A*a^2 - 8*A*(a + b*x^2)^2 + 4*A*a*(a + b*x^2))/(
3*a^3*x*(a + b*x^2)^(3/2))
```

3.42 $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

3.42.1 Optimal result

Integrand size = 20, antiderivative size = 129

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\frac{A+Bx}{3ax^2\left(a+bx^2\right)^{3/2}}+\frac{5A+4Bx}{3a^2x^2\sqrt{a+bx^2}}-\frac{5A\sqrt{a+bx^2}}{2a^3x^2}-\frac{8B\sqrt{a+bx^2}}{3a^3x}+\frac{5Ab\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{7/2}}$$

output
```
1/3*(B*x+A)/a/x^2/(b*x^2+a)^(3/2)+5/2*A*b*arctanh((b*x^2+a)^(1/2)/a^(1/2))
/a^(7/2)+1/3*(4*B*x+5*A)/a^2/x^2/(b*x^2+a)^(1/2)-5/2*A*(b*x^2+a)^(1/2)/a^3
/x^2-8/3*B*(b*x^2+a)^(1/2)/a^3/x
```

3.42.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx=\frac{-3a^2(A+2Bx)-4abx^2(5A+6Bx)-b^2x^4(15A+16Bx)}{6a^3x^2\left(a+bx^2\right)^{3/2}}-\frac{5Ab\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
(-3*a^2*(A + 2*B*x) - 4*a*b*x^2*(5*A + 6*B*x) - b^2*x^4*(15*A + 16*B*x))/(
6*a^3*x^2*(a + b*x^2)^(3/2)) - (5*A*b*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2]
)/Sqrt[a]])/a^(7/2)
```

3.42.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {532, 25, 2336, 27, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}dx$$

$$\Big\downarrow \text{532}$$

$$-\frac{\int -\frac{-\frac{2bBx^3}{a}-\frac{3Abx^2}{a}+3Bx+3A}{x^3(bx^2+a)^{3/2}}dx}{3a}-\frac{b(A+Bx)}{3a^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{2bBx^3}{a}-\frac{3Abx^2}{a}+3Bx+3A}{x^3(bx^2+a)^{3/2}}dx}{3a}-\frac{b(A+Bx)}{3a^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{\int -\frac{3\left(-\frac{2Abx^2}{a}+Bx+A\right)}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{-\frac{2Abx^2}{a}+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{2338}$$

$$\frac{\frac{3\left(-\frac{\int -\frac{2aB-5Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2(a+bx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{3\left(\frac{\int \frac{2aB-5Abx}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{534}$$

$$\frac{\frac{3\left(\frac{-5Ab\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{243}$$

$$\frac{\frac{3\left(\frac{-\frac{5}{2}Ab\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{\frac{3\left(\frac{-5A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2\left(a+bx^2\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{3\left(\frac{\frac{5Ab\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{b(6A+5Bx)}{a^2\sqrt{a+bx^2}}}{3a}-\frac{b(A+Bx)}{3a^2\left(a+bx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^3*(a + b*x^2)^(5/2)),x]
```

output
```
-1/3*(b*(A + B*x))/(a^2*(a + b*x^2)^(3/2)) + (-((b*(6*A + 5*B*x))/(a^2*Sqr
t[a + b*x^2])) + (3*(-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*
x^2])/x + (5*A*b*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a)))/a)/(3*
a)
```

3.42. $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

3.42.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.42.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.13

method	result
default	$B\left(-\frac{1}{ax(b\,x^2+a)^{\frac{3}{2}}}-\frac{4b\left(\frac{x}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{b\,x^2+a}}\right)}{a}\right)+A\left(-\frac{1}{2a\,x^2(b\,x^2+a)^{\frac{3}{2}}}-\frac{5b\left(\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(2 \right.}{}}{}\right.}{2a}\right.$
risch	$-\frac{\sqrt{b\,x^2+a}\,(2Bx+A)}{2a^3x^2}+\frac{5\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)Ab}{2a^{\frac{7}{2}}}+\frac{13\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}\,Ab}{12a^3\sqrt{-ab}\,\left(x+\frac{\sqrt{-ab}}{b}\right)}-\frac{5\sqrt{\left(x+\frac{\sqrt{-ab}}{b}\right)^2b-2\sqrt{-ab}}}{6a^3\left(x+\frac{\sqrt{-ab}}{b}\right.}$

input
```
int((B*x+A)/x^3/(b*x^2+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
B*(-1/a/x/(b*x^2+a)^(3/2)-4*b/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+
a)^(1/2)))+A*(-1/2/a/x^2/(b*x^2+a)^(3/2)-5/2*b/a*(1/3/a/(b*x^2+a)^(3/2)+1/
a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.38

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = \left[\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{a}\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\left(16\,Bab^2x^5+15\right.}{12\left(a^4b^2x^6+2\,a^5bx^4+a\right.}\right.$$

$$-\frac{15\left(Ab^3x^6+2\,Aab^2x^4+Aa^2bx^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+\left(16\,Bab^2x^5+15\,Aab^2x^4+24\,Ba^2bx^3+20\,Aa^2b\right.}{6\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="fricas")
```

output
```
[1/12*(15*(A*b^3*x^6 + 2*A*a*b^2*x^4 + A*a^2*b*x^2)*sqrt(a)*log(-(b*x^2 +
2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(16*B*a*b^2*x^5 + 15*A*a*b^2*x^4
 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^3)*sqrt(b*x^2 + a))
/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2), -1/6*(15*(A*b^3*x^6 + 2*A*a*b^2*x^
4 + A*a^2*b*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (16*B*a*b^2*x
^5 + 15*A*a*b^2*x^4 + 24*B*a^2*b*x^3 + 20*A*a^2*b*x^2 + 6*B*a^3*x + 3*A*a^
3)*sqrt(b*x^2 + a))/(a^4*b^2*x^6 + 2*a^5*b*x^4 + a^6*x^2)]
```

3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(122) = 244.

Time = 7.94 (sec) , antiderivative size = 1034, normalized size of antiderivative = 8.02

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = A\left(-\frac{6a^{17}\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}\right.$$

$$-\frac{46a^{16}bx^2\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{15a^{16}bx^2\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$+\frac{30a^{16}bx^2\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{70a^{15}b^2x^4\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{45a^{15}b^2x^4\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$+\frac{90a^{15}b^2x^4\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{30a^{14}b^3x^6\sqrt{1+\frac{bx^2}{a}}}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{45a^{14}b^3x^6\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$+\frac{90a^{14}b^3x^6\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$-\frac{15a^{13}b^4x^8\log\left(\frac{bx^2}{a}\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}$$

$$\left.+\frac{30a^{13}b^4x^8\log\left(\sqrt{1+\frac{bx^2}{a}}+1\right)}{12a^{\frac{39}{2}}x^2+36a^{\frac{37}{2}}bx^4+36a^{\frac{35}{2}}b^2x^6+12a^{\frac{33}{2}}b^3x^8}\right)+B\left(-\frac{3a^2b^{\frac{9}{2}}\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}-\frac{12ab^{\frac{11}{2}}x^2\sqrt{\frac{a}{bx^2}+}}{3a^5b^4+6a^4b^5x^2+3a}\right.$$

$$\left.-\frac{8b^{\frac{13}{2}}x^4\sqrt{\frac{a}{bx^2}+1}}{3a^5b^4+6a^4b^5x^2+3a^3b^6x^4}\right)$$

3.42. $\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx$

input
```
integrate((B*x+A)/x**3/(b*x**2+a)**(5/2),x)
```

output
```
A*(-6*a**17*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 +
36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 46*a**16*b*x**2*sqrt(1
+ b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x
**6 + 12*a**(33/2)*b**3*x**8) - 15*a**16*b*x**2*log(b*x**2/a)/(12*a**(39/2
)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*
x**8) + 30*a**16*b*x**2*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 3
6*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 70
*a**15*b**2*x**4*sqrt(1 + b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x*
*4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 45*a**15*b**2*x**4
*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**
2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**15*b**2*x**4*log(sqrt(1 + b*x**2/
a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6
+ 12*a**(33/2)*b**3*x**8) - 30*a**14*b**3*x**6*sqrt(1 + b*x**2/a)/(12*a**(
39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b
**3*x**8) - 45*a**14*b**3*x**6*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(3
7/2)*b*x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) + 90*a**14*
b**3*x**6*log(sqrt(1 + b*x**2/a) + 1)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*
x**4 + 36*a**(35/2)*b**2*x**6 + 12*a**(33/2)*b**3*x**8) - 15*a**13*b**4*x*
*8*log(b*x**2/a)/(12*a**(39/2)*x**2 + 36*a**(37/2)*b*x**4 + 36*a**(35/2)*b
**2*x**6 + 12*a**(33/2)*b**3*x**8) + 30*a**13*b**4*x**8*log(sqrt(1 + b*...
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^3\left(a+bx^2\right)^{5/2}}\,dx = -\frac{8\,Bbx}{3\,\sqrt{bx^2+a}a^3} - \frac{4\,Bbx}{3\,(bx^2+a)^{\frac{3}{2}}a^2} + \frac{5\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{7}{2}}} - \frac{5\,Ab}{2\,\sqrt{bx^2+a}a^3} - \frac{5\,Ab}{6\,(bx^2+a)^{\frac{3}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{3}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{3}{2}}ax^2}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="maxima")
```

output
```
-8/3*B*b*x/(sqrt(b*x^2 + a)*a^3) - 4/3*B*b*x/((b*x^2 + a)^(3/2)*a^2) + 5/2
*A*b*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 5/2*A*b/(sqrt(b*x^2 + a)*a^3)
 - 5/6*A*b/((b*x^2 + a)^(3/2)*a^2) - B/((b*x^2 + a)^(3/2)*a*x) - 1/2*A/((b
*x^2 + a)^(3/2)*a*x^2)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx = -\frac{\left(\left(\frac{5\,Bb^2x}{a^3}+\frac{6\,Ab^2}{a^3}\right)x+\frac{6\,Bb}{a^2}\right)x+\frac{7\,Ab}{a^2}}{3\,(bx^2+a)^{\frac{3}{2}}}-\frac{5\,Ab\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^3}$$

input
```
integrate((B*x+A)/x^3/(b*x^2+a)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(((5*B*b^2*x/a^3 + 6*A*b^2/a^3)*x + 6*B*b/a^2)*x + 7*A*b/a^2)/(b*x^2
+ a)^(3/2) - 5*A*b*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-
a)*a^3) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqrt(b*x^2
 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^2*sqrt(
b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^3)
```

3.42.9 Mupad [B] (verification not implemented)

Time = 6.74 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^3(a+bx^2)^{5/2}}\,dx = \frac{B\,a^2-8\,B\,(b\,x^2+a)^2+4\,B\,a\,(b\,x^2+a)}{3\,a^3\,x\,(b\,x^2+a)^{3/2}}-\frac{10\,A\,b}{3\,a^2\,(b\,x^2+a)^{3/2}}$$
$$-\frac{A}{2\,a\,x^2\,(b\,x^2+a)^{3/2}}+\frac{5\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{7/2}}-\frac{5\,A\,b^2\,x^2}{2\,a^3\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x)/(x^3*(a + b*x^2)^(5/2)),x)
```

output
```
(B*a^2 - 8*B*(a + b*x^2)^2 + 4*B*a*(a + b*x^2))/(3*a^3*x*(a + b*x^2)^(3/2)
) - (10*A*b)/(3*a^2*(a + b*x^2)^(3/2)) - A/(2*a*x^2*(a + b*x^2)^(3/2)) + (
5*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(7/2)) - (5*A*b^2*x^2)/(2*a^3
*(a + b*x^2)^(3/2))
```

3.43 $\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx$

3.43.1 Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{\arcsin(x)}{2}$$

output
```
-1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

3.43.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}+\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[((1 - x)*x)/Sqrt[1 - x^2],x]
```

output
```
-1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.43.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {533, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \ 533$$

$$\frac{1}{2}\int -\frac{1-2x}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}x\sqrt{1-x^2}-\frac{1}{2}\int \frac{1-2x}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \ 455$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{1-x^2}}dx-2\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \ 223$$

$$\frac{1}{2}\left(-\arcsin(x)-2\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[((1 - x)*x)/Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + (-2*Sqrt[1 - x^2] - ArcSin[x])/2
```

3.43.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

3.43.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$	29
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\sqrt{-x^2+1}}{2\sqrt{\pi}}-\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\arcsin(x)\right)}{2\sqrt{\pi}}$	58

input
```
int((1-x)*x/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx=\frac{1}{2}\sqrt{-x^2+1}(x-2)+\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.43.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((1-x)*x/(-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.43.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

3.43.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

input
```
integrate((1-x)*x/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.43.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}\,dx = \left(\frac{x}{2}-1\right)\sqrt{1-x^2}-\frac{\operatorname{asin}(x)}{2}$$

input
```
int(-(x*(x - 1))/(1 - x^2)^(1/2),x)
```

output
```
(x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.44 $\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx$

3.44.1 Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}-\frac{\arcsin(x)}{2}$$

output
```
-1/2*arcsin(x)-1/2*(2-x)*(-x^2+1)^(1/2)
```

3.44.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = -\frac{1}{2}(2-x)\sqrt{1-x^2}+\arctan\left(\frac{\sqrt{1-x^2}}{1+x}\right)$$

input
```
Integrate[(x - x^2)/Sqrt[1 - x^2],x]
```

output
```
-1/2*((2 - x)*Sqrt[1 - x^2]) + ArcTan[Sqrt[1 - x^2]/(1 + x)]
```

3.44.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {2027, 533, 25, 455, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-x^2}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{(1-x)x}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{533}$$

$$\frac{1}{2}\int -\frac{1-2x}{\sqrt{1-x^2}}dx+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}x\sqrt{1-x^2}-\frac{1}{2}\int \frac{1-2x}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{455}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{1-x^2}}dx-2\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{1-x^2}x$$

$$\Big\downarrow \text{223}$$

$$\frac{1}{2}\left(-\arcsin(x)-2\sqrt{1-x^2}\right)+\frac{1}{2}\sqrt{1-x^2}x$$

input
```
Int[(x - x^2)/Sqrt[1 - x^2],x]
```

output
```
(x*Sqrt[1 - x^2])/2 + (-2*Sqrt[1 - x^2] - ArcSin[x])/2
```

3.44.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.44.4 Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

method	result	size
risch	$-\frac{(-2+x)(x^2-1)}{2\sqrt{-x^2+1}}-\frac{\arcsin(x)}{2}$	25
default	$\frac{x\sqrt{-x^2+1}}{2}-\frac{\arcsin(x)}{2}-\sqrt{-x^2+1}$	29
trager	$\left(-1+\frac{x}{2}\right)\sqrt{-x^2+1}+\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)}{2}$	45
meijerg	$-\frac{-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{-x^2+1}}{2\sqrt{\pi}}-\frac{i\left(i\sqrt{\pi}\,x\sqrt{-x^2+1}-i\sqrt{\pi}\,\arcsin(x)\right)}{2\sqrt{\pi}}$	58

input
```
int((-x^2+x)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-2+x)*(x^2-1)/(-x^2+1)^(1/2)-1/2*arcsin(x)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.15

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) + \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

input
```
integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) + arctan((sqrt(-x^2 + 1) - 1)/x)
```

3.44.6 Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.89

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{x\sqrt{1-x^2}}{2} - \sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2}$$

input
```
integrate((-x**2+x)/(-x**2+1)**(1/2),x)
```

output
```
x*sqrt(1 - x**2)/2 - sqrt(1 - x**2) - asin(x)/2
```

3.44.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 1)*x - sqrt(-x^2 + 1) - 1/2*arcsin(x)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.70

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

input
```
integrate((-x^2+x)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 1)*(x - 2) - 1/2*arcsin(x)
```

3.44.9 Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x-x^2}{\sqrt{1-x^2}}\,dx = \left(\frac{x}{2}-1\right)\sqrt{1-x^2} - \frac{\operatorname{asin}(x)}{2}$$

input
```
int((x - x^2)/(1 - x^2)^(1/2),x)
```

output
```
(x/2 - 1)*(1 - x^2)^(1/2) - asin(x)/2
```

3.45 $\int \frac{3+x^2}{-3+x^2}\,dx$

3.45.1 Optimal result

Integrand size = 13, antiderivative size = 17

$$\int \frac{3+x^2}{-3+x^2}\,dx = x-2\sqrt{3}\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)$$

output
```
x-2*arctanh(1/3*x*3^(1/2))*3^(1/2)
```

3.45.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.94

$$\int \frac{3+x^2}{-3+x^2}\,dx = x+\sqrt{3}\log\left(\sqrt{3}-x\right)-\sqrt{3}\log\left(\sqrt{3}+x\right)$$

input
```
Integrate[(3 + x^2)/(-3 + x^2),x]
```

output
```
x + Sqrt[3]*Log[Sqrt[3] - x] - Sqrt[3]*Log[Sqrt[3] + x]
```

3.45.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {299, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+3}{x^2-3}\,dx$$

$$\Big\downarrow \text{ 299}$$

$$6\int \frac{1}{x^2-3}dx+x$$

$$\Big\downarrow \text{ 220}$$

$$x-2\sqrt{3}\text{arctanh}\left(\frac{x}{\sqrt{3}}\right)$$

input
```
Int[(3 + x^2)/(-3 + x^2),x]
```

output
```
x - 2*Sqrt[3]*ArcTanh[x/Sqrt[3]]
```

3.45.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.45.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.88

method	result	size
default	$x-2\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}$	15
risch	$x+\sqrt{3}\,\ln\left(x-\sqrt{3}\right)-\sqrt{3}\,\ln\left(x+\sqrt{3}\right)$	26
meijerg	$-\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\sqrt{3}-\frac{i\sqrt{3}\left(\frac{2i\sqrt{3}\,x}{3}-2i\,\operatorname{arctanh}\left(\frac{\sqrt{3}\,x}{3}\right)\right)}{2}$	38

input
```
int((x^2+3)/(x^2-3),x,method=_RETURNVERBOSE)
```

output
```
x-2*arctanh(1/3*3^(1/2)*x)*3^(1/2)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.53

$$\int \frac{3+x^2}{-3+x^2}\,dx=\sqrt{3}\log\left(\frac{x^2-2\,\sqrt{3}x+3}{x^2-3}\right)+x$$

input
```
integrate((x^2+3)/(x^2-3),x, algorithm="fricas")
```

output
```
sqrt(3)*log((x^2 - 2*sqrt(3)*x + 3)/(x^2 - 3)) + x
```

3.45.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.59

$$\int \frac{3+x^2}{-3+x^2}\,dx=x+\sqrt{3}\log\left(x-\sqrt{3}\right)-\sqrt{3}\log\left(x+\sqrt{3}\right)$$

input
```
integrate((x**2+3)/(x**2-3),x)
```

output
```
x + sqrt(3)*log(x - sqrt(3)) - sqrt(3)*log(x + sqrt(3))
```

3.45.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.29

$$\int \frac{3+x^2}{-3+x^2} \, dx = \sqrt{3} \log \left(\frac{x-\sqrt{3}}{x+\sqrt{3}} \right) + x$$

input
```
integrate((x^2+3)/(x^2-3),x, algorithm="maxima")
```

output
```
sqrt(3)*log((x - sqrt(3))/(x + sqrt(3))) + x
```

3.45.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int \frac{3+x^2}{-3+x^2} \, dx = \sqrt{3} \log \left(\frac{|2\,x-2\,\sqrt{3}|}{|2\,x+2\,\sqrt{3}|} \right) + x$$

input
```
integrate((x^2+3)/(x^2-3),x, algorithm="giac")
```

output
```
sqrt(3)*log(abs(2*x - 2*sqrt(3))/abs(2*x + 2*sqrt(3))) + x
```

3.45.9 Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

$$\int \frac{3+x^2}{-3+x^2} \, dx = x - 2\,\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,x}{3} \right)$$

input
```
int((x^2 + 3)/(x^2 - 3),x)
```

output
```
x - 2*3^(1/2)*atanh((3^(1/2)*x)/3)
```

3.46 $\int \frac{-1+x^2}{1+x^2} \, dx$

3.46.1 Optimal result

Integrand size = 13, antiderivative size = 6

$$\int \frac{-1+x^2}{1+x^2} \, dx = x - 2\arctan(x)$$

output
```
x-2*arctan(x)
```

3.46.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2} \, dx = x - 2\arctan(x)$$

input
```
Integrate[(-1 + x^2)/(1 + x^2),x]
```

output
```
x - 2*ArcTan[x]
```

3.46.3 Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2-1}{x^2+1}dx$$

$$\Big\downarrow \text{299}$$

$$x-2\int \frac{1}{x^2+1}dx$$

$$\Big\downarrow \text{216}$$

$$x-2\arctan(x)$$

input
```
Int[(-1 + x^2)/(1 + x^2),x]
```

output
```
x - 2*ArcTan[x]
```

3.46.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.46.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$x - 2\arctan\left(x\right)$	7
meijerg	$x - 2\arctan\left(x\right)$	7
risch	$x - 2\arctan\left(x\right)$	7
parallelrisch	$x + i\ln\left(x - i\right) - i\ln\left(x + i\right)$	19

input
```
int((x^2-1)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
x-2*arctan(x)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\,\arctan\left(x\right)$$

input
```
integrate((x^2-1)/(x^2+1),x, algorithm="fricas")
```

output
```
x - 2*arctan(x)
```

3.46.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.83

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\,\mathrm{atan}\left(x\right)$$

input
```
integrate((x**2-1)/(x**2+1),x)
```

output
```
x - 2*atan(x)
```

3.46.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

input
```
integrate((x^2-1)/(x^2+1),x, algorithm="maxima")
```

output
```
x - 2*arctan(x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\arctan(x)$$

input
```
integrate((x^2-1)/(x^2+1),x, algorithm="giac")
```

output
```
x - 2*arctan(x)
```

3.46.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{-1+x^2}{1+x^2}\,dx = x - 2\,\mathrm{atan}(x)$$

input
```
int((x^2 - 1)/(x^2 + 1),x)
```

output
```
x - 2*atan(x)
```

3.47 $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.47.1 Optimal result

Integrand size = 25, antiderivative size = 213

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx=-\frac{x^7(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}}-\frac{x^5(7aB-(Ab-8aC)x)}{35ab^2(a+bx^2)^{5/2}}-\frac{x^3(35aB-6(Ab-8aC)x)}{105ab^3(a+bx^2)^{3/2}}-\frac{x(35aB-8(Ab-8aC)x)}{35ab^4\sqrt{a+bx^2}}-\frac{16(Ab-8aC)\sqrt{a+bx^2}}{35ab^5}+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
-1/7*x^7*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^5*(7*B*a-(A*b-8*C*a)
*x)/a/b^2/(b*x^2+a)^(5/2)-1/105*x^3*(35*B*a-6*(A*b-8*C*a)*x)/a/b^3/(b*x^2+
a)^(3/2)+B*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)-1/35*x*(35*B*a-8*(A*
b-8*C*a)*x)/a/b^4/(b*x^2+a)^(1/2)-16/35*(A*b-8*C*a)*(b*x^2+a)^(1/2)/a/b^5
```

3.47.2 Mathematica [A] (verified)

Time = 1.05 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.73

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx=\frac{384a^4C-3a^3b(16A+7x(5B-64Cx))+14a^2b^2x^2(-12A+5x(-5B+24Cx)}{}$$

input
```
Integrate[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(384*a^4*C - 3*a^3*b*(16*A + 7*x*(5*B - 64*C*x)) + 14*a^2*b^2*x^2*(-12*A +
 5*x*(-5*B + 24*C*x)) + 14*a*b^3*x^4*(-15*A + x*(-29*B + 60*C*x)) + b^4*x^
6*(-105*A + x*(-176*B + 105*C*x)) - 105*Sqrt[b]*B*(a + b*x^2)^(7/2)*Log[-(
Sqrt[b]*x) + Sqrt[a + b*x^2]])/(105*b^5*(a + b*x^2)^(7/2))
```

3.47.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2335, 25, 530, 25, 2345, 2345, 27, 455, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7 \left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^6(7aB-(Ab-8aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^6(7aB-(Ab-8aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{\int -\frac{-5a\left(A-\frac{8aC}{b}\right)x^5+\frac{35a^2Bx^4}{b}+\frac{5a^2(Ab-8aC)x^3}{b^2}-\frac{35a^3Bx^2}{b^2}-\frac{5a^3(Ab-8aC)x}{b^3}+\frac{7a^4B}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-5a\left(A-\frac{8aC}{b}\right)x^5+\frac{35a^2Bx^4}{b}+\frac{5a^2(Ab-8aC)x^3}{b^2}-\frac{35a^3Bx^2}{b^2}-\frac{5a^3(Ab-8aC)x}{b^3}+\frac{7a^4B}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{\frac{56Ba^4}{b^3}-\frac{105Bx^2a^3}{b^2}-\frac{30(Ab-8aC)xa^3}{b^3}+\frac{15(Ab-8aC)x^3a^2}{b^2}}{\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2345$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{\int \frac{15a^3(7aB-(Ab-8aC)x)}{b^3\sqrt{bx^2+a}}dx}{a}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\int \frac{7aB-(Ab-8aC)x}{\sqrt{bx^2+a}}dx}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 455$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(7aB\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 224$$

3.47. $\int \frac{x^7\left(A+Bx+Cx^2\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(7aB\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{a^3(15(Ab-8aC)+77bBx)}{3b^4\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^3(45(Ab-8aC)+161bBx)}{b^4\sqrt{a+bx^2}}-\frac{15a^2\left(\frac{7aB\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}(Ab-8aC)}{b}\right)}{b^3}}{3a}}{5a}-\frac{a^3(-8aC+Ab+7bBx)}{5b^4(a+bx^2)^{5/2}}}{7ab}-\frac{x^7(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^7*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^3*(A*b
 - 8*a*C + 7*b*B*x))/(b^4*(a + b*x^2)^(5/2)) + ((a^3*(15*(A*b - 8*a*C) + 7
7*b*B*x))/(3*b^4*(a + b*x^2)^(3/2)) - ((a^3*(45*(A*b - 8*a*C) + 161*b*B*x)
)/(b^4*Sqrt[a + b*x^2]) - (15*a^2*(-(((A*b - 8*a*C)*Sqrt[a + b*x^2])/b) +
(7*a*B*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b]))/b^3)/(3*a))/(5*a))/
(7*a*b)
```

3.47.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.47. $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.47.4 Maple [A] (verified)

Time = 3.71 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.38

method	result
default	$C\left(\frac{x^{8}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{8a\left(-\frac{x^{6}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)}{b}\right)+B\left(-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right.$
risch	Expression too large to display

input
```
int(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
C*(x^8/b/(b*x^2+a)^(7/2)-8*a/b*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(
b*x^2+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7
/2)))))+B*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*
(-1/3*x^3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/
2)+(b*x^2+a)^(1/2))))))+A*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2
+a)^(7/2)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2)))
)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.45

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{b}\right.}{}\right.$$

$$-\frac{105\,(Bb^4x^8+4\,Bab^3x^6+6\,Ba^2b^2x^4+4\,Ba^3bx^2+Ba^4)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(105\,Cb^4x^8-176\,Bb^4x^7}{10}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 +
 B*a^4)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + 2*(105*C
*b^4*x^8 - 176*B*b^4*x^7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*
a*b^3 - A*b^4)*x^6 - 105*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2
*b^2 - A*a*b^3)*x^4 + 168*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b
^9*x^8 + 4*a*b^8*x^6 + 6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5), -1/105*(1
05*(B*b^4*x^8 + 4*B*a*b^3*x^6 + 6*B*a^2*b^2*x^4 + 4*B*a^3*b*x^2 + B*a^4)*s
qrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (105*C*b^4*x^8 - 176*B*b^4*x^
7 - 406*B*a*b^3*x^5 - 350*B*a^2*b^2*x^3 + 105*(8*C*a*b^3 - A*b^4)*x^6 - 10
5*B*a^3*b*x + 384*C*a^4 - 48*A*a^3*b + 210*(8*C*a^2*b^2 - A*a*b^3)*x^4 + 1
68*(8*C*a^3*b - A*a^2*b^2)*x^2)*sqrt(b*x^2 + a))/(b^9*x^8 + 4*a*b^8*x^6 +
6*a^2*b^7*x^4 + 4*a^3*b^6*x^2 + a^4*b^5)]
```

3.47.6 Sympy [A] (verification not implemented)

Time = 34.30 (sec) , antiderivative size = 3806, normalized size of antiderivative = 17.87

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**7*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5*x**2*
sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*sqrt(a
+ b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b**5
*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6*s
qrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**
2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*
x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**2) + 105
*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b
**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True)) + B*(105
*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205
/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 +
 b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**
(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x*
*8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a)
 + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2)*b*
*46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(9
9/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x**2/a
) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/2)*b
**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sqrt(1
 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 105...
```

3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(189) = 378$.

Time = 0.22 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.04

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{Cx^8}{(bx^2+a)^{\frac{7}{2}}b} - \frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b} + \frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2} + \frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3} + \frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Bx + \frac{8\,Cax^6}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b} + \frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2} + \frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Bx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b} + \frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} + \frac{16\,Ca^2x^4}{(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bax^3}{(bx^2+a)^{\frac{5}{2}}b^3} + \frac{64\,Ca^3x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{139\,Bx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Bax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Ba^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{B\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}} + \frac{128\,Ca^4}{35\,(bx^2+a)^{\frac{7}{2}}b^5} - \frac{16\,Aa^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
C*x^8/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^
4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((
b*x^2 + a)^(7/2)*b^4))*B*x + 8*C*a*x^6/((b*x^2 + a)^(7/2)*b^2) - A*x^6/((b
*x^2 + a)^(7/2)*b) - 1/15*B*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b
*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/3*B*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 + 16*C*a^2*x^4/((b
*x^2 + a)^(7/2)*b^3) - 2*A*a*x^4/((b*x^2 + a)^(7/2)*b^2) - B*a*x^3/((b*x^2
 + a)^(5/2)*b^3) + 64/5*C*a^3*x^2/((b*x^2 + a)^(7/2)*b^4) - 8/5*A*a^2*x^2/
((b*x^2 + a)^(7/2)*b^3) + 139/105*B*x/(sqrt(b*x^2 + a)*b^4) + 17/105*B*a*x
/((b*x^2 + a)^(3/2)*b^4) - 29/35*B*a^2*x/((b*x^2 + a)^(5/2)*b^4) + B*arcsi
nh(b*x/sqrt(a*b))/b^(9/2) + 128/35*C*a^4/((b*x^2 + a)^(7/2)*b^5) - 16/35*A
*a^3/((b*x^2 + a)^(7/2)*b^4)
```

3.47. $\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.47.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.96

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(\left(\left(\frac{105\,Cx}{b}-\frac{176\,B}{b}\right)x+\frac{105\,(8\,Ca^4b^7-Aa^3b^8)}{a^3b^9}\right)x-\frac{406\,Ba}{b^2}\right)x+\frac{210\,(8\,Ca^5b^6-Aa^4b^7)}{a^3b^9}\right.\right.\right.\right.}{105\,(bx^2+a)} - \frac{B\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^7*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((((((((105*C*x/b - 176*B/b)*x + 105*(8*C*a^4*b^7 - A*a^3*b^8)/(a^3*
b^9))*x - 406*B*a/b^2)*x + 210*(8*C*a^5*b^6 - A*a^4*b^7)/(a^3*b^9))*x - 35
0*B*a^2/b^3)*x + 168*(8*C*a^6*b^5 - A*a^5*b^6)/(a^3*b^9))*x - 105*B*a^3/b^
4)*x + 48*(8*C*a^7*b^4 - A*a^6*b^5)/(a^3*b^9))/(b*x^2 + a)^(7/2) - B*log(a
bs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^7(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^7\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^7*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.48 $\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

3.48.1 Optimal result

Integrand size = 25, antiderivative size = 150

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^6(aB-(Ab-aC)x)}{7ab\left(a+bx^2\right)^{7/2}} - \frac{x^4(6B+7Cx)}{35b^2\left(a+bx^2\right)^{5/2}} - \frac{x^2(24B+35Cx)}{105b^3\left(a+bx^2\right)^{3/2}} - \frac{16B+35Cx}{35b^4\sqrt{a+bx^2}} + \frac{C\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
-1/7*x^6*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(7*C*x+6*B)/b^2/(b
*x^2+a)^(5/2)-1/105*x^2*(35*C*x+24*B)/b^3/(b*x^2+a)^(3/2)+C*arctanh(x*b^(1
/2)/(b*x^2+a)^(1/2))/b^(9/2)+1/35*(-35*C*x-16*B)/b^4/(b*x^2+a)^(1/2)
```

3.48.2 Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{15Ab^4x^7 - 14a^3bx^2(12B+25Cx) - 14a^2b^2x^4(15B+29Cx) - 3a^4(16B+35C}{105ab^4\left(a+bx^2\right)^{7/2}} - \frac{C\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(15*A*b^4*x^7 - 14*a^3*b*x^2*(12*B + 25*C*x) - 14*a^2*b^2*x^4*(15*B + 29*C
*x) - 3*a^4*(16*B + 35*C*x) - a*b^3*x^6*(105*B + 176*C*x))/(105*a*b^4*(a +
 b*x^2)^(7/2)) - (C*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

3.48.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2335, 25, 27, 530, 25, 2345, 2345, 27, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6 (A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{ax^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{ax^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{x^5(6B+7Cx)}{(bx^2+a)^{7/2}}dx}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{\int -\frac{\frac{35aCx^4}{b}+\frac{30aBx^3}{b}-\frac{35a^2Cx^2}{b^2}-\frac{30a^2Bx}{b^2}+\frac{7a^3C}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{\frac{35aCx^4}{b}+\frac{30aBx^3}{b}-\frac{35a^2Cx^2}{b^2}-\frac{30a^2Bx}{b^2}+\frac{7a^3C}{b^3}}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 2345

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{\frac{56Ca^3}{b^3}-\frac{105Cx^2a^2}{b^2}-\frac{90Bxa^2}{b^2}}{\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 2345

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{\int \frac{105a^3C}{b^3\sqrt{bx^2+a}}dx}{a}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\int \frac{1}{\sqrt{bx^2+a}}dx}{b^3}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b^3}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{a^2(60B+77Cx)}{3b^3\left(a+bx^2\right)^{3/2}}-\frac{\frac{a^2(90B+161Cx)}{b^3\sqrt{a+bx^2}}-\frac{105a^2C\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{7/2}}}{3a}}{5a}-\frac{a^2(6B+7Cx)}{5b^3(a+bx^2)^{5/2}}}{7b}-\frac{x^6(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^6*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^2*(6*B
 + 7*C*x))/(b^3*(a + b*x^2)^(5/2)) + ((a^2*(60*B + 77*C*x))/(3*b^3*(a + b*
x^2)^(3/2)) - ((a^2*(90*B + 161*C*x))/(b^3*Sqrt[a + b*x^2]) - (105*a^2*C*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(7/2))/(3*a))/(5*a))/(7*b)
```

3.48. $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.48.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.48. $\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.48.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(128) = 256$.

Time = 3.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.23

method	result
default	$C\left(-\frac{x^{7}}{7b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5b\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3b\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^{2}+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+B\left(-\frac{x^{6}}{b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{6a\left(-\frac{\ldots}{3b\left(\ldots\right.}\right.}{\ldots}\right.$

3.48. $\int \frac{x^6\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

input
```
int(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
C*(-1/7*x^7/b/(b*x^2+a)^(7/2)+1/b*(-1/5*x^5/b/(b*x^2+a)^(5/2)+1/b*(-1/3*x^
3/b/(b*x^2+a)^(3/2)+1/b*(-x/b/(b*x^2+a)^(1/2)+1/b^(3/2)*ln(x*b^(1/2)+(b*x^
2+a)^(1/2))))))+B*(-x^6/b/(b*x^2+a)^(7/2)+6*a/b*(-1/3*x^4/b/(b*x^2+a)^(7/2
)+4/3*a/b*(-1/5*x^2/b/(b*x^2+a)^(7/2)-2/35*a/b^2/(b*x^2+a)^(7/2))))+A*(-1/
2*x^5/b/(b*x^2+a)^(7/2)+5/2*a/b*(-1/4*x^3/b/(b*x^2+a)^(7/2)+3/4*a/b*(-1/6*
x/b/(b*x^2+a)^(7/2)+1/6*a/b*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2
+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))))
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 467, normalized size of antiderivative = 3.11

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{b}\log\left(-2\,bx^2-2\right.}{}\right.$$

$$-\frac{105\left(Cab^4x^8+4\,Ca^2b^3x^6+6\,Ca^3b^2x^4+4\,Ca^4bx^2+Ca^5\right)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Bab^4x^6+406\,Ca^2}{105\left(ab^9x^8+4\,a^2b^8x^6+\right.}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3*b^2*x^4 + 4*C*a^4*b*x
^2 + C*a^5)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(1
05*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3*b^2*x^3
 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*x + 48*B
*a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^4 + 4*a^
4*b^6*x^2 + a^5*b^5), -1/105*(105*(C*a*b^4*x^8 + 4*C*a^2*b^3*x^6 + 6*C*a^3
*b^2*x^4 + 4*C*a^4*b*x^2 + C*a^5)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 +
a)) + (105*B*a*b^4*x^6 + 406*C*a^2*b^3*x^5 + 210*B*a^2*b^3*x^4 + 350*C*a^3
*b^2*x^3 + 168*B*a^3*b^2*x^2 + (176*C*a*b^4 - 15*A*b^5)*x^7 + 105*C*a^4*b*
x + 48*B*a^4*b)*sqrt(b*x^2 + a))/(a*b^9*x^8 + 4*a^2*b^8*x^6 + 6*a^3*b^7*x^
4 + 4*a^4*b^6*x^2 + a^5*b^5)]
```

3.48.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(133) = 266$.

Time = 44.84 (sec) , antiderivative size = 3448, normalized size of antiderivative = 22.99

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**6*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2
/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt
(1 + b*x**2/a)) + B*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 1
05*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35
*b**7*x**6*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**
2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a +
 b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a +
b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt
(a + b*x**2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(
a + b*x**2) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)
), True)) + C*(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqr
t(a))/(105*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(10
1/2)*x**2*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*
x**2/a) + 2100*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(19
7/2)*b**(107/2)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*
sqrt(1 + b*x**2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) +
 630*a**(203/2)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(10
5*a**(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2
*sqrt(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a...
```

3.48.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(127) = 254$.

Time = 0.22 (sec) , antiderivative size = 447, normalized size of antiderivative = 2.98

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx =$$
$$-\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Cx$$
$$-\frac{Bx^6}{(bx^2+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b}$$
$$-\frac{Ax^5}{2\,(bx^2+a)^{\frac{7}{2}}b}-\frac{Cx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{2\,Bax^4}{(bx^2+a)^{\frac{7}{2}}b^2}$$
$$-\frac{Cax^3}{(bx^2+a)^{\frac{5}{2}}b^3}-\frac{5\,Aax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{8\,Ba^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{139\,Cx}{105\,\sqrt{bx^2+a}b^4}$$
$$+\frac{17\,Cax}{105\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{29\,Ca^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4}+\frac{Ax}{14\,(bx^2+a)^{\frac{3}{2}}b^3}+\frac{Ax}{7\,\sqrt{bx^2+a}ab^3}$$
$$+\frac{3\,Aax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Aa^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{C\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}-\frac{16\,Ba^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*x -
B*x^6/((b*x^2 + a)^(7/2)*b) - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*
a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*A*x
^5/((b*x^2 + a)^(7/2)*b) - 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*
x^2 + a)^(3/2)*b^2))/b^2 - 2*B*a*x^4/((b*x^2 + a)^(7/2)*b^2) - C*a*x^3/((b
*x^2 + a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 8/5*B*a^2*x^2
/((b*x^2 + a)^(7/2)*b^3) + 139/105*C*x/(sqrt(b*x^2 + a)*b^4) + 17/105*C*a*
x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*A
*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*A*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*A*a*x/
((b*x^2 + a)^(5/2)*b^3) - 15/56*A*a^2*x/((b*x^2 + a)^(7/2)*b^3) + C*arcsin
h(b*x/sqrt(a*b))/b^(9/2) - 16/35*B*a^3/((b*x^2 + a)^(7/2)*b^4)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(\left(\left(\left(\left(x\left(\frac{105\,B}{b}+\frac{(176\,Ca^3b^7-15\,Aa^2b^8)x}{a^3b^8}\right)+\frac{406\,Ca}{b^2}\right)x+\frac{210\,Ba}{b^2}\right)x+\frac{350\,Ca^2}{b^3}\right)x+\frac{168\,Ba^2}{b^3}\right)x+\frac{105\,Ca^3}{b^4}\right)x+\frac{48\,Ba^3}{b^4}}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{C\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^6*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((((((x*(105*B/b + (176*C*a^3*b^7 - 15*A*a^2*b^8)*x/(a^3*b^8)) + 40
6*C*a/b^2)*x + 210*B*a/b^2)*x + 350*C*a^2/b^3)*x + 168*B*a^2/b^3)*x + 105*
C*a^3/b^4)*x + 48*B*a^3/b^4)/(b*x^2 + a)^(7/2) - C*log(abs(-sqrt(b)*x + sq
rt(b*x^2 + a)))/b^(9/2)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^6\,(C\,x^2+B\,x+A)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^6*(A + B*x + C*x^2))/(a + b*x^2)^(9/2), x)
```

3.49 $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.49.1 Optimal result

Integrand size = 25, antiderivative size = 132

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^5(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^4(Ab+6aC-5bBx)}{35ab^2\,(a+bx^2)^{5/2}} + \frac{4(Ab+6aC)}{105b^4\,(a+bx^2)^{3/2}} - \frac{4(Ab+6aC)}{35ab^4\sqrt{a+bx^2}}$$

output
```
-1/7*x^5*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^4*(-5*B*b*x+A*b+6*C*
a)/a/b^2/(b*x^2+a)^(5/2)+4/105*(A*b+6*C*a)/b^4/(b*x^2+a)^(3/2)-4/35*(A*b+6
*C*a)/a/b^4/(b*x^2+a)^(1/2)
```

3.49.2 Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-48a^4C+15b^4Bx^7-35ab^3x^4(A+3Cx^2)-14a^2b^2x^2(2A+15Cx^2)-8a^3b(A+}{105ab^4\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-48*a^4*C + 15*b^4*B*x^7 - 35*a*b^3*x^4*(A + 3*C*x^2) - 14*a^2*b^2*x^2*(2
*A + 15*C*x^2) - 8*a^3*b*(A + 21*C*x^2))/(105*a*b^4*(a + b*x^2)^(7/2))
```

3.49.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {2335, 25, 530, 27, 2345, 27, 453}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 (A+Bx+Cx^2)}{(a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^4(5aB+(Ab+6aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4(5aB+(Ab+6aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{-\frac{\int \frac{5\left(\frac{Ba^3}{b^2}-\frac{5Bx^2a^2}{b}+\frac{(Ab+6aC)xa^2}{b^2}-\left(A+\frac{6aC}{b}\right)x^3a\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\int \frac{\frac{Ba^3}{b^2}-\frac{5Bx^2a^2}{b}+\frac{(Ab+6aC)xa^2}{b^2}-\left(A+\frac{6aC}{b}\right)x^3a}{\left(bx^2+a\right)^{5/2}}dx}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{-\frac{-\frac{\int \frac{3a^2(aB+(Ab+6aC)x)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\frac{a\int \frac{aB+(Ab+6aC)x}{\left(bx^2+a\right)^{3/2}}dx}{b^2}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 453$$

$$\frac{-\frac{a^2(6aC+Ab-5bBx)}{5b^3(a+bx^2)^{5/2}}-\frac{\frac{a(6aC+Ab-bBx)}{b^3\sqrt{a+bx^2}}-\frac{2a^2(6aC+Ab-3bBx)}{3b^3\left(a+bx^2\right)^{3/2}}}{a}}{7ab}-\frac{x^5(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^5*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(a^2*(A*b
 + 6*a*C - 5*b*B*x))/(b^3*(a + b*x^2)^(5/2)) - ((-2*a^2*(A*b + 6*a*C - 3*b
*B*x))/(3*b^3*(a + b*x^2)^(3/2)) + (a*(A*b + 6*a*C - b*B*x))/(b^3*Sqrt[a +
 b*x^2]))/a)/(7*a*b)
```

3.49.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 453
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-(a*
d - b*c*x)/(a*b*Sqrt[a + b*x^2]), x] /; FreeQ[{a, b, c, d}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.49.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.72

method	result
gosper	$-\frac{-15x^7 B b^4+105C x^6 a b^3+35Aa b^3 x^4+210C a^2 b^2 x^4+28A a^2 b^2 x^2+168C a^3 b x^2+8A a^3 b+48C a^4}{105(b x^2+a)^{\frac{7}{2}} a b^4}$
trager	$-\frac{-15x^7 B b^4+105C x^6 a b^3+35Aa b^3 x^4+210C a^2 b^2 x^4+28A a^2 b^2 x^2+168C a^3 b x^2+8A a^3 b+48C a^4}{105(b x^2+a)^{\frac{7}{2}} a b^4}$
default	$C\left(-\frac{x^6}{b(b x^2+a)^{\frac{7}{2}}}+\frac{6a\left(-\frac{x^4}{3b\left(b x^2+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^2}{5b\left(b x^2+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^2\left(b x^2+a\right)^{\frac{7}{2}}}\right)}{3b}\right)}{b}\right)+B\left(-\frac{x^5}{2b(b x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b x^2+\cdots\right.}\cdots\right.}{\cdots}\right.$

3.49. $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

input
```
int(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-15*B*b^4*x^7+105*C*a*b^3*x^6+35*A*a*b^3*x^4+210*C*a^2*b^2*x^4+28*
A*a^2*b^2*x^2+168*C*a^3*b*x^2+8*A*a^3*b+48*C*a^4)/(b*x^2+a)^(7/2)/a/b^4
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(15\,Bb^4x^7 - 105\,Cab^3x^6 - 48\,Ca^4 - 8\,Aa^3b - 35\,(6\,Ca^2b^2 + Aab^3)x^4 - 28\,(6\,Ca}{105\,(ab^8x^8 + 4\,a^2b^7x^6 + 6\,a^3b^6x^4 + 4\,a^4b^5x^2 + a^5b^4)}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(15*B*b^4*x^7 - 105*C*a*b^3*x^6 - 48*C*a^4 - 8*A*a^3*b - 35*(6*C*a^2
*b^2 + A*a*b^3)*x^4 - 28*(6*C*a^3*b + A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a*b
^8*x^8 + 4*a^2*b^7*x^6 + 6*a^3*b^6*x^4 + 4*a^4*b^5*x^2 + a^5*b^4)
```

3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(121) = 242.

Time = 20.66 (sec) , antiderivative size = 740, normalized size of antiderivative = 5.61

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\left\{\begin{matrix} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{}{105a^3b^3\sqrt{a+bx^2}+} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{matrix}\right.\right.$$
$$+ \frac{Bx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}} + 21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}} + 7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$
$$+C\left(\left\{\begin{matrix} -\frac{16a^3}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+35b^7x^6\sqrt{a+bx^2}} - \frac{56a^2bx^2}{35a^3b^4\sqrt{a+bx^2}+105a^2b^5x^2\sqrt{a+bx^2}+105ab^6x^4\sqrt{a+bx^2}+} \\ \frac{x^8}{8a^{\frac{9}{2}}} \end{matrix}\right.\right.$$

input
```
integrate(x**5*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*
sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a
 + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*
x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*s
qrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2
*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*
x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + B*x**7/(7*
a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a
**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**
2/a)) + C*Piecewise((-16*a**3/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a**2*b*
*5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**7*x**6
*sqrt(a + b*x**2)) - 56*a**2*b*x**2/(35*a**3*b**4*sqrt(a + b*x**2) + 105*a
**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) + 35*b**
7*x**6*sqrt(a + b*x**2)) - 70*a*b**2*x**4/(35*a**3*b**4*sqrt(a + b*x**2) +
 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2) +
35*b**7*x**6*sqrt(a + b*x**2)) - 35*b**3*x**6/(35*a**3*b**4*sqrt(a + b*x**
2) + 105*a**2*b**5*x**2*sqrt(a + b*x**2) + 105*a*b**6*x**4*sqrt(a + b*x**2
) + 35*b**7*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**8/(8*a**(9/2)), True))
```

3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(116) = 232$.

Time = 0.21 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.82

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^6}{(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{2\,Cax^4}{(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Bax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2x^2}{5\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{4\,Aax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Bx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Bx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Bax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ba^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{16\,Ca^3}{35\,(bx^2+a)^{\frac{7}{2}}b^4} - \frac{8\,Aa^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-C*x^6/((b*x^2 + a)^(7/2)*b) - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) - 2*C*a*x^4
/((b*x^2 + a)^(7/2)*b^2) - 1/3*A*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*B*a*x^3/(
(b*x^2 + a)^(7/2)*b^2) - 8/5*C*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) - 4/15*A*a*
x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(
sqrt(b*x^2 + a)*a*b^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*
x/((b*x^2 + a)^(7/2)*b^3) - 16/35*C*a^3/((b*x^2 + a)^(7/2)*b^4) - 8/105*A*
a^2/((b*x^2 + a)^(7/2)*b^3)
```

3.49.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(5\left(3\left(\frac{Bx}{a}-\frac{7C}{b}\right)x^2-\frac{7(6Ca^4b^2+Aa^3b^3)}{a^3b^4}\right)x^2-\frac{28(6Ca^5b+Aa^4b^2)}{a^3b^4}\right)x^2-\frac{8(6Ca^6+Aa^5b)}{a^3b^4}}{105(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^5*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((5*(3*(B*x/a - 7*C/b)*x^2 - 7*(6*C*a^4*b^2 + A*a^3*b^3)/(a^3*b^4))*
x^2 - 28*(6*C*a^5*b + A*a^4*b^2)/(a^3*b^4))*x^2 - 8*(6*C*a^6 + A*a^5*b)/(a
^3*b^4))/(b*x^2 + a)^(7/2)
```

3.49.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.48

$$\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{C}{3b^3}-\frac{7Ab-14Ca}{21ab^3}\right)}{b}-\frac{3Bx}{7b^3}}{(bx^2+a)^{3/2}} - \frac{\frac{a^2\left(\frac{A}{7b}-\frac{Ca}{7b^2}\right)}{b^2}+\frac{Ba^2x}{7b^3}}{(bx^2+a)^{7/2}} - \frac{\frac{C}{b^4}-\frac{Bx}{7ab^3}}{\sqrt{bx^2+a}} - \frac{\frac{a\left(\frac{7Ca^2-7Aab}{35ab^3}+\frac{a\left(\frac{C}{5b^2}-\frac{7Ab^2-7Cab}{35ab^3}\right)}{b}\right)}{b}-\frac{3Bax}{7b^3}}{(bx^2+a)^{5/2}}$$

input
```
int((x^5*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
((a*(C/(3*b^3) - (7*A*b - 14*C*a)/(21*a*b^3)))/b - (3*B*x)/(7*b^3))/(a + b
*x^2)^(3/2) - ((a^2*(A/(7*b) - (C*a)/(7*b^2)))/b^2 + (B*a^2*x)/(7*b^3))/(a
 + b*x^2)^(7/2) - (C/b^4 - (B*x)/(7*a*b^3))/(a + b*x^2)^(1/2) - ((a*((7*C*
a^2 - 7*A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a*b^3)
))/b))/b - (3*B*a*x)/(7*b^3))/(a + b*x^2)^(5/2)
```

3.49. $\int \frac{x^5(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.50 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.50.1 Optimal result

Integrand size = 25, antiderivative size = 149

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^4(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}} - \frac{x^2(4aB+(2Ab+5aC)x)}{35ab^2\,(a+bx^2)^{5/2}} - \frac{8aB+3(2Ab+5aC)x}{105ab^3\,(a+bx^2)^{3/2}} + \frac{(2Ab+5aC)x}{35a^2b^3\sqrt{a+bx^2}}$$

output
```
-1/7*x^4*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x^2*(4*B*a+(2*A*b+5*C*
a)*x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(-8*B*a-3*(2*A*b+5*C*a)*x)/a/b^3/(b*x^2+
a)^(3/2)+1/35*(2*A*b+5*C*a)*x/a^2/b^3/(b*x^2+a)^(1/2)
```

3.50.2 Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^4B-28a^3bBx^2-35a^2b^2Bx^4+21aAb^3x^5+6Ab^4x^7+15ab^3Cx^7}{105a^2b^3\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-8*a^4*B - 28*a^3*b*B*x^2 - 35*a^2*b^2*B*x^4 + 21*a*A*b^3*x^5 + 6*A*b^4*x
^7 + 15*a*b^3*C*x^7)/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

3.50.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2335, 25, 530, 2345, 27, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx+Cx^2)}{(a+bx^2)^{9/2}} dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^3(4aB+(2Ab+5aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^3(4aB+(2Ab+5aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{530}$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{\frac{(2Ab+5aC)a^2}{b^2}-\frac{20Bxa^2}{b}-5\left(2A+\frac{5aC}{b}\right)x^2a}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{2345}$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{3a^2(2Ab+5aC)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{a(5aC+2Ab)\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{b^2}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\frac{a\left(bx\left(\frac{5aC}{b}+2A\right)+4aB\right)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{2a\left(3bx\left(\frac{5aC}{b}+2A\right)+10aB\right)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{x(5aC+2Ab)}{b^2\sqrt{a+bx^2}}}{5a}}{7ab}-\frac{x^4(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^4*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + ((a*(4*a*B + b*
(2*A + (5*a*C)/b)*x))/(5*b^2*(a + b*x^2)^(5/2)) - ((2*a*(10*a*B + 3*b*(2*A
 + (5*a*C)/b)*x))/(3*b^2*(a + b*x^2)^(3/2)) - ((2*A*b + 5*a*C)*x)/(b^2*Sqr
t[a + b*x^2]))/(5*a))/(7*a*b)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.50.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

method	result
gosper	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35a^2B\,b^2x^4-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
trager	$\frac{6A\,b^4x^7+15Ca\,x^7b^3+21Aa\,b^3x^5-35a^2B\,b^2x^4-28B\,a^3b\,x^2-8B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^2b^3}$
default	$C\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\cdots\right.}{7a}}{a}\cdots\right.}{6b}\cdots\right.}{4b}\cdots\right.}{2b}\cdots\right.$

3.50. $\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

input
```
int(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(6*A*b^4*x^7+15*C*a*b^3*x^7+21*A*a*b^3*x^5-35*B*a^2*b^2*x^4-28*B*a^3
*b*x^2-8*B*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.82

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(21\,Aab^3x^5-35\,Ba^2b^2x^4+3\,(5\,Cab^3+2\,Ab^4)x^7-28\,Ba^3bx^2-8\,Ba^4)\sqrt{bx^2+}}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(21*A*a*b^3*x^5 - 35*B*a^2*b^2*x^4 + 3*(5*C*a*b^3 + 2*A*b^4)*x^7 - 2
8*B*a^3*b*x^2 - 8*B*a^4)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3*b^6*x^6 + 6*
a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

3.50.6 Sympy [A] (verification not implemented)

Time = 31.27 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.86

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1}}\right.$$
$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$
$$+B\left(\begin{cases}-\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}}-\frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+}}\\ \frac{x^6}{6a^{\frac{9}{2}}}\end{cases}\right.$$
$$+\frac{Cx^7}{7a^{\frac{9}{2}}\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{7}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+21a^{\frac{5}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+7a^{\frac{3}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}$$

input
```
integrate(x**4*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1
+ b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3
*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 10
5*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*Piecewise((-8*a**2
/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 3
15*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b
*x**2/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2
) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 3
5*b**2*x**4/(105*a**3*b**3*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a +
b*x**2) + 315*a*b**5*x**4*sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2
)), Ne(b, 0)), (x**6/(6*a**(9/2)), True)) + C*x**7/(7*a**(9/2)*sqrt(1 + b*
x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.70

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Bax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} + \frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Cx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Ax}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Ax}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Ax}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Aax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ba^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3*B*x^4/((b*x^2 + a)^(7/2)*b) - 5/8*C
*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*B*
a*x^2/((b*x^2 + a)^(7/2)*b^2) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x
/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^
2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x
/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a
*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*B*a^2/((b*x^2 + a)^(7/2)*b^3)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.54

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\,x\left(\frac{7\,A}{a}+\frac{(5\,Ca^2b^3+2\,Aab^4)x^2}{a^3b^3}\right)-\frac{35\,B}{b}\right)x^2-\frac{28\,Ba}{b^2}\right)x^2-\frac{8\,Ba^2}{b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^4*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((3*x*(7*A/a + (5*C*a^2*b^3 + 2*A*a*b^4)*x^2/(a^3*b^3)) - 35*B/b)*x
^2 - 28*B*a/b^2)*x^2 - 8*B*a^2/b^3)/(b*x^2 + a)^(7/2)
```

3.50.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{x\left(\frac{C\,a^2-A\,a\,b}{35\,a\,b^3}+\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b^2-7\,C\,a\,b}{35\,a\,b^3}\right)}{b}\right)+\frac{2\,B\,a}{5\,b^3}}{(b\,x^2+a)^{5/2}} - \frac{\frac{B}{3\,b^3}+x\left(\frac{C}{3\,b^3}-\frac{3\,A\,b-10\,C\,a}{105\,a\,b^3}\right)}{(b\,x^2+a)^{3/2}} - \frac{\frac{B\,a^2}{7\,b^3}-\frac{a\,x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}}{(b\,x^2+a)^{7/2}} + \frac{x\,(2\,A\,b+5\,C\,a)}{35\,a^2\,b^3\,\sqrt{b\,x^2+a}}$$

input
```
int((x^4*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(x*((C*a^2 - A*a*b)/(35*a*b^3) + (a*(C/(5*b^2) - (7*A*b^2 - 7*C*a*b)/(35*a
*b^3)))/b) + (2*B*a)/(5*b^3))/(a + b*x^2)^(5/2) - (B/(3*b^3) + x*(C/(3*b^3
) - (3*A*b - 10*C*a)/(105*a*b^3)))/(a + b*x^2)^(3/2) - ((B*a^2)/(7*b^3) -
(a*x*(A/(7*b) - (C*a)/(7*b^2)))/b)/(a + b*x^2)^(7/2) + (x*(2*A*b + 5*C*a))
/(35*a^2*b^3*(a + b*x^2)^(1/2))
```

3.51 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.51.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x^3(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}} - \frac{x(3aB+(3Ab+4aC)x)}{35ab^2(a+bx^2)^{5/2}} - \frac{2(3Ab+4aC)-3bBx}{105ab^3(a+bx^2)^{3/2}} + \frac{2Bx}{35a^2b^2\sqrt{a+bx^2}}$$

output
```
-1/7*x^3*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)-1/35*x*(3*B*a+(3*A*b+4*C*a)
*x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(3*B*b*x-6*A*b-8*C*a)/a/b^3/(b*x^2+a)^(3/2
)+2/35*B*x/a^2/b^2/(b*x^2+a)^(1/2)
```

3.51.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.60

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-8a^4C+21ab^3Bx^5+6b^4Bx^7-7a^2b^2x^2(3A+5Cx^2)-2a^3b(3A+14Cx^2)}{105a^2b^3(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-8*a^4*C + 21*a*b^3*B*x^5 + 6*b^4*B*x^7 - 7*a^2*b^2*x^2*(3*A + 5*C*x^2) -
 2*a^3*b*(3*A + 14*C*x^2))/(105*a^2*b^3*(a + b*x^2)^(7/2))
```

3.51.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2335, 25, 530, 25, 27, 454, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 (A+Bx+Cx^2)}{(a+bx^2)^{9/2}} \, dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^2(3aB+(3Ab+4aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^2(3aB+(3Ab+4aC)x)}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 530}$$

$$\frac{\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}-\frac{\int -\frac{a\left(3aB+5b\left(3A+\frac{4aC}{b}\right)x\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{a(3aB+5(3Ab+4aC)x)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\int \frac{3aB+5(3Ab+4aC)x}{\left(bx^2+a\right)^{5/2}}dx}{5b}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{2B\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx-\frac{5(4aC+3Ab)-3bBx}{3b\left(a+bx^2\right)^{3/2}}}{5b}+\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \textbf{208}$$

$$\frac{\frac{a(4aC+3Ab-3bBx)}{5b^2(a+bx^2)^{5/2}}+\frac{\frac{2Bx}{a\sqrt{a+bx^2}}-\frac{5(4aC+3Ab)-3bBx}{3b\left(a+bx^2\right)^{3/2}}}{5b}}{7ab}-\frac{x^3(aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^3*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + ((a*(3*A*b + 4*
a*C - 3*b*B*x))/(5*b^2*(a + b*x^2)^(5/2)) + (-1/3*(5*(3*A*b + 4*a*C) - 3*b
*B*x)/(b*(a + b*x^2)^(3/2)) + (2*B*x)/(a*Sqrt[a + b*x^2]))/(5*b))/(7*a*b)
```

3.51.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.51.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.61

method	result
gosper	$-\frac{-6x^{7}B\,b^{4}-21x^{5}Ba\,b^{3}+35C\,a^{2}b^{2}x^{4}+21A\,a^{2}b^{2}x^{2}+28C\,a^{3}b\,x^{2}+6A\,a^{3}b+8C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}b^{3}}$
trager	$-\frac{-6x^{7}B\,b^{4}-21x^{5}Ba\,b^{3}+35C\,a^{2}b^{2}x^{4}+21A\,a^{2}b^{2}x^{2}+28C\,a^{3}b\,x^{2}+6A\,a^{3}b+8C\,a^{4}}{105\left(b\,x^{2}+a\right)^{\frac{7}{2}}a^{2}b^{3}}$
default	$C\left(-\frac{x^{4}}{3b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{4a\left(-\frac{x^{2}}{5b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{2a}{35b^{2}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}\right)}{3b}\right)+B\left(-\frac{x^{3}}{4b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)\cdots}\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-6*B*b^4*x^7-21*B*a*b^3*x^5+35*C*a^2*b^2*x^4+21*A*a^2*b^2*x^2+28*C
*a^3*b*x^2+6*A*a^3*b+8*C*a^4)/(b*x^2+a)^(7/2)/a^2/b^3
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(6\,Bb^4x^7+21\,Bab^3x^5-35\,Ca^2b^2x^4-8\,Ca^4-6\,Aa^3b-7\,(4\,Ca^3b+3\,Aa^2b^2)x^2}{105\,(a^2b^7x^8+4\,a^3b^6x^6+6\,a^4b^5x^4+4\,a^5b^4x^2+a^6b^3)}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(6*B*b^4*x^7 + 21*B*a*b^3*x^5 - 35*C*a^2*b^2*x^4 - 8*C*a^4 - 6*A*a^3
*b - 7*(4*C*a^3*b + 3*A*a^2*b^2)*x^2)*sqrt(b*x^2 + a)/(a^2*b^7*x^8 + 4*a^3
*b^6*x^6 + 6*a^4*b^5*x^4 + 4*a^5*b^4*x^2 + a^6*b^3)
```

3.51.6 Sympy [A] (verification not implemented)

Time = 20.05 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.75

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{}{35a^3b^2\sqrt{a+bx^2}+105} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{cases}\right.$$

$$+B\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$

$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$

$$+C\left(\begin{cases} -\frac{8a^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+bx^2}+105b^6x^6\sqrt{a+bx^2}} - \frac{28abx^2}{105a^3b^3\sqrt{a+bx^2}+315a^2b^4x^2\sqrt{a+bx^2}+315ab^5x^4\sqrt{a+}} \\ \frac{x^6}{6a^{\frac{9}{2}}} \end{cases}\right.$$

input
```
integrate(x**3*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True)) + B*(7*a*x**5/(35*a**(11/2)*
sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)
*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))
+ 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 +
 b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*
x**6*sqrt(1 + b*x**2/a))) + C*Piecewise((-8*a**2/(105*a**3*b**3*sqrt(a + b
*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a + b*
x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 28*a*b*x**2/(105*a**3*b**3*sqrt(
a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*sqrt(a
 + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)) - 35*b**2*x**4/(105*a**3*b**3
*sqrt(a + b*x**2) + 315*a**2*b**4*x**2*sqrt(a + b*x**2) + 315*a*b**5*x**4*
sqrt(a + b*x**2) + 105*b**6*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**6/(6*a*
*(9/2)), True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.29

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^4}{3\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{4\,Cax^2}{15\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{8\,Ca^2}{105\,(bx^2+a)^{\frac{7}{2}}b^3} - \frac{2\,Aa}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/3*C*x^4/((b*x^2 + a)^(7/2)*b) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) - 4/15*
C*a*x^2/((b*x^2 + a)^(7/2)*b^2) - 1/5*A*x^2/((b*x^2 + a)^(7/2)*b) + 3/140*
B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*
x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 8/105*C
*a^2/((b*x^2 + a)^(7/2)*b^3) - 2/35*A*a/((b*x^2 + a)^(7/2)*b^2)
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(3\left(\frac{2\,Bbx^2}{a^2}+\frac{7\,B}{a}\right)x-\frac{35\,C}{b}\right)x^2-\frac{7\,(4\,Ca^4b+3\,Aa^3b^2)}{a^3b^3}\right)x^2-\frac{2\,(4\,Ca^5+3\,Aa^4b)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^3*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((3*(2*B*b*x^2/a^2 + 7*B/a)*x - 35*C/b)*x^2 - 7*(4*C*a^4*b + 3*A*a^
3*b^2)/(a^3*b^3))*x^2 - 2*(4*C*a^5 + 3*A*a^4*b)/(a^3*b^3))/(b*x^2 + a)^(7/
2)
```

3.51.9 Mupad [B] (verification not implemented)

Time = 5.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\frac{a\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)}{b}+\frac{B\,a\,x}{7\,b^2}}{(b\,x^2+a)^{7/2}}-\frac{\frac{C}{3\,b^3}-\frac{B\,x}{35\,a\,b^2}}{(b\,x^2+a)^{3/2}} + \frac{\frac{a\left(\frac{C}{5\,b^2}-\frac{7\,A\,b-7\,C\,a}{35\,a\,b^2}\right)}{b}-\frac{8\,B\,x}{35\,b^2}}{(b\,x^2+a)^{5/2}}+\frac{2\,B\,x}{35\,a^2\,b^2\,\sqrt{b\,x^2+a}}$$

input
```
int((x^3*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
((a*(A/(7*b) - (C*a)/(7*b^2)))/b + (B*a*x)/(7*b^2))/(a + b*x^2)^(7/2) - (C
/(3*b^3) - (B*x)/(35*a*b^2))/(a + b*x^2)^(3/2) + ((a*(C/(5*b^2) - (7*A*b -
 7*C*a)/(35*a*b^2)))/b - (8*B*x)/(35*b^2))/(a + b*x^2)^(5/2) + (2*B*x)/(35
*a^2*b^2*(a + b*x^2)^(1/2))
```

3.52 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.52.1 Optimal result

Integrand size = 25, antiderivative size = 139

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx=-\frac{x^2(aB-(Ab-aC)x)}{7ab\,(a+bx^2)^{7/2}}-\frac{2aB+(4Ab+3aC)x}{35ab^2\,(a+bx^2)^{5/2}}+\frac{(4Ab+3aC)x}{105a^2b^2\,(a+bx^2)^{3/2}}+\frac{2(4Ab+3aC)x}{105a^3b^2\sqrt{a+bx^2}}$$

output
```
-1/7*x^2*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(-2*B*a-(4*A*b+3*C*a)*
x)/a/b^2/(b*x^2+a)^(5/2)+1/105*(4*A*b+3*C*a)*x/a^2/b^2/(b*x^2+a)^(3/2)+2/1
05*(4*A*b+3*C*a)*x/a^3/b^2/(b*x^2+a)^(1/2)
```

3.52.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.63

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx=\frac{-6a^4B-21a^3bBx^2+8Ab^4x^7+7a^2b^2x^3(5A+3Cx^2)+2ab^3x^5(14A+3Cx^2)}{105a^3b^2\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-6*a^4*B - 21*a^3*b*B*x^2 + 8*A*b^4*x^7 + 7*a^2*b^2*x^3*(5*A + 3*C*x^2) +
 2*a*b^3*x^5*(14*A + 3*C*x^2))/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

3.52.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {2335, 25, 530, 25, 27, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+B x+C x^2\right)}{\left(a+b x^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \ \textbf{2335}$$

$$-\frac{\int -\frac{x(2aB+(4Ab+3aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{\int \frac{x(2aB+(4Ab+3aC)x)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{530}$$

$$\frac{-\frac{\int -\frac{a\left(4A+\frac{3aC}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{x(3aC+4Ab)+2aB}{5b\left(a+bx^2\right)^{5/2}}}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{25}$$

$$\frac{\frac{\int \frac{a\left(4A+\frac{3aC}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{x(3aC+4Ab)+2aB}{5b\left(a+bx^2\right)^{5/2}}}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{27}$$

$$\frac{\frac{1}{5}\left(\frac{3aC}{b}+4A\right)\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx-\frac{x(3aC+4Ab)+2aB}{5b\left(a+bx^2\right)^{5/2}}}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{209}$$

$$\frac{\frac{1}{5}\left(\frac{3aC}{b}+4A\right)\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)-\frac{x(3aC+4Ab)+2aB}{5b\left(a+bx^2\right)^{5/2}}}{7ab}-\frac{x^2 (aB-x(Ab-aC))}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ \textbf{208}$$

$$\frac{\frac{1}{5}\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)\left(\frac{3aC}{b}+4A\right)-\frac{x(3aC+4Ab)+2aB}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x^2(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

input
```
Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x^2*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(2*a*B +
(4*A*b + 3*a*C)*x)/(b*(a + b*x^2)^(5/2)) + ((4*A + (3*a*C)/b)*(x/(3*a*(a +
 b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/5)/(7*a*b)
```

3.52.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 530
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symb
ol] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Co
eff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Po
lynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x
)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(a
 + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Qx + e*(2*p + 3), x], x], x]] /;
FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && IGtQ[m, 0] && LtQ[p, -1] && EqQ[n,
1] && IntegerQ[2*p]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.52.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{8A\,b^{4}x^{7}+6Ca\,x^{7}b^{3}+28Aa\,b^{3}x^{5}+21C\,a^{2}x^{5}b^{2}+35A\,a^{2}b^{2}x^{3}-21B\,a^{3}b\,x^{2}-6B\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{3}b^{2}}$
trager	$\frac{8A\,b^{4}x^{7}+6Ca\,x^{7}b^{3}+28Aa\,b^{3}x^{5}+21C\,a^{2}x^{5}b^{2}+35A\,a^{2}b^{2}x^{3}-21B\,a^{3}b\,x^{2}-6B\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{3}b^{2}}$
default	$C\left(-\frac{x^{3}}{4b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)}{6b}\right)}{4b}\right)+B\Big($

input
```
int(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(8*A*b^4*x^7+6*C*a*b^3*x^7+28*A*a*b^3*x^5+21*C*a^2*b^2*x^5+35*A*a^2*
b^2*x^3-21*B*a^3*b*x^2-6*B*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(35\,Aa^2b^2x^3+2\,(3\,Cab^3+4\,Ab^4)x^7-21\,Ba^3bx^2+7\,(3\,Ca^2b^2+4\,Aab^3)x^5-6\,}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(35*A*a^2*b^2*x^3 + 2*(3*C*a*b^3 + 4*A*b^4)*x^7 - 21*B*a^3*b*x^2 + 7
*(3*C*a^2*b^2 + 4*A*a*b^3)*x^5 - 6*B*a^4)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4
*a^4*b^5*x^6 + 6*a^5*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

3.52.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(129) = 258$.

Time = 30.65 (sec) , antiderivative size = 904, normalized size of antiderivative = 6.50

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x\right.$$

$$+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+B\left(\begin{cases}-\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}}-\frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}} \\ \frac{x^4}{4a^{\frac{9}{2}}}\end{cases}\right.$$

$$+C\left(\frac{7ax^5}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right.$$

$$\left.+\frac{2bx^7}{35a^{\frac{11}{2}}\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{9}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{7}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+35a^{\frac{5}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}}\right)$$

input
```
integrate(x**2*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

3.52. $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

output
```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*Piecewise((-2*a/(35*a**3*b**2*sqrt(a
+ b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a +
 b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a
+ b*x**2) + 105*a**2*b**3*x**2*sqrt(a + b*x**2) + 105*a*b**4*x**4*sqrt(a +
 b*x**2) + 35*b**5*x**6*sqrt(a + b*x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)),
True)) + C*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**
2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(
5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*...
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.42

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ba}{35\,(bx^2+a)^{\frac{7}{2}}b^2}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*C*x^3/((b*x^2 + a)^(7/2)*b) - 1/5*B*x^2/((b*x^2 + a)^(7/2)*b) + 3/140
*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C
*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*
x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((
b*x^2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) - 2/35*B*a/((b*
x^2 + a)^(7/2)*b^2)
```

3.52.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.68

$$\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{2\,(3\,Cab^4+4\,Ab^5)x^2}{a^3b^3}+\frac{7\,(3\,Ca^2b^3+4\,Aab^4)}{a^3b^3}\right)+\frac{35\,A}{a}\right)x-\frac{21\,B}{b}\right)x^2-\frac{6\,Ba}{b^2}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x^2*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((x^2*(2*(3*C*a*b^4 + 4*A*b^5)*x^2/(a^3*b^3) + 7*(3*C*a^2*b^3 + 4*A
*a*b^4)/(a^3*b^3)) + 35*A/a)*x - 21*B/b)*x^2 - 6*B*a/b^2)/(b*x^2 + a)^(7/2
)
```

3.52.9 Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.96

$$\begin{aligned}\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx &= \frac{x\,(4\,A\,b+3\,C\,a)}{105\,a^2\,b^2\,(b\,x^2+a)^{3/2}}\\ &\quad-\frac{\frac{B}{5\,b^2}+x\left(\frac{C}{5\,b^2}-\frac{A\,b-C\,a}{35\,a\,b^2}\right)}{(b\,x^2+a)^{5/2}}-\frac{x\left(\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}\right)-\frac{B\,a}{7\,b^2}}{(b\,x^2+a)^{7/2}}+\frac{x\,(8\,A\,b+6\,C\,a)}{105\,a^3\,b^2\,\sqrt{b\,x^2+a}}\end{aligned}$$

input
```
int((x^2*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(x*(4*A*b + 3*C*a))/(105*a^2*b^2*(a + b*x^2)^(3/2)) - (B/(5*b^2) + x*(C/(5
*b^2) - (A*b - C*a)/(35*a*b^2)))/(a + b*x^2)^(5/2) - (x*(A/(7*b) - (C*a)/(
7*b^2)) - (B*a)/(7*b^2))/(a + b*x^2)^(7/2) + (x*(8*A*b + 6*C*a))/(105*a^3*
b^2*(a + b*x^2)^(1/2))
```

3.53 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx$

3.53.1 Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{x(aB-(Ab-aC)x)}{7ab(a+bx^2)^{7/2}} - \frac{5Ab+2aC-bBx}{35ab^2(a+bx^2)^{5/2}} + \frac{4Bx}{105a^2b(a+bx^2)^{3/2}} + \frac{8Bx}{105a^3b\sqrt{a+bx^2}}$$

output
```
-1/7*x*(B*a-(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(B*b*x-5*A*b-2*C*a)/a/b^
2/(b*x^2+a)^(5/2)+4/105*B*x/a^2/b/(b*x^2+a)^(3/2)+8/105*B*x/a^3/b/(b*x^2+a
)^(1/2)
```

3.53.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.64

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^3Ab-6a^4C-21a^3bCx^2+35a^2b^2Bx^3+28ab^3Bx^5+8b^4Bx^7}{105a^3b^2(a+bx^2)^{7/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
(-15*a^3*A*b - 6*a^4*C - 21*a^3*b*C*x^2 + 35*a^2*b^2*B*x^3 + 28*a*b^3*B*x^
5 + 8*b^4*B*x^7)/(105*a^3*b^2*(a + b*x^2)^(7/2))
```

3.53.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2335, 25, 454, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \; 2335$$

$$-\frac{\int -\frac{aB+(5Ab+2aC)x}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{aB+(5Ab+2aC)x}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 454$$

$$\frac{\frac{4}{5}B\int \frac{1}{(bx^2+a)^{5/2}}dx-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$\frac{\frac{4}{5}B\left(\frac{2\int \frac{1}{(bx^2+a)^{3/2}}dx}{3a}+\frac{x}{3a(a+bx^2)^{3/2}}\right)-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 208$$

$$\frac{\frac{4}{5}B\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a(a+bx^2)^{3/2}}\right)-\frac{2aC+5Ab-bBx}{5b(a+bx^2)^{5/2}}}{7ab}-\frac{x(aB-x(Ab-aC))}{7ab\,(a+bx^2)^{7/2}}$$

input
```
Int[(x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(x*(a*B - (A*b - a*C)*x))/(a*b*(a + b*x^2)^(7/2)) + (-1/5*(5*A*b + 2*
a*C - b*B*x)/(b*(a + b*x^2)^(5/2)) + (4*B*(x/(3*a*(a + b*x^2)^(3/2)) + (2*
x)/(3*a^2*Sqrt[a + b*x^2])))/5)/(7*a*b)
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.53.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result
gosper	$-\frac{-8x^{7}B\,b^{4}-28x^{5}Ba\,b^{3}-35B\,a^{2}b^{2}x^{3}+21C\,a^{3}b\,x^{2}+15A\,a^{3}b+6C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{3}b^{2}}$
trager	$-\frac{-8x^{7}B\,b^{4}-28x^{5}Ba\,b^{3}-35B\,a^{2}b^{2}x^{3}+21C\,a^{3}b\,x^{2}+15A\,a^{3}b+6C\,a^{4}}{105(b\,x^{2}+a)^{\frac{7}{2}}a^{3}b^{2}}$
default	$C\left(-\frac{x^{2}}{5b(b\,x^{2}+a)^{\frac{7}{2}}}-\frac{2a}{35b^{2}(b\,x^{2}+a)^{\frac{7}{2}}}\right)+B\left(-\frac{x}{6b(b\,x^{2}+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8\dots}{15a^{2}\sqrt{\dots}}\right)}{7a}}{a}\dots\right)}{6b}\dots\right.$

input
```
int(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/105*(-8*B*b^4*x^7-28*B*a*b^3*x^5-35*B*a^2*b^2*x^3+21*C*a^3*b*x^2+15*A*a
^3*b+6*C*a^4)/(b*x^2+a)^(7/2)/a^3/b^2
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,Bb^4x^7+28\,Bab^3x^5+35\,Ba^2b^2x^3-21\,Ca^3bx^2-6\,Ca^4-15\,Aa^3b)\sqrt{bx^2+a}}{105\,(a^3b^6x^8+4\,a^4b^5x^6+6\,a^5b^4x^4+4\,a^6b^3x^2+a^7b^2)}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*B*b^4*x^7 + 28*B*a*b^3*x^5 + 35*B*a^2*b^2*x^3 - 21*C*a^3*b*x^2 -
6*C*a^4 - 15*A*a^3*b)*sqrt(b*x^2 + a)/(a^3*b^6*x^8 + 4*a^4*b^5*x^6 + 6*a^5
*b^4*x^4 + 4*a^6*b^3*x^2 + a^7*b^2)
```

3.53.6 Sympy [A] (verification not implemented)

Time = 19.63 (sec) , antiderivative size = 796, normalized size of antiderivative = 6.69

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = A\left(\begin{cases} -\frac{1}{7a^3b\sqrt{a+bx^2}+21a^2b^2x^2\sqrt{a+bx^2}+21ab^3x^4\sqrt{a+bx^2}+7b^4x^6\sqrt{a+bx^2}} & \text{for } b\neq 0 \\ \frac{x^2}{2a^{\frac{9}{2}}} & \text{otherwise} \end{cases}\right)$$

$$+B\left(\frac{35a^5x^3}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{}}\right.$$

$$+\frac{63a^4bx^5}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{36a^3b^2x^7}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+\frac{8a^2b^3x^9}{105a^{\frac{19}{2}}\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{17}{2}}bx^2\sqrt{1+\frac{bx^2}{a}}+630a^{\frac{15}{2}}b^2x^4\sqrt{1+\frac{bx^2}{a}}+420a^{\frac{13}{2}}b^3x^6\sqrt{1+\frac{bx^2}{a}}+105a^{\frac{11}{2}}b^4x^8\sqrt{1+}}$$

$$+C\left(\begin{cases} -\frac{2a}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}+35b^5x^6\sqrt{a+bx^2}} - \frac{7bx^2}{35a^3b^2\sqrt{a+bx^2}+105a^2b^3x^2\sqrt{a+bx^2}+105ab^4x^4\sqrt{a+bx^2}} \\ \frac{x^4}{4a^{\frac{9}{2}}} \end{cases}\right.$$

input
```
integrate(x*(C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*Piecewise((-1/(7*a**3*b*sqrt(a + b*x**2) + 21*a**2*b**2*x**2*sqrt(a + b*
x**2) + 21*a*b**3*x**4*sqrt(a + b*x**2) + 7*b**4*x**6*sqrt(a + b*x**2)), N
e(b, 0)), (x**2/(2*a**(9/2)), True)) + B*(35*a**5*x**3/(105*a**(19/2)*sqrt
(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b
**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) +
 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 63*a**4*b*x**5/(105*a**(19/
2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(
15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x*
*2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 36*a**3*b**2*x**7/(1
05*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a)
+ 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqr
t(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a)) + 8*a**2*b**
3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*sqrt(1 + b
*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(13/2)*b**3
*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**2/a))) +
C*Piecewise((-2*a/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)) - 7*b*x**2/(35*a**3*b**2*sqrt(a + b*x**2) + 105*a**2*b**3*x**2*sqrt
(a + b*x**2) + 105*a*b**4*x**4*sqrt(a + b*x**2) + 35*b**5*x**6*sqrt(a + b*
x**2)), Ne(b, 0)), (x**4/(4*a**(9/2)), True))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.03

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = -\frac{Cx^2}{5\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{2\,Ca}{35\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/5*C*x^2/((b*x^2 + a)^(7/2)*b) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B
*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*
x/((b*x^2 + a)^(5/2)*a*b) - 2/35*C*a/((b*x^2 + a)^(7/2)*b^2) - 1/7*A/((b*x
^2 + a)^(7/2)*b)
```

3.53.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.69

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\left(\frac{2\,Bb^2x^2}{a^3}+\frac{7\,Bb}{a^2}\right)x^2+\frac{35\,B}{a}\right)x-\frac{21\,C}{b}\right)x^2-\frac{3\,(2\,Ca^4b+5\,Aa^3b^2)}{a^3b^3}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate(x*(C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*(2*B*b^2*x^2/a^3 + 7*B*b/a^2)*x^2 + 35*B/a)*x - 21*C/b)*x^2 - 3
*(2*C*a^4*b + 5*A*a^3*b^2)/(a^3*b^3))/(b*x^2 + a)^(7/2)
```

3.53.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2)}{(a+bx^2)^{9/2}}\,dx = \frac{8\,B\,x}{105\,a^3\,b\,\sqrt{b\,x^2+a}} - \frac{\frac{A}{7\,b}-\frac{C\,a}{7\,b^2}+\frac{B\,x}{7\,b}}{(b\,x^2+a)^{7/2}} - \frac{\frac{C}{5\,b^2}-\frac{B\,x}{35\,a\,b}}{(b\,x^2+a)^{5/2}} + \frac{4\,B\,x}{105\,a^2\,b\,(b\,x^2+a)^{3/2}}$$

input
```
int((x*(A + B*x + C*x^2))/(a + b*x^2)^(9/2),x)
```

output
```
(8*B*x)/(105*a^3*b*(a + b*x^2)^(1/2)) - (A/(7*b) - (C*a)/(7*b^2) + (B*x)/(
7*b))/(a + b*x^2)^(7/2) - (C/(5*b^2) - (B*x)/(35*a*b))/(a + b*x^2)^(5/2) +
 (4*B*x)/(105*a^2*b*(a + b*x^2)^(3/2))
```

3.54 $\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}} dx$

3.54.1 Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = -\frac{aB-(Ab-aC)x}{7ab\,(a+bx^2)^{7/2}} + \frac{(6Ab+aC)x}{35a^2b\,(a+bx^2)^{5/2}} + \frac{4(6Ab+aC)x}{105a^3b\,(a+bx^2)^{3/2}} + \frac{8(6Ab+aC)x}{105a^4b\sqrt{a+bx^2}}$$

output
```
1/7*(-B*a+(A*b-C*a)*x)/a/b/(b*x^2+a)^(7/2)+1/35*(6*A*b+C*a)*x/a^2/b/(b*x^2
+a)^(5/2)+4/105*(6*A*b+C*a)*x/a^3/b/(b*x^2+a)^(3/2)+8/105*(6*A*b+C*a)*x/a^
4/b/(b*x^2+a)^(1/2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{-15a^4B+48Ab^4x^7+35a^3bx(3A+Cx^2)+8ab^3x^5(21A+Cx^2)+14a^2b^2x^3(15A+}{105a^4b\,(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
(-15*a^4*B + 48*A*b^4*x^7 + 35*a^3*b*x*(3*A + C*x^2) + 8*a*b^3*x^5*(21*A +
 C*x^2) + 14*a^2*b^2*x^3*(15*A + 2*C*x^2))/(105*a^4*b*(a + b*x^2)^(7/2))
```

3.54.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {2345, 25, 27, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{6Ab+aC}{b(bx^2+a)^{7/2}}dx}{7a}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aC+6Ab)\int \frac{1}{(bx^2+a)^{7/2}}dx}{7ab}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{209}$$

$$\frac{(aC+6Ab)\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{208}$$

$$\frac{\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a(a+bx^2)^{5/2}}\right)(aC+6Ab)}{7ab}-\frac{aB-x(Ab-aC)}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(a + b*x^2)^(9/2),x]
```

output
```
-1/7*(a*B - (A*b - a*C)*x)/(a*b*(a + b*x^2)^(7/2)) + ((6*A*b + a*C)*(x/(5*
a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a
 + b*x^2])))/(5*a)))/(7*a*b)
```

3.54.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.54.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.76

method	result
gosper	$\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$
trager	$\frac{48A\,b^4x^7+8Ca\,x^7b^3+168Aa\,b^3x^5+28C\,a^2x^5b^2+210A\,a^2b^2x^3+35C\,a^3x^3b+105A\,a^3bx-15B\,a^4}{105(b\,x^2+a)^{\frac{7}{2}}a^4b}$
default	$A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+C\left(-\frac{x}{6b(b\,x^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(b\,x^2+a\right)^{\frac{7}{2}}}+\frac{6}{35a\left(b\,x\right.}\right.}{}\right.$

input
```
int((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*(48*A*b^4*x^7+8*C*a*b^3*x^7+168*A*a*b^3*x^5+28*C*a^2*b^2*x^5+210*A*a
^2*b^2*x^3+35*C*a^3*b*x^3+105*A*a^3*b*x-15*B*a^4)/(b*x^2+a)^(7/2)/a^4/b
```

3.54.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{(8\,(Cab^3+6\,Ab^4)x^7+105\,Aa^3bx+28\,(Ca^2b^2+6\,Aab^3)x^5-15\,Ba^4+35\,(Ca^3b+6}{105\,(a^4b^5x^8+4\,a^5b^4x^6+6\,a^6b^3x^4+4\,a^7b^2x^2+a^8b)}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(C*a*b^3 + 6*A*b^4)*x^7 + 105*A*a^3*b*x + 28*(C*a^2*b^2 + 6*A*a*b
^3)*x^5 - 15*B*a^4 + 35*(C*a^3*b + 6*A*a^2*b^2)*x^3)*sqrt(b*x^2 + a)/(a^4*
b^5*x^8 + 4*a^5*b^4*x^6 + 6*a^6*b^3*x^4 + 4*a^7*b^2*x^2 + a^8*b)
```

3.54.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. 2(117) = 234.

Time = 24.19 (sec) , antiderivative size = 1880, normalized size of antiderivative = 14.80

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

3.54.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx &= \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} \\ &+ \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} \\ &+ \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}b}\end{aligned}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2 + a)^(7/2)*a) - 1/7*C*x/((b*
x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*C*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5/2)*a*b) - 1/7*B/((b*x^2 + a)^
(7/2)*b)
```

3.54.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(4\,x^2\left(\frac{2\,(Cab^5+6\,Ab^6)x^2}{a^4b^3}+\frac{7\,(Ca^2b^4+6\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ca^3b^3+6\,Aa^2b^4)}{a^4b^3}\right)x^2+\frac{105\,A}{a}\right)x-\frac{15\,B}{b}}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((C*x^2+B*x+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((4*x^2*(2*(C*a*b^5 + 6*A*b^6)*x^2/(a^4*b^3) + 7*(C*a^2*b^4 + 6*A*a
*b^5)/(a^4*b^3)) + 35*(C*a^3*b^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*
x - 15*B/b)/(b*x^2 + a)^(7/2)
```

3.54.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2}{(a+bx^2)^{9/2}}\,dx = \frac{x\,(6\,A\,b+C\,a)}{35\,a^2\,b\,(b\,x^2+a)^{5/2}} - \frac{\frac{B}{7\,b}-x\left(\frac{A}{7\,a}-\frac{C}{7\,b}\right)}{(b\,x^2+a)^{7/2}} + \frac{x\,(24\,A\,b+4\,C\,a)}{105\,a^3\,b\,(b\,x^2+a)^{3/2}} + \frac{x\,(48\,A\,b+8\,C\,a)}{105\,a^4\,b\,\sqrt{b\,x^2+a}}$$

input
```
int((A + B*x + C*x^2)/(a + b*x^2)^(9/2),x)
```

output
```
(x*(6*A*b + C*a))/(35*a^2*b*(a + b*x^2)^(5/2)) - (B/(7*b) - x*(A/(7*a) - C
/(7*b)))/(a + b*x^2)^(7/2) + (x*(24*A*b + 4*C*a))/(105*a^3*b*(a + b*x^2)^(
3/2)) + (x*(48*A*b + 8*C*a))/(105*a^4*b*(a + b*x^2)^(1/2))
```

3.55 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx$

3.55.1 Optimal result

Integrand size = 25, antiderivative size = 138

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=\frac{Ab-aC+bBx}{7ab\left(a+bx^2\right)^{7/2}}+\frac{7A+6Bx}{35a^2\left(a+bx^2\right)^{5/2}}+\frac{35A+24Bx}{105a^3\left(a+bx^2\right)^{3/2}}+\frac{35A+16Bx}{35a^4\sqrt{a+bx^2}}-\frac{A\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
1/7*(B*b*x+A*b-C*a)/a/b/(b*x^2+a)^(7/2)+1/35*(6*B*x+7*A)/a^2/(b*x^2+a)^(5/
2)+1/105*(24*B*x+35*A)/a^3/(b*x^2+a)^(3/2)-A*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(9/2)+1/35*(16*B*x+35*A)/a^4/(b*x^2+a)^(1/2)
```

3.55.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx=\frac{-15a^4C+14ab^3x^4(25A+12Bx)+14a^2b^2x^2(29A+15Bx)+3b^4x^6(35A+16Bx)+}{105a^4b\left(a+bx^2\right)^{7/2}}+\frac{2A\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

output
```
(-15*a^4*C + 14*a*b^3*x^4*(25*A + 12*B*x) + 14*a^2*b^2*x^2*(29*A + 15*B*x)
 + 3*b^4*x^6*(35*A + 16*B*x) + a^3*b*(176*A + 105*B*x))/(105*a^4*b*(a + b*
x^2)^(7/2)) + (2*A*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/a^(9/2)
```

3.55.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {2336, 25, 532, 25, 532, 27, 532, 27, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{7A+6Bx}{x\left(bx^2+a\right)^{7/2}}dx}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{7A+6Bx}{x\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{7A+6Bx}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{35A+24Bx}{x\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{35A+24Bx}{x\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{7A+6Bx}{5a\left(a+bx^2\right)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{532}$$

$$\frac{\frac{\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3(35A+16Bx)}{x\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{7A+6Bx}{5a\left(a+bx^2\right)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\int \frac{35A+16Bx}{x\left(bx^2+a\right)^{3/2}}dx}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 532$$

$$\frac{\frac{\frac{\frac{35A+16Bx}{a\sqrt{a+bx^2}}-\frac{\int -\frac{35A}{x\sqrt{bx^2+a}}dx}{a}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{x\sqrt{bx^2+a}}dx}{a}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 243$$

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2}{2a}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{\frac{\frac{\frac{35A\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{ab}+\frac{35A+16Bx}{a\sqrt{a+bx^2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{\frac{\frac{35A+16Bx}{a\sqrt{a+bx^2}}-\frac{35A\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}}{a}+\frac{35A+24Bx}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7A+6Bx}{5a(a+bx^2)^{5/2}}}{7a}+\frac{-aC+Ab+bBx}{7ab\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x]
```

3.55. $\int \frac{A+Bx+Cx^2}{x(a+bx^2)^{9/2}}\,dx$

output
```
(A*b - a*C + b*B*x)/(7*a*b*(a + b*x^2)^(7/2)) + ((7*A + 6*B*x)/(5*a*(a + b
*x^2)^(5/2)) + ((35*A + 24*B*x)/(3*a*(a + b*x^2)^(3/2)) + ((35*A + 16*B*x)
/(a*Sqrt[a + b*x^2]) - (35*A*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2))/a)
/(5*a))/(7*a)
```

3.55.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.55.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.40

method	result
default	$B\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)-\frac{C}{7b(b\,x^2+a)^{\frac{7}{2}}}+A\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{1}{5a\left(b\,x^2+a\right)}\right.$

input
```
int((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
B*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))-1/7*C/b/(b*x^2+a)^(7/2)+A*(1/7
/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1
/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))
)
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 465, normalized size of antiderivative = 3.37

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Ab^5x^8+4\,Aab^4x^6+6\,Aa^2b^3x^4+4\,Aa^3b^2x^2+Aa^4b\right)\sqrt{a}\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+}{x^2}\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^3*x^4 + 4*A*a^3*b^2*x^2
 + A*a^4*b)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) +
2*(48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3*x^
4 + 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 + 176
*A*a^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4 +
4*a^8*b^2*x^2 + a^9*b), 1/105*(105*(A*b^5*x^8 + 4*A*a*b^4*x^6 + 6*A*a^2*b^
3*x^4 + 4*A*a^3*b^2*x^2 + A*a^4*b)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a
)) + (48*B*a*b^4*x^7 + 105*A*a*b^4*x^6 + 168*B*a^2*b^3*x^5 + 350*A*a^2*b^3
*x^4 + 210*B*a^3*b^2*x^3 + 406*A*a^3*b^2*x^2 + 105*B*a^4*b*x - 15*C*a^5 +
176*A*a^4*b)*sqrt(b*x^2 + a))/(a^5*b^5*x^8 + 4*a^6*b^4*x^6 + 6*a^7*b^3*x^4
 + 4*a^8*b^2*x^2 + a^9*b)]
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5251 vs. $2(122) = 244$.

Time = 32.01 (sec) , antiderivative size = 6613, normalized size of antiderivative = 47.92

$$\int \frac{A + Bx + Cx^2}{x\left(a + bx^2\right)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x/(b*x**2+a)**(9/2),x)
```

output
```
A*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9
450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4
*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a*
*(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18
 + 210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) + 2
100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**
6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/
2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2
100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqrt
(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2
)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 5292
0*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a**(59/2)*b**7
*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**18 + 210*a**(5
3/2)*b**10*x**20) + 2924*a**31*b*x**2*sqrt(1 + b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x*
*6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) + 1050*a**31*b*x**2
*log(b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**
2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**4*x**8 + 52920*...
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx+Cx^2}{x\,(a+bx^2)^{9/2}}\,dx = \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{A}{\sqrt{bx^2+a}a^4} + \frac{A}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{A}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}b}$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b*x^2 + a)^(7/2)*a) - A*arcsinh(a/
(sqrt(a*b)*abs(x)))/a^(9/2) + A/(sqrt(b*x^2 + a)*a^4) + 1/3*A/((b*x^2 + a)
^(3/2)*a^3) + 1/5*A/((b*x^2 + a)^(5/2)*a^2) + 1/7*A/((b*x^2 + a)^(7/2)*a)
- 1/7*C/((b*x^2 + a)^(7/2)*b)
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.10

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(3\left(\left(\frac{16\,Bb^3x}{a^4}+\frac{35\,Ab^3}{a^4}\right)x+\frac{56\,Bb^2}{a^3}\right)x+\frac{350\,Ab^2}{a^3}\right)x+\frac{210\,Bb}{a^2}\right)x+\frac{406\,Ab}{a^2}\right)x+\frac{105\,B}{a}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,A\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4}$$

input
```
integrate((C*x^2+B*x+A)/x/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((((3*((16*B*b^3*x/a^4 + 35*A*b^3/a^4)*x + 56*B*b^2/a^3)*x + 350*A*
b^2/a^3)*x + 210*B*b/a^2)*x + 406*A*b/a^2)*x + 105*B/a)*x - (15*C*a^14*b^2
 - 176*A*a^13*b^3)/(a^14*b^3))/(b*x^2 + a)^(7/2) + 2*A*arctan(-(sqrt(b)*x
- sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4)
```

3.55.9 Mupad [B] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.15

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{A}{7\,a}+\frac{A\,\left(b\,x^2+a\right)^2}{3\,a^3}+\frac{A\,\left(b\,x^2+a\right)^3}{a^4}+\frac{A\,\left(b\,x^2+a\right)}{5\,a^2}}{\left(b\,x^2+a\right)^{7/2}} - \frac{C}{7\,b\,\left(b\,x^2+a\right)^{7/2}} - \frac{A\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{16\,B\,x}{35\,a^4\,\sqrt{b\,x^2+a}} + \frac{8\,B\,x}{35\,a^3\,\left(b\,x^2+a\right)^{3/2}} + \frac{6\,B\,x}{35\,a^2\,\left(b\,x^2+a\right)^{5/2}} + \frac{B\,x}{7\,a\,\left(b\,x^2+a\right)^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x*(a + b*x^2)^(9/2)),x)
```

output
```
(A/(7*a) + (A*(a + b*x^2)^2)/(3*a^3) + (A*(a + b*x^2)^3)/a^4 + (A*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - C/(7*b*(a + b*x^2)^(7/2)) - (A*atanh((a
+ b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*B*x)/(35*a^4*(a + b*x^2)^(1/2)) + (
8*B*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*B*x)/(35*a^2*(a + b*x^2)^(5/2)) + (
B*x)/(7*a*(a + b*x^2)^(7/2))
```

3.56 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx$

3.56.1 Optimal result

Integrand size = 25, antiderivative size = 188

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx=\frac{B-\left(\frac{Ab}{a}-C\right)x}{7a(a+bx^2)^{7/2}}+\frac{7B-\left(\frac{13Ab}{a}-6C\right)x}{35a^2(a+bx^2)^{5/2}}+\frac{35B-3\left(\frac{29Ab}{a}-8C\right)x}{105a^3(a+bx^2)^{3/2}}+\frac{35B-\left(\frac{93Ab}{a}-16C\right)x}{35a^4\sqrt{a+bx^2}}-\frac{A\sqrt{a+bx^2}}{a^5x}-\frac{B\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
1/7*(B-(A*b/a-C)*x)/a/(b*x^2+a)^(7/2)+1/35*(7*B-(13*A*b/a-6*C)*x)/a^2/(b*x
^2+a)^(5/2)+1/105*(35*B-3*(29*A*b/a-8*C)*x)/a^3/(b*x^2+a)^(3/2)-B*arctanh(
(b*x^2+a)^(1/2)/a^(1/2))/a^(9/2)+1/35*(35*B-(93*A*b/a-16*C)*x)/a^4/(b*x^2+
a)^(1/2)-A*(b*x^2+a)^(1/2)/a^5/x
```

3.56.2 Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx=\frac{-384Ab^4x^8+14a^2b^2x^4(-120A+x(25B+12Cx))+14a^3bx^2(-60A+x(29B+15}{}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 14*a^2*b^2*x^4*(-120*A + x*(25*B + 12*C*x)) + 14*a^3*b*x
^2*(-60*A + x*(29*B + 15*C*x)) + 3*a*b^3*x^6*(-448*A + x*(35*B + 16*C*x))
+ a^4*(-105*A + x*(176*B + 105*C*x)) + 210*Sqrt[a]*B*x*(a + b*x^2)^(7/2)*A
rcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sqrt[a]])/(105*a^5*x*(a + b*x^2)^(7/2
))
```

3.56.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {2336, 25, 2336, 25, 2336, 27, 2336, 27, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \ 2336$$

$$\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{-6\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^2\left(bx^2+a\right)^{7/2}}dx}{7a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-6\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^2\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int -\frac{-4\left(\frac{13Ab}{a}-6C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{-4\left(\frac{13Ab}{a}-6C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a\left(a+bx^2\right)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2336$$

$$\frac{\frac{\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{3\left(-2\left(\frac{29Ab}{a}-8C\right)x^2+35Bx+35A\right)}{x^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{-2\left(\frac{29Ab}{a}-8C\right)x^2+35Bx+35A}{x^2\left(bx^2+a\right)^{3/2}}dx}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2336$$

$$\frac{\frac{\frac{\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}-\frac{\int -\frac{35(A+Bx)}{x^2\sqrt{bx^2+a}}dx}{a}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\frac{35\int \frac{A+Bx}{x^2\sqrt{bx^2+a}}dx}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 534$$

$$\frac{\frac{\frac{\frac{35\left(B\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 243$$

$$\frac{\frac{\frac{\frac{35\left(\frac{1}{2}B\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 73$$

3.56. $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{\frac{35\left(\frac{B\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}-\frac{A\sqrt{a+bx^2}}{ax}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{\frac{35\left(-\frac{A\sqrt{a+bx^2}}{ax}-\frac{B\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}\right)}{a}+\frac{35B-x\left(\frac{93Ab}{a}-16C\right)}{a\sqrt{a+bx^2}}}{a}+\frac{35B-3x\left(\frac{29Ab}{a}-8C\right)}{3a\left(a+bx^2\right)^{3/2}}}{5a}+\frac{7B-x\left(\frac{13Ab}{a}-6C\right)}{5a(a+bx^2)^{5/2}}}{7a}+\frac{B-x\left(\frac{Ab}{a}-C\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(B - ((A*b)/a - C)*x)/(7*a*(a + b*x^2)^(7/2)) + ((7*B - ((13*A*b)/a - 6*C)
*x)/(5*a*(a + b*x^2)^(5/2)) + ((35*B - 3*((29*A*b)/a - 8*C)*x)/(3*a*(a + b
*x^2)^(3/2)) + ((35*B - ((93*A*b)/a - 16*C)*x)/(a*Sqrt[a + b*x^2]) + (35*(
-((A*Sqrt[a + b*x^2])/(a*x)) - (B*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a
]))/a)/a)/(5*a))/(7*a)
```

3.56.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.56.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.47

method	result
default	$C\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+B\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}\right.$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
C*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(
b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2))))+B*(1/7/a/(b*x^2+a)^(7/2)+1/a*(
1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-
1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x)))))+A*(-1/a/x/(b*x^2+a)^(7
/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1
/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(1/2)))))
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 525, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \left[\frac{105\left(Bb^4x^9+4\,Bab^3x^7+6\,Ba^2b^2x^5+4\,Ba^3bx^3+Ba^4x\right)\sqrt{a}\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}-}{x^2}\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 +
 B*a^4*x)*sqrt(a)*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*
(105*B*a*b^3*x^7 + 350*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*
a^3*b*x^3 + 168*(C*a^2*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 21
0*(C*a^3*b - 8*A*a^2*b^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 +
a))/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x), 1
/105*(105*(B*b^4*x^9 + 4*B*a*b^3*x^7 + 6*B*a^2*b^2*x^5 + 4*B*a^3*b*x^3 + B
*a^4*x)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (105*B*a*b^3*x^7 + 350
*B*a^2*b^2*x^5 + 48*(C*a*b^3 - 8*A*b^4)*x^8 + 406*B*a^3*b*x^3 + 168*(C*a^2
*b^2 - 8*A*a*b^3)*x^6 + 176*B*a^4*x - 105*A*a^4 + 210*(C*a^3*b - 8*A*a^2*b
^2)*x^4 + 105*(C*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 + a))/(a^5*b^4*x^9 + 4*a
^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)]
```

3.56.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6922 vs. $2(155) = 310$.

Time = 42.51 (sec) , antiderivative size = 6922, normalized size of antiderivative = 36.82

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(352*a**32*sqrt(1 + b*x**2/a)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 +
9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x**6 + 44100*a**(65/2)*b**
4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61/2)*b**6*x**12 + 25200*a
**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 + 2100*a**(55/2)*b**9*x**1
8 + 210*a**(53/2)*b**10*x**20) + 105*a**32*log(b*x**2/a)/(210*a**(73/2) +
2100*a**(71/2)*b*x**2 + 9450*a**(69/2)*b**2*x**4 + 25200*a**(67/2)*b**3*x*
*6 + 44100*a**(65/2)*b**4*x**8 + 52920*a**(63/2)*b**5*x**10 + 44100*a**(61
/2)*b**6*x**12 + 25200*a**(59/2)*b**7*x**14 + 9450*a**(57/2)*b**8*x**16 +
2100*a**(55/2)*b**9*x**18 + 210*a**(53/2)*b**10*x**20) - 210*a**32*log(sqr
t(1 + b*x**2/a) + 1)/(210*a**(73/2) + 2100*a**(71/2)*b*x**2 + 9450*a**(...
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx = \frac{16\,Cx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Cx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Cx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Abx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{B}{\sqrt{bx^2+a}a^4} + \frac{B}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{B}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{B}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
C*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 128/35*A*b*x
/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*A*b*x
/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(7/2)*a^2) - B*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(9/2) + B/(sqrt(b*x^2 + a)*a^4) + 1/3*B/((b*x^2 + a
)^(3/2)*a^3) + 1/5*B/((b*x^2 + a)^(5/2)*a^2) + 1/7*B/((b*x^2 + a)^(7/2)*a)
 - A/((b*x^2 + a)^(7/2)*a*x)
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2}{x^2(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(3\left(x\left(\frac{35\,Bb^3}{a^4} + \frac{(16\,Ca^{20}b^6-93\,Aa^{19}b^7)x}{a^{24}b^3}\right) + \frac{28\,(2\,Ca^{21}b^5-11\,Aa^{20}b^6)}{a^{24}b^3}\right)x + \frac{350\,Bb^2}{a^3}\right)x + \frac{210}{}\right.\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}} + \frac{2\,B\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((C*x^2+B*x+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*(((((3*(x*(35*B*b^3/a^4 + (16*C*a^20*b^6 - 93*A*a^19*b^7)*x/(a^24*b^
3)) + 28*(2*C*a^21*b^5 - 11*A*a^20*b^6)/(a^24*b^3))*x + 350*B*b^2/a^3)*x +
 210*(C*a^22*b^4 - 5*A*a^21*b^5)/(a^24*b^3))*x + 406*B*b/a^2)*x + 105*(C*a
^23*b^3 - 4*A*a^22*b^4)/(a^24*b^3))*x + 176*B/a)/(b*x^2 + a)^(7/2) + 2*B*a
rctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^4) + 2*A*sqrt(b
)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.56.9 Mupad [B] (verification not implemented)

Time = 6.75 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{B}{7\,a}+\frac{B\,{\left(b\,x^2+a\right)}^2}{3\,a^3}+\frac{B\,{\left(b\,x^2+a\right)}^3}{a^4}+\frac{B\,\left(b\,x^2+a\right)}{5\,a^2}}{{\left(b\,x^2+a\right)}^{7/2}}-\frac{\frac{A}{a^4}+\frac{128\,A\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{B\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}}+\frac{16\,C\,x}{35\,a^4\,\sqrt{b\,x^2+a}}+\frac{8\,C\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{3/2}}+\frac{6\,C\,x}{35\,a^2\,{\left(b\,x^2+a\right)}^{5/2}} + \frac{C\,x}{7\,a\,{\left(b\,x^2+a\right)}^{7/2}}-\frac{29\,A\,b\,x}{35\,a^4\,{\left(b\,x^2+a\right)}^{3/2}}-\frac{13\,A\,b\,x}{35\,a^3\,{\left(b\,x^2+a\right)}^{5/2}}-\frac{A\,b\,x}{7\,a^2\,{\left(b\,x^2+a\right)}^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
(B/(7*a) + (B*(a + b*x^2)^2)/(3*a^3) + (B*(a + b*x^2)^3)/a^4 + (B*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - (A/a^4 + (128*A*b*x^2)/(35*a^5))/(x*(a +
 b*x^2)^(1/2)) - (B*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(9/2) + (16*C*x)/(
35*a^4*(a + b*x^2)^(1/2)) + (8*C*x)/(35*a^3*(a + b*x^2)^(3/2)) + (6*C*x)/(
35*a^2*(a + b*x^2)^(5/2)) + (C*x)/(7*a*(a + b*x^2)^(7/2)) - (29*A*b*x)/(35
*a^4*(a + b*x^2)^(3/2)) - (13*A*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (A*b*x)/
(7*a^2*(a + b*x^2)^(7/2))
```

3.57 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

3.57.1 Optimal result

Integrand size = 25, antiderivative size = 219

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx=&-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}-\frac{7(2Ab-aC)+13bBx}{35a^3\left(a+bx^2\right)^{5/2}}\\&-\frac{35(3Ab-aC)+87bBx}{105a^4\left(a+bx^2\right)^{3/2}}-\frac{35(4Ab-aC)+93bBx}{35a^5\sqrt{a+bx^2}}\\&-\frac{A\sqrt{a+bx^2}}{2a^5x^2}-\frac{B\sqrt{a+bx^2}}{a^5x}+\frac{(9Ab-2aC)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{11/2}}\end{aligned}$$

output
```
1/7*(-a*(A*b/a-C)-B*b*x)/a^2/(b*x^2+a)^(7/2)+1/35*(-13*B*b*x-14*A*b+7*C*a)
/a^3/(b*x^2+a)^(5/2)+1/105*(-87*B*b*x-105*A*b+35*C*a)/a^4/(b*x^2+a)^(3/2)+
1/2*(9*A*b-2*C*a)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(11/2)+1/35*(-93*B*b*
x-140*A*b+35*C*a)/a^5/(b*x^2+a)^(1/2)-1/2*A*(b*x^2+a)^(1/2)/a^5/x^2-B*(b*x
^2+a)^(1/2)/a^5/x
```

3.57.2 Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \frac{-3b^4x^8(315A+256Bx)+a^4(-105A-210Bx+352Cx^2)-4a^3bx^2(396A+7x(60}{210a^5} + \frac{(-9Ab+2aC)\operatorname{arctanh}\left(\frac{\sqrt{b}x-\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
(-3*b^4*x^8*(315*A + 256*B*x) + a^4*(-105*A - 210*B*x + 352*C*x^2) - 4*a^3
*b*x^2*(396*A + 7*x*(60*B - 29*C*x)) + 42*a*b^3*x^6*(-75*A + x*(-64*B + 5*
C*x)) + 14*a^2*b^2*x^4*(-261*A + 10*x*(-24*B + 5*C*x)))/(210*a^5*x^2*(a +
b*x^2)^(7/2)) + ((-9*A*b + 2*a*C)*ArcTanh[(Sqrt[b]*x - Sqrt[a + b*x^2])/Sq
rt[a]])/a^(11/2)
```

3.57.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.12, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.560$, Rules used = {2336, 25, 2336, 25, 2336, 27, 2336, 27, 2338, 25, 534, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\frac{6bBx^3}{a}-7\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^3(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{6bBx^3}{a}-7\left(\frac{Ab}{a}-C\right)x^2+7Bx+7A}{x^3(bx^2+a)^{7/2}}dx}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2336}$$

$$\frac{-\frac{\int -\frac{-\frac{52bBx^3}{a}-35\left(\frac{2Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{-\frac{52bBx^3}{a}-35\left(\frac{2Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{5/2}}dx}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 2336

$$\frac{\frac{-\frac{\int -\frac{3\left(-\frac{58bBx^3}{a}-35\left(\frac{3Ab}{a}-C\right)x^2+35Bx+35A\right)}{x^3\left(bx^2+a\right)^{3/2}}dx}{3a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{-\frac{58bBx^3}{a}-35\left(\frac{3Ab}{a}-C\right)x^2+35Bx+35A}{x^3\left(bx^2+a\right)^{3/2}}dx}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 2336

$$\frac{\frac{\frac{-\frac{\int -\frac{35\left(-\left(\left(\frac{4Ab}{a}-C\right)x^2\right)+Bx+A\right)}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{35\int \frac{-\left(\left(\frac{4Ab}{a}-C\right)x^2\right)+Bx+A}{x^3\sqrt{bx^2+a}}dx}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 2338

3.57. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{35\left(-\frac{\int -\frac{2aB-(9Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{35\left(\frac{\int \frac{2aB-(9Ab-2aC)x}{x^2\sqrt{bx^2+a}}dx}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 534

$$\frac{\frac{\frac{35\left(\frac{-(9Ab-2aC)\int \frac{1}{x\sqrt{bx^2+a}}dx-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 243

$$\frac{\frac{\frac{35\left(\frac{-\frac{1}{2}(9Ab-2aC)\int \frac{1}{x^2\sqrt{bx^2+a}}dx^2-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 73

3.57. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{35\left(-\frac{\frac{(9Ab-2aC)\int \frac{1}{\frac{x^4}{b}-\frac{a}{b}}d\sqrt{bx^2+a}}{b}}{2a}-\frac{2B\sqrt{a+bx^2}}{x}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{35\left(\frac{\frac{(9Ab-2aC)\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2B\sqrt{a+bx^2}}{x}}{2a}-\frac{A\sqrt{a+bx^2}}{2ax^2}\right)}{a}-\frac{35(4Ab-aC)+93bBx}{a^2\sqrt{a+bx^2}}}{a}-\frac{35(3Ab-aC)+87bBx}{3a^2\left(a+bx^2\right)^{3/2}}}{5a}-\frac{7(2Ab-aC)+13bBx}{5a^2(a+bx^2)^{5/2}}}{7a}-\frac{a\left(\frac{Ab}{a}-C\right)+bBx}{7a^2\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x]
```

output
```
-1/7*(a*((A*b)/a - C) + b*B*x)/(a^2*(a + b*x^2)^(7/2)) + (-1/5*(7*(2*A*b -
 a*C) + 13*b*B*x)/(a^2*(a + b*x^2)^(5/2)) + (-1/3*(35*(3*A*b - a*C) + 87*b
*B*x)/(a^2*(a + b*x^2)^(3/2)) + (-((35*(4*A*b - a*C) + 93*b*B*x)/(a^2*Sqrt
[a + b*x^2])) + (35*(-1/2*(A*Sqrt[a + b*x^2])/(a*x^2) + ((-2*B*Sqrt[a + b*
x^2])/x + ((9*A*b - 2*a*C)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a
)))/a)/a)/(5*a))/(7*a)
```

3.57.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 534
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[(-c)*x^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1))), x] + Simp[d   Int[
x^(m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x] && ILtQ[m,
0] && GtQ[p, -1] && EqQ[m + 2*p + 3, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.57. $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx$

3.57.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.49

method	result
default	$C\left(\frac{1}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{1}{5a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{\frac{1}{3a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{\frac{1}{a\sqrt{b\,x^2+a}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^2+a}}{x}\right)}{a^{\frac{3}{2}}}}{a}}{a}}{a}\right)+B\left(-\frac{1}{ax(b\,x^2+a)^{\frac{7}{2}}}-\frac{8b\left(\frac{}{7a\left(b\,x\right.}\right.}{}\right.$
risch	Expression too large to display

input
```
int((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
C*(1/7/a/(b*x^2+a)^(7/2)+1/a*(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(
3/2)+1/a*(1/a/(b*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))
/x)))))+B*(-1/a/x/(b*x^2+a)^(7/2)-8*b/a*(1/7*x/a/(b*x^2+a)^(7/2)+6/7/a*(1/
5*x/a/(b*x^2+a)^(5/2)+4/5/a*(1/3*x/a/(b*x^2+a)^(3/2)+2/3*x/a^2/(b*x^2+a)^(
1/2)))))+A*(-1/2/a/x^2/(b*x^2+a)^(7/2)-9/2*b/a*(1/7/a/(b*x^2+a)^(7/2)+1/a*
(1/5/a/(b*x^2+a)^(5/2)+1/a*(1/3/a/(b*x^2+a)^(3/2)+1/a*(1/a/(b*x^2+a)^(1/2)
-1/a^(3/2)*ln((2*a+2*a^(1/2)*(b*x^2+a)^(1/2))/x))))))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.14

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \left[-\frac{105\left((2\,Cab^4-9\,Ab^5)x^{10}+4\,(2\,Ca^2b^3-9\,Aab^4)x^8+6\,(2\,Ca^3b^2-9\,Aa^2b^3)x^6+\right.}{}\right.$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[-1/420*(105*((2*C*a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8
 + 6*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (
2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(a)*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a)
+ 2*a)/x^2) + 2*(768*B*a*b^4*x^9 + 2688*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5
 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 210*B*a^5*x - 35
0*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b
^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 +
4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b*x^4 + a^10*x^2), 1/210*(105*((2*C*
a*b^4 - 9*A*b^5)*x^10 + 4*(2*C*a^2*b^3 - 9*A*a*b^4)*x^8 + 6*(2*C*a^3*b^2 -
 9*A*a^2*b^3)*x^6 + 4*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 + (2*C*a^5 - 9*A*a^4*b
)*x^2)*sqrt(-a)*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (768*B*a*b^4*x^9 + 2688
*B*a^2*b^3*x^7 + 3360*B*a^3*b^2*x^5 + 1680*B*a^4*b*x^3 - 105*(2*C*a^2*b^3
- 9*A*a*b^4)*x^8 + 210*B*a^5*x - 350*(2*C*a^3*b^2 - 9*A*a^2*b^3)*x^6 + 105
*A*a^5 - 406*(2*C*a^4*b - 9*A*a^3*b^2)*x^4 - 176*(2*C*a^5 - 9*A*a^4*b)*x^2
)*sqrt(b*x^2 + a))/(a^6*b^4*x^10 + 4*a^7*b^3*x^8 + 6*a^8*b^2*x^6 + 4*a^9*b
*x^4 + a^10*x^2)]
```

3.57.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11198 vs. $2(196) = 392$.

Time = 62.49 (sec) , antiderivative size = 11198, normalized size of antiderivative = 51.13

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((C*x**2+B*x+A)/x**3/(b*x**2+a)**(9/2),x)
```

output
```
A*(-70*a**49*sqrt(1 + b*x**2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x
**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(9
9/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14
+ 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*
b**9*x**20 + 140*a**(87/2)*b**10*x**22) - 1476*a**48*b*x**2*sqrt(1 + b*x**
2/a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*
x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*b**4*x**10 + 35280*a**
(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**1
6 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*
b**10*x**22) - 315*a**48*b*x**2*log(b*x**2/a)/(140*a**(107/2)*x**2 + 1400*
a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8
 + 29400*a**(99/2)*b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/
2)*b**6*x**14 + 16800*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1
400*a**(89/2)*b**9*x**20 + 140*a**(87/2)*b**10*x**22) + 630*a**48*b*x**2*l
og(sqrt(1 + b*x**2/a) + 1)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 +
 6300*a**(103/2)*b**2*x**6 + 16800*a**(101/2)*b**3*x**8 + 29400*a**(99/2)*
b**4*x**10 + 35280*a**(97/2)*b**5*x**12 + 29400*a**(95/2)*b**6*x**14 + 168
00*a**(93/2)*b**7*x**16 + 6300*a**(91/2)*b**8*x**18 + 1400*a**(89/2)*b**9*
x**20 + 140*a**(87/2)*b**10*x**22) - 9822*a**47*b**2*x**4*sqrt(1 + b*x**2/
a)/(140*a**(107/2)*x**2 + 1400*a**(105/2)*b*x**4 + 6300*a**(103/2)*b**2...
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx = -\frac{128\,Bbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Bbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Bbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Bbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{C\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{a^{\frac{9}{2}}} + \frac{9\,Ab\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{11}{2}}} + \frac{C}{\sqrt{bx^2+a}a^4} + \frac{C}{3\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{C}{5\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{C}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{9\,Ab}{2\,\sqrt{bx^2+a}a^5} - \frac{3\,Ab}{2\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{9\,Ab}{10\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{9\,Ab}{14\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{B}{(bx^2+a)^{\frac{7}{2}}ax} - \frac{A}{2\,(bx^2+a)^{\frac{7}{2}}ax^2}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-128/35*B*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*B*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2)
- C*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 9/2*A*b*arcsinh(a/(sqrt(a*b)*a
bs(x)))/a^(11/2) + C/(sqrt(b*x^2 + a)*a^4) + 1/3*C/((b*x^2 + a)^(3/2)*a^3)
 + 1/5*C/((b*x^2 + a)^(5/2)*a^2) + 1/7*C/((b*x^2 + a)^(7/2)*a) - 9/2*A*b/(
sqrt(b*x^2 + a)*a^5) - 3/2*A*b/((b*x^2 + a)^(3/2)*a^4) - 9/10*A*b/((b*x^2
+ a)^(5/2)*a^3) - 9/14*A*b/((b*x^2 + a)^(7/2)*a^2) - B/((b*x^2 + a)^(7/2)*
a*x) - 1/2*A/((b*x^2 + a)^(7/2)*a*x^2)
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2\right)^{9/2}}\,dx =$$
$$-\frac{\left(\left(\left(\left(3\left(\left(\frac{93\,Bb^4x}{a^5}-\frac{35\,\left(Ca^{24}b^6-4\,Aa^{23}b^7\right)}{a^{28}b^3}\right)x+\frac{308\,Bb^3}{a^4}\right)x-\frac{35\,\left(10\,Ca^{25}b^5-39\,Aa^{24}b^6\right)}{a^{28}b^3}\right)x+\frac{1050\,Bb^2}{a^3}\right)x-\frac{14\,\left(29\,Ca^{26}b^4-1\right.}{a^{28}b^3}}{105\,\left(bx^2+a\right)^{\frac{7}{2}}}$$
$$+\frac{\left(2\,Ca-9\,Ab\right)\arctan\left(-\frac{\sqrt{b}x-\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^5}$$
$$+\frac{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^3Ab+2\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba\sqrt{b}+\left(\sqrt{b}x-\sqrt{bx^2+a}\right)Aab-2\,Ba^2\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^2a^5}$$

input
```
integrate((C*x^2+B*x+A)/x^3/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*(((((3*((93*B*b^4*x/a^5 - 35*(C*a^24*b^6 - 4*A*a^23*b^7)/(a^28*b^3)
)*x + 308*B*b^3/a^4)*x - 35*(10*C*a^25*b^5 - 39*A*a^24*b^6)/(a^28*b^3))*x
+ 1050*B*b^2/a^3)*x - 14*(29*C*a^26*b^4 - 108*A*a^25*b^5)/(a^28*b^3))*x +
420*B*b/a^2)*x - 2*(88*C*a^27*b^3 - 291*A*a^26*b^4)/(a^28*b^3))/(b*x^2 + a
)^(7/2) + (2*C*a - 9*A*b)*arctan(-(sqrt(b)*x - sqrt(b*x^2 + a))/sqrt(-a))/
(sqrt(-a)*a^5) + ((sqrt(b)*x - sqrt(b*x^2 + a))^3*A*b + 2*(sqrt(b)*x - sqr
t(b*x^2 + a))^2*B*a*sqrt(b) + (sqrt(b)*x - sqrt(b*x^2 + a))*A*a*b - 2*B*a^
2*sqrt(b))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^2*a^5)
```

3.57.9 Mupad [B] (verification not implemented)

Time = 7.46 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx+Cx^2}{x^3(a+bx^2)^{9/2}}\,dx = \frac{\frac{C}{7a}+\frac{C\,(b\,x^2+a)^2}{3\,a^3}+\frac{C\,(b\,x^2+a)^3}{a^4}+\frac{C\,(b\,x^2+a)}{5\,a^2}}{(b\,x^2+a)^{7/2}} - \frac{\frac{A\,b}{7\,a}+\frac{9\,A\,b\,(b\,x^2+a)}{35\,a^2}+\frac{3\,A\,b\,(b\,x^2+a)^2}{5\,a^3}+\frac{3\,A\,b\,(b\,x^2+a)^3}{a^4}-\frac{9\,A\,b\,(b\,x^2+a)^4}{2\,a^5}}{a\,(b\,x^2+a)^{7/2}-(b\,x^2+a)^{9/2}} - \frac{\frac{B}{a^4}+\frac{128\,B\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}} - \frac{C\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{a^{9/2}} + \frac{9\,A\,b\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{11/2}} - \frac{29\,B\,b\,x}{35\,a^4\,(b\,x^2+a)^{3/2}} - \frac{13\,B\,b\,x}{35\,a^3\,(b\,x^2+a)^{5/2}} - \frac{B\,b\,x}{7\,a^2\,(b\,x^2+a)^{7/2}}$$

input
```
int((A + B*x + C*x^2)/(x^3*(a + b*x^2)^(9/2)),x)
```

output
```
(C/(7*a) + (C*(a + b*x^2)^2)/(3*a^3) + (C*(a + b*x^2)^3)/a^4 + (C*(a + b*x
^2))/(5*a^2))/(a + b*x^2)^(7/2) - ((A*b)/(7*a) + (9*A*b*(a + b*x^2))/(35*a
^2) + (3*A*b*(a + b*x^2)^2)/(5*a^3) + (3*A*b*(a + b*x^2)^3)/a^4 - (9*A*b*(
a + b*x^2)^4)/(2*a^5))/(a*(a + b*x^2)^(7/2) - (a + b*x^2)^(9/2)) - (B/a^4
+ (128*B*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - (C*atanh((a + b*x^2)^(1/
2)/a^(1/2)))/a^(9/2) + (9*A*b*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(11/2
)) - (29*B*b*x)/(35*a^4*(a + b*x^2)^(3/2)) - (13*B*b*x)/(35*a^3*(a + b*x^2
)^(5/2)) - (B*b*x)/(7*a^2*(a + b*x^2)^(7/2))
```

3.58 $\int \frac{A(cx)^m}{a+bx^2}\,dx$

3.58.1 Optimal result

Integrand size = 16, antiderivative size = 45

$$\int \frac{A(cx)^m}{a+bx^2}\,dx=\frac{A(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ac(1+m)}$$

output
```
A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)
```

3.58.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A(cx)^m}{a+bx^2}\,dx=\frac{Ax(cx)^m\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},1+\frac{1+m}{2},-\frac{bx^2}{a}\right)}{a(1+m)}$$

input
```
Integrate[(A*(c*x)^m)/(a + b*x^2),x]
```

output
```
(A*x*(c*x)^m*Hypergeometric2F1[1, (1 + m)/2, 1 + (1 + m)/2, -((b*x^2)/a)])
/(a*(1 + m))
```

3.58.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {27, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A(cx)^m}{a+bx^2} dx$$
$$\Big\downarrow \; 27$$
$$A\int \frac{(cx)^m}{bx^2+a}dx$$
$$\Big\downarrow \; 278$$
$$\frac{A(cx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)}$$

input
```
Int[(A*(c*x)^m)/(a + b*x^2),x]
```

output
```
(A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
/(a*c*(1 + m))
```

3.58.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

3.58.4 Maple [F]

$$\int \frac{A(cx)^m}{b\,x^2+a}dx$$

input
```
int(A*(c*x)^m/(b*x^2+a),x)
```

output
```
int(A*(c*x)^m/(b*x^2+a),x)
```

3.58.5 Fricas [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m\,A}{bx^2+a}\,dx$$

input
```
integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((c*x)^m*A/(b*x^2 + a), x)
```

3.58.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.16

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = A\left(\frac{c^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{c^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}\right)$$

input
```
integrate(A*(c*x)**m/(b*x**2+a),x)
```

output
```
A*(c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamm
a(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + c**m*x**(m + 1)*lerchphi(b*x**2*exp_
polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)))
```

3.58.7 Maxima [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m A}{bx^2+a}\,dx$$

input
```
integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="maxima")
```

output
```
A*integrate((c*x)^m/(b*x^2 + a), x)
```

3.58.8 Giac [F]

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{(cx)^m A}{bx^2+a}\,dx$$

input
```
integrate(A*(c*x)^m/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((c*x)^m*A/(b*x^2 + a), x)
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A(cx)^m}{a+bx^2}\,dx = \int \frac{A\,(c\,x)^m}{b\,x^2+a}\,dx$$

input
```
int((A*(c*x)^m)/(a + b*x^2),x)
```

output
```
int((A*(c*x)^m)/(a + b*x^2), x)
```

3.59 $\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx$

3.59.1 Optimal result

Integrand size = 20, antiderivative size = 91

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \frac{A(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{ac(1+m)} + \frac{B(cx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

output
```
A*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/c/(1+m)+B*(
c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-b*x^2/a)/a/c^2/(2+m)
```

3.59.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \frac{x(cx)^m\left(B(1+m)x\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-\frac{bx^2}{a}\right)+A(2+m)\,\text{Hypergeometric2F1}\left(1,\frac{1+}{2}\right.\right.}{a(1+m)(2+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*(B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 + m/2, -((b*x^2)/a
)] + A*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)]))/
(a*(1 + m)*(2 + m))
```

3.59.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {557, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(cx)^m}{a+bx^2}\,dx$$

$$\Big\downarrow \text{557}$$

$$A\int \frac{(cx)^m}{bx^2+a}dx+\frac{B\int \frac{(cx)^{m+1}}{bx^2+a}dx}{c}$$

$$\Big\downarrow \text{278}$$

$$\frac{A(cx)^{m+1}\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{ac(m+1)}+\frac{B(cx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{ac^2(m+2)}$$

input
```
Int[((c*x)^m*(A + B*x))/(a + b*x^2),x]
```

output
```
(A*(c*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])
/(a*c*(1 + m)) + (B*(c*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, -((b*x^2)/a)])/(a*c^2*(2 + m))
```

3.59.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 557
```
Int[((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Sym
bol] :> Simp[c   Int[(e*x)^m*(a + b*x^2)^p, x], x] + Simp[d/e   Int[(e*x)^(
m + 1)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x]
```

3.59.4 Maple [F]

$$\int \frac{(cx)^{m} (Bx+A)}{b\,x^{2}+a}dx$$

input
```
int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

output
```
int((c*x)^m*(B*x+A)/(b*x^2+a),x)
```

3.59.5 Fricas [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.59.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.08

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \frac{Ac^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)}$$

input
```
integrate((c*x)**m*(B*x+A)/(b*x**2+a),x)
```

output
```
A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*
c**m*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + B*c**m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I
*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2))
```

3.59.7 Maxima [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx = \int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.59.8 Giac [F]

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx=\int \frac{(Bx+A)(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m*(B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m(A+Bx)}{a+bx^2}\,dx=\int \frac{(c\,x)^m\,(A+B\,x)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^m*(A + B*x))/(a + b*x^2),x)
```

output
```
int(((c*x)^m*(A + B*x))/(a + b*x^2), x)
```

3.60 $\int \frac{(cx)^m(A+Cx^2)}{a+bx^2}\,dx$

3.60.1 Optimal result

Integrand size = 22, antiderivative size = 76

$$\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)} + \frac{(Ab-aC)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abc(1+m)}$$

output
```
C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/
2+1/2*m],-b*x^2/a)/a/b/c/(1+m)
```

3.60.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{(cx)^m (A+Cx^2)}{a+bx^2}\,dx = \frac{x(cx)^m \left(aC+(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)\right)}{ab(1+m)}$$

input
```
Integrate[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*(a*C + (A*b - a*C)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -
((b*x^2)/a)]))/(a*b*(1 + m))
```

3.60.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {363, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Cx^2)(cx)^m}{a+bx^2}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{(Ab-aC)\int \frac{(cx)^m}{bx^2+a}dx}{b}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(cx)^{m+1}(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

input
```
Int[((c*x)^m*(A + C*x^2))/(a + b*x^2),x]
```

output
```
(C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m))
```

3.60.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

3.60.4 Maple [F]

$$\int \frac{(cx)^m (C\,x^2 + A)}{b\,x^2 + a} dx$$

input
```
int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

output
```
int((c*x)^m*(C*x^2+A)/(b*x^2+a),x)
```

3.60.5 Fricas [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2}\, dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a}\, dx$$

input
```
integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

3.60.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.82 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.64

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2}\, dx = \frac{Ac^m m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Ac^m x^{m+1} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{1}{2}\right) \Gamma\left(\frac{m}{2} + \frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{Cc^m m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)} + \frac{3Cc^m x^{m+3} \Phi\left(\frac{bx^2 e^{i\pi}}{a}, 1, \frac{m}{2} + \frac{3}{2}\right) \Gamma\left(\frac{m}{2} + \frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2} + \frac{5}{2}\right)}$$

input
```
integrate((c*x)**m*(C*x**2+A)/(b*x**2+a),x)
```

output
```
A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + C*
c**m*m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m
/2 + 3/2)/(4*a*gamma(m/2 + 5/2)) + 3*C*c**m*x**(m + 3)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.60.7 Maxima [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a} dx$$

input
```
integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

3.60.8 Giac [F]

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} dx = \int \frac{(Cx^2 + A)(cx)^m}{bx^2 + a} dx$$

input
```
integrate((c*x)^m*(C*x^2+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((C*x^2 + A)*(c*x)^m/(b*x^2 + a), x)
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A + Cx^2)}{a + bx^2} dx = \int \frac{(C x^2 + A) (c x)^m}{b x^2 + a} dx$$

input
```
int(((A + C*x^2)*(c*x)^m)/(a + b*x^2),x)
```

output
```
int(((A + C*x^2)*(c*x)^m)/(a + b*x^2), x)
```

3.61 $\int \frac{(cx)^m(A+Bx+Cx^2)}{a+bx^2}\,dx$

3.61.1 Optimal result

Integrand size = 25, antiderivative size = 121

$$\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx = \frac{C(cx)^{1+m}}{bc(1+m)}+\frac{(Ab-aC)(cx)^{1+m}\,\text{Hypergeometric2F1}\left(1,\frac{1+m}{2},\frac{3+m}{2},-\frac{bx^2}{a}\right)}{abc(1+m)} + \frac{B(cx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},-\frac{bx^2}{a}\right)}{ac^2(2+m)}$$

output
```
C*(c*x)^(1+m)/b/c/(1+m)+(A*b-C*a)*(c*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/
2+1/2*m],-b*x^2/a)/a/b/c/(1+m)+B*(c*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2
*m],-b*x^2/a)/a/c^2/(2+m)
```

3.61.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82

$$\int \frac{(cx)^m\left(A+Bx+Cx^2\right)}{a+bx^2}\,dx = \frac{x(cx)^m\left(aC(2+m)+bB(1+m)x\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},-\frac{bx^2}{a}\right)+(Ab-aC)(2+m)\,\text{Hy}\right.}{ab(1+m)(2+m)}$$

input
```
Integrate[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

output
```
(x*(c*x)^m*(a*C*(2 + m) + b*B*(1 + m)*x*Hypergeometric2F1[1, 1 + m/2, 2 +
m/2, -((b*x^2)/a)] + (A*b - a*C)*(2 + m)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*(2 + m))
```

3.61.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{(cx)^m(-aC+Ab+bBx)}{b(a+bx^2)}+\frac{C(cx)^m}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(cx)^{m+1}(Ab-aC)\,\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},-\frac{bx^2}{a}\right)}{abc(m+1)}+\frac{B(cx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},-\frac{bx^2}{a}\right)}{ac^2(m+2)}+\frac{C(cx)^{m+1}}{bc(m+1)}$$

input
```
Int[((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x]
```

output
```
(C*(c*x)^(1 + m))/(b*c*(1 + m)) + ((A*b - a*C)*(c*x)^(1 + m)*Hypergeometri
c2F1[1, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(a*b*c*(1 + m)) + (B*(c*x)^(2
 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, -((b*x^2)/a)])/(a*c^2*(2
+ m))
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.61.4 Maple [F]

$$\int \frac{(cx)^m (C\,x^2 + Bx + A)}{b\,x^2 + a}dx$$

input
```
int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

output
```
int((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x)
```

3.61.5 Fricas [F]

$$\int \frac{(cx)^m (A + Bx + Cx^2)}{a + bx^2}\,dx = \int \frac{(Cx^2 + Bx + A)(cx)^m}{bx^2 + a}\,dx$$

input
```
integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.61.6 Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.40

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \frac{Ac^m m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Ac^m x^{m+1}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{1}{2}\right)\Gamma\left(\frac{m}{2}+\frac{1}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)} + \frac{Bc^m m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{4a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Bc^m x^{m+2}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+1\right)\Gamma\left(\frac{m}{2}+1\right)}{2a\Gamma\left(\frac{m}{2}+2\right)} + \frac{Cc^m m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)} + \frac{3Cc^m x^{m+3}\Phi\left(\frac{bx^2e^{i\pi}}{a},1,\frac{m}{2}+\frac{3}{2}\right)\Gamma\left(\frac{m}{2}+\frac{3}{2}\right)}{4a\Gamma\left(\frac{m}{2}+\frac{5}{2}\right)}$$

input
```
integrate((c*x)**m*(C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
A*c**m*m*x**(m + 1)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1/2)*gamma
(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + A*c**m*x**(m + 1)*lerchphi(b*x**2*exp
_polar(I*pi)/a, 1, m/2 + 1/2)*gamma(m/2 + 1/2)/(4*a*gamma(m/2 + 3/2)) + B*
c**m*m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 1)*gamma(m/2
 + 1)/(4*a*gamma(m/2 + 2)) + B*c**m*x**(m + 2)*lerchphi(b*x**2*exp_polar(I
*pi)/a, 1, m/2 + 1)*gamma(m/2 + 1)/(2*a*gamma(m/2 + 2)) + C*c**m*m*x**(m +
 3)*lerchphi(b*x**2*exp_polar(I*pi)/a, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a
*gamma(m/2 + 5/2)) + 3*C*c**m*x**(m + 3)*lerchphi(b*x**2*exp_polar(I*pi)/a
, 1, m/2 + 3/2)*gamma(m/2 + 3/2)/(4*a*gamma(m/2 + 5/2))
```

3.61.7 Maxima [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \int \frac{(Cx^2+Bx+A)(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.61.8 Giac [F]

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \int \frac{(Cx^2+Bx+A)(cx)^m}{bx^2+a}\,dx$$

input
```
integrate((c*x)^m*(C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate((C*x^2 + B*x + A)*(c*x)^m/(b*x^2 + a), x)
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(cx)^m (A+Bx+Cx^2)}{a+bx^2}\,dx = \int \frac{(c\,x)^m\,(C\,x^2+B\,x+A)}{b\,x^2+a}\,dx$$

input
```
int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2),x)
```

output
```
int(((c*x)^m*(A + B*x + C*x^2))/(a + b*x^2), x)
```

3.62 $\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.62.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

output
```
1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/
9*b*D*x^9
```

3.62.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^3\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{4}aAx^4+\frac{1}{5}aBx^5+\frac{1}{6}(Ab+aC)x^6+\frac{1}{7}(bB+aD)x^7+\frac{1}{8}bCx^8+\frac{1}{9}bDx^9$$

input
```
Integrate[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b
*C*x^8)/8 + (b*D*x^9)/9
```

3.62.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right) \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^5 (aC+Ab)+aAx^3+x^6 (aD+bB)+aBx^4+bCx^7+bDx^8\right) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6} x^6 (aC+Ab)+\frac{1}{4} aAx^4+\frac{1}{7} x^7 (aD+bB)+\frac{1}{5} aBx^5+\frac{1}{8} bCx^8+\frac{1}{9} bDx^9$$

input
```
Int[x^3*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^4)/4 + (a*B*x^5)/5 + ((A*b + a*C)*x^6)/6 + ((b*B + a*D)*x^7)/7 + (b
*C*x^8)/8 + (b*D*x^9)/9
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.62.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^{4}}{4}+\frac{aB\,x^{5}}{5}+\frac{(Ab+Ca)x^{6}}{6}+\frac{(Bb+Da)x^{7}}{7}+\frac{bC\,x^{8}}{8}+\frac{bDx^{9}}{9}$	54
norman	$\frac{bDx^{9}}{9}+\frac{bC\,x^{8}}{8}+\left(\frac{Bb}{7}+\frac{Da}{7}\right)x^{7}+\left(\frac{Ab}{6}+\frac{Ca}{6}\right)x^{6}+\frac{aB\,x^{5}}{5}+\frac{aA\,x^{4}}{4}$	56
gosper	$\frac{1}{9}bDx^{9}+\frac{1}{8}bC\,x^{8}+\frac{1}{7}bB\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ca+\frac{1}{5}aB\,x^{5}+\frac{1}{4}aA\,x^{4}$	58
parallelrisch	$\frac{1}{9}bDx^{9}+\frac{1}{8}bC\,x^{8}+\frac{1}{7}bB\,x^{7}+\frac{1}{7}x^{7}Da+\frac{1}{6}x^{6}Ab+\frac{1}{6}x^{6}Ca+\frac{1}{5}aB\,x^{5}+\frac{1}{4}aA\,x^{4}$	58

input
```
int(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*A*x^4+1/5*a*B*x^5+1/6*(A*b+C*a)*x^6+1/7*(B*b+D*a)*x^7+1/8*b*C*x^8+1/
9*b*D*x^9
```

3.62.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7+\frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
 A*b)*x^6 + 1/4*A*a*x^4
```

3.62.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^4}{4}+\frac{Bax^5}{5}+\frac{Cbx^8}{8}+\frac{Dbx^9}{9}+x^7\left(\frac{Bb}{7}+\frac{Da}{7}\right)+x^6\left(\frac{Ab}{6}+\frac{Ca}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**4/4 + B*a*x**5/5 + C*b*x**8/8 + D*b*x**9/9 + x**7*(B*b/7 + D*a/7) +
 x**6*(A*b/6 + C*a/6)
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,(Da+Bb)x^7+\frac{1}{5}\,Bax^5+\frac{1}{6}\,(Ca+Ab)x^6+\frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*(D*a + B*b)*x^7 + 1/5*B*a*x^5 + 1/6*(C*a +
 A*b)*x^6 + 1/4*A*a*x^4
```

3.62.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Dbx^9+\frac{1}{8}\,Cbx^8+\frac{1}{7}\,Dax^7+\frac{1}{7}\,Bbx^7+\frac{1}{6}\,Cax^6+\frac{1}{6}\,Abx^6+\frac{1}{5}\,Bax^5+\frac{1}{4}\,Aax^4$$

input
```
integrate(x^3*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b*x^9 + 1/8*C*b*x^8 + 1/7*D*a*x^7 + 1/7*B*b*x^7 + 1/6*C*a*x^6 + 1/6*
A*b*x^6 + 1/5*B*a*x^5 + 1/4*A*a*x^4
```

3.62.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^3(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a\,x^7\,D}{7}+\frac{b\,x^9\,D}{9}+\frac{A\,a\,x^4}{4}+\frac{B\,a\,x^5}{5}+\frac{A\,b\,x^6}{6}+\frac{C\,a\,x^6}{6}+\frac{B\,b\,x^7}{7}+\frac{C\,b\,x^8}{8}$$

input
```
int(x^3*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^7*D)/7 + (b*x^9*D)/9 + (A*a*x^4)/4 + (B*a*x^5)/5 + (A*b*x^6)/6 + (C*a
*x^6)/6 + (B*b*x^7)/7 + (C*b*x^8)/8
```

3.63 $\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.63.1 Optimal result

Integrand size = 26, antiderivative size = 65

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

output
```
1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/
8*b*D*x^8
```

3.63.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x^2\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}(bB+aD)x^6+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

input
```
Integrate[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b
*C*x^7)/7 + (b*D*x^8)/8
```

3.63.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2) (A+Bx+Cx^2+Dx^3) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^4(aC+Ab)+aAx^2+x^5(aD+bB)+aBx^3+bCx^6+bDx^7\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{6}x^6(aD+bB)+\frac{1}{4}aBx^4+\frac{1}{7}bCx^7+\frac{1}{8}bDx^8$$

input
```
Int[x^2*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + ((b*B + a*D)*x^6)/6 + (b
*C*x^7)/7 + (b*D*x^8)/8
```

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.63.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^3}{3}+\frac{Ba\,x^4}{4}+\frac{(Ab+Ca)x^5}{5}+\frac{(Bb+Da)x^6}{6}+\frac{bC\,x^7}{7}+\frac{bDx^8}{8}$	54
norman	$\frac{bDx^8}{8}+\frac{bC\,x^7}{7}+\left(\frac{Bb}{6}+\frac{Da}{6}\right)x^6+\left(\frac{Ab}{5}+\frac{Ca}{5}\right)x^5+\frac{Ba\,x^4}{4}+\frac{aA\,x^3}{3}$	56
gosper	$\frac{1}{8}bDx^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}x^6Da+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}Ba\,x^4+\frac{1}{3}aA\,x^3$	58
parallelrisch	$\frac{1}{8}bDx^8+\frac{1}{7}bC\,x^7+\frac{1}{6}bB\,x^6+\frac{1}{6}x^6Da+\frac{1}{5}x^5Ab+\frac{1}{5}x^5Ca+\frac{1}{4}Ba\,x^4+\frac{1}{3}aA\,x^3$	58

input
```
int(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*A*x^3+1/4*B*a*x^4+1/5*(A*b+C*a)*x^5+1/6*(B*b+D*a)*x^6+1/7*b*C*x^7+1/
8*b*D*x^8
```

3.63.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{8}\,Dbx^8+\frac{1}{7}\,Cbx^7+\frac{1}{6}\,(Da+Bb)x^6+\frac{1}{4}\,Bax^4+\frac{1}{5}\,(Ca+Ab)x^5+\frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
 A*b)*x^5 + 1/3*A*a*x^3
```

3.63.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^3}{3}+\frac{Bax^4}{4}+\frac{Cbx^7}{7}+\frac{Dbx^8}{8}+x^6\left(\frac{Bb}{6}+\frac{Da}{6}\right)+x^5\left(\frac{Ab}{5}+\frac{Ca}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**3/3 + B*a*x**4/4 + C*b*x**7/7 + D*b*x**8/8 + x**6*(B*b/6 + D*a/6) +
 x**5*(A*b/5 + C*a/5)
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Dbx^8 + \frac{1}{7}\,Cbx^7 + \frac{1}{6}\,(Da+Bb)x^6 + \frac{1}{4}\,Bax^4 + \frac{1}{5}\,(Ca+Ab)x^5 + \frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*(D*a + B*b)*x^6 + 1/4*B*a*x^4 + 1/5*(C*a +
 A*b)*x^5 + 1/3*A*a*x^3
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Dbx^8 + \frac{1}{7}\,Cbx^7 + \frac{1}{6}\,Dax^6 + \frac{1}{6}\,Bbx^6 + \frac{1}{5}\,Cax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{4}\,Bax^4 + \frac{1}{3}\,Aax^3$$

input
```
integrate(x^2*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b*x^8 + 1/7*C*b*x^7 + 1/6*D*a*x^6 + 1/6*B*b*x^6 + 1/5*C*a*x^5 + 1/5*
A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

3.63.9 Mupad [B] (verification not implemented)

Time = 6.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x^2(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a\,x^6\,D}{6}+\frac{b\,x^8\,D}{8}+\frac{A\,a\,x^3}{3}+\frac{B\,a\,x^4}{4}+\frac{A\,b\,x^5}{5}+\frac{C\,a\,x^5}{5}+\frac{B\,b\,x^6}{6}+\frac{C\,b\,x^7}{7}$$

input
```
int(x^2*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^6*D)/6 + (b*x^8*D)/8 + (A*a*x^3)/3 + (B*a*x^4)/4 + (A*b*x^5)/5 + (C*a
*x^5)/5 + (B*b*x^6)/6 + (C*b*x^7)/7
```

3.64 $\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.64.1 Optimal result

Integrand size = 24, antiderivative size = 65

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

output
```
1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/
7*b*D*x^7
```

3.64.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}(bB+aD)x^5+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

input
```
Integrate[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b
*C*x^6)/6 + (b*D*x^7)/7
```

3.64.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x^3(aC+Ab)+aAx+x^4(aD+bB)+aBx^2+bCx^5+bDx^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{5}x^5(aD+bB)+\frac{1}{3}aBx^3+\frac{1}{6}bCx^6+\frac{1}{7}bDx^7$$

input
```
Int[x*(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + ((b*B + a*D)*x^5)/5 + (b
*C*x^6)/6 + (b*D*x^7)/7
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.64.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{aA\,x^{2}}{2}+\frac{Ba\,x^{3}}{3}+\frac{(Ab+Ca)x^{4}}{4}+\frac{(Bb+Da)x^{5}}{5}+\frac{bC\,x^{6}}{6}+\frac{bDx^{7}}{7}$	54
norman	$\frac{bDx^{7}}{7}+\frac{bC\,x^{6}}{6}+\left(\frac{Bb}{5}+\frac{Da}{5}\right)x^{5}+\left(\frac{Ab}{4}+\frac{Ca}{4}\right)x^{4}+\frac{Ba\,x^{3}}{3}+\frac{aA\,x^{2}}{2}$	56
gosper	$\frac{1}{7}bDx^{7}+\frac{1}{6}bC\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Da+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ca+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	58
parallelrisch	$\frac{1}{7}bDx^{7}+\frac{1}{6}bC\,x^{6}+\frac{1}{5}bB\,x^{5}+\frac{1}{5}x^{5}Da+\frac{1}{4}x^{4}Ab+\frac{1}{4}x^{4}Ca+\frac{1}{3}Ba\,x^{3}+\frac{1}{2}aA\,x^{2}$	58

input
```
int(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*A*x^2+1/3*B*a*x^3+1/4*(A*b+C*a)*x^4+1/5*(B*b+D*a)*x^5+1/6*b*C*x^6+1/
7*b*D*x^7
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5+\frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
 A*b)*x^4 + 1/2*A*a*x^2
```

3.64.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.92

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aax^2}{2}+\frac{Bax^3}{3}+\frac{Cbx^6}{6}+\frac{Dbx^7}{7}+x^5\left(\frac{Bb}{5}+\frac{Da}{5}\right)+x^4\left(\frac{Ab}{4}+\frac{Ca}{4}\right)$$

input
```
integrate(x*(b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x**2/2 + B*a*x**3/3 + C*b*x**6/6 + D*b*x**7/7 + x**5*(B*b/5 + D*a/5) +
 x**4*(A*b/4 + C*a/4)
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,(Da+Bb)x^5 + \frac{1}{3}\,Bax^3+\frac{1}{4}\,(Ca+Ab)x^4+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*(D*a + B*b)*x^5 + 1/3*B*a*x^3 + 1/4*(C*a +
 A*b)*x^4 + 1/2*A*a*x^2
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{7}\,Dbx^7+\frac{1}{6}\,Cbx^6+\frac{1}{5}\,Dax^5+\frac{1}{5}\,Bbx^5 + \frac{1}{4}\,Cax^4+\frac{1}{4}\,Abx^4+\frac{1}{3}\,Bax^3+\frac{1}{2}\,Aax^2$$

input
```
integrate(x*(b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/7*D*b*x^7 + 1/6*C*b*x^6 + 1/5*D*a*x^5 + 1/5*B*b*x^5 + 1/4*C*a*x^4 + 1/4*
A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

3.64.9 Mupad [B] (verification not implemented)

Time = 6.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a\,x^5\,D}{5}+\frac{b\,x^7\,D}{7}+\frac{A\,a\,x^2}{2}+\frac{B\,a\,x^3}{3}+\frac{A\,b\,x^4}{4}+\frac{C\,a\,x^4}{4}+\frac{B\,b\,x^5}{5}+\frac{C\,b\,x^6}{6}$$

input
```
int(x*(a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^5*D)/5 + (b*x^7*D)/7 + (A*a*x^2)/2 + (B*a*x^3)/3 + (A*b*x^4)/4 + (C*a
*x^4)/4 + (B*b*x^5)/5 + (C*b*x^6)/6
```

3.65 $\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx$

3.65.1 Optimal result

Integrand size = 23, antiderivative size = 60

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3 + \frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

output
```
a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

3.65.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3 + \frac{1}{4}(bB+aD)x^4+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Integrate[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

3.65.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(x^2(aC+Ab)+aA+x^3(aD+bB)+aBx+bCx^4+bDx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{4}x^4(aD+bB)+\frac{1}{2}aBx^2+\frac{1}{5}bCx^5+\frac{1}{6}bDx^6$$

input
```
Int[(a + b*x^2)*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + ((b*B + a*D)*x^4)/4 + (b*C*x^5
)/5 + (b*D*x^6)/6
```

3.65.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.65.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

method	result	size
default	$aAx+\frac{Ba\,x^{2}}{2}+\frac{(Ab+Ca)x^{3}}{3}+\frac{(Bb+Da)x^{4}}{4}+\frac{bC\,x^{5}}{5}+\frac{bDx^{6}}{6}$	51
norman	$\frac{bDx^{6}}{6}+\frac{bC\,x^{5}}{5}+\left(\frac{Bb}{4}+\frac{Da}{4}\right)x^{4}+\left(\frac{Ab}{3}+\frac{Ca}{3}\right)x^{3}+\frac{Ba\,x^{2}}{2}+aAx$	53
gosper	$\frac{1}{6}bDx^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	55
parallelrisch	$\frac{1}{6}bDx^{6}+\frac{1}{5}bC\,x^{5}+\frac{1}{4}bB\,x^{4}+\frac{1}{4}x^{4}Da+\frac{1}{3}Ab\,x^{3}+\frac{1}{3}x^{3}Ca+\frac{1}{2}Ba\,x^{2}+aAx$	55

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
a*A*x+1/2*B*a*x^2+1/3*(A*b+C*a)*x^3+1/4*(B*b+D*a)*x^4+1/5*b*C*x^5+1/6*b*D*
x^6
```

3.65.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{6}\,Dbx^6+\frac{1}{5}\,Cbx^5+\frac{1}{4}\,(Da+Bb)x^4+\frac{1}{2}\,Bax^2+\frac{1}{3}\,(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

3.65.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx=Aax+\frac{Bax^2}{2}+\frac{Cbx^5}{5}+\frac{Dbx^6}{6}+x^4\left(\frac{Bb}{4}+\frac{Da}{4}\right)+x^3\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a*x + B*a*x**2/2 + C*b*x**5/5 + D*b*x**6/6 + x**4*(B*b/4 + D*a/4) + x**3
*(A*b/3 + C*a/3)
```

3.65.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}(Da+Bb)x^4+\frac{1}{2}Bax^2+\frac{1}{3}(Ca+Ab)x^3+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*(D*a + B*b)*x^4 + 1/2*B*a*x^2 + 1/3*(C*a +
 A*b)*x^3 + A*a*x
```

3.65.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2)(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{6}Dbx^6+\frac{1}{5}Cbx^5+\frac{1}{4}Dax^4+\frac{1}{4}Bbx^4+\frac{1}{3}Cax^3+\frac{1}{3}Abx^3+\frac{1}{2}Bax^2+Aax$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/6*D*b*x^6 + 1/5*C*b*x^5 + 1/4*D*a*x^4 + 1/4*B*b*x^4 + 1/3*C*a*x^3 + 1/3*
A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

3.65.9 Mupad [B] (verification not implemented)

Time = 5.88 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int (a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a\,x^4\,D}{4}+\frac{b\,x^6\,D}{6}+A\,a\,x+\frac{B\,a\,x^2}{2}+\frac{A\,b\,x^3}{3}+\frac{C\,a\,x^3}{3}+\frac{B\,b\,x^4}{4}+\frac{C\,b\,x^5}{5}$$

input
```
int((a + b*x^2)*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a*x^4*D)/4 + (b*x^6*D)/6 + A*a*x + (B*a*x^2)/2 + (A*b*x^3)/3 + (C*a*x^3)/
3 + (B*b*x^4)/4 + (C*b*x^5)/5
```

3.66 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx$

3.66.1 Optimal result

Integrand size = 26, antiderivative size = 56

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5 + aA\log(x)$$

output
```
a*B*x+1/2*(A*b+C*a)*x^2+1/3*(B*b+D*a)*x^3+1/4*b*C*x^4+1/5*b*D*x^5+a*A*ln(x
)
```

3.66.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x} \, dx = aBx + \frac{1}{2}(Ab+aC)x^2 + \frac{1}{3}(bB+aD)x^3 + \frac{1}{4}bCx^4 + \frac{1}{5}bDx^5 + aA\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5
)/5 + a*A*Log[x]
```

3.66.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\,(A+Bx+Cx^2+Dx^3)}{x}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(x(aC+Ab)+\frac{aA}{x}+x^2(aD+bB)+aB+bCx^3+bDx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+\frac{1}{3}x^3(aD+bB)+aBx+\frac{1}{4}bCx^4+\frac{1}{5}bDx^5$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a*B*x + ((A*b + a*C)*x^2)/2 + ((b*B + a*D)*x^3)/3 + (b*C*x^4)/4 + (b*D*x^5
)/5 + a*A*Log[x]
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.66.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

method	result	size
norman	$\left(\frac{Ab}{2}+\frac{Ca}{2}\right)x^{2}+\left(\frac{Bb}{3}+\frac{Da}{3}\right)x^{3}+Bax+\frac{bC\,x^{4}}{4}+\frac{bD x^{5}}{5}+aA\ln\left(x\right)$	51
default	$\frac{bD x^{5}}{5}+\frac{bC\,x^{4}}{4}+\frac{bB\,x^{3}}{3}+\frac{Da\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	53
parallelrisch	$\frac{bD x^{5}}{5}+\frac{bC\,x^{4}}{4}+\frac{bB\,x^{3}}{3}+\frac{Da\,x^{3}}{3}+\frac{Ab\,x^{2}}{2}+\frac{Ca\,x^{2}}{2}+Bax+aA\ln\left(x\right)$	53

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/2*A*b+1/2*C*a)*x^2+(1/3*B*b+1/3*D*a)*x^3+B*a*x+1/4*b*C*x^4+1/5*b*D*x^5+
a*A*ln(x)
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx=\frac{1}{5}\,Dbx^{5}+\frac{1}{4}\,Cbx^{4}+\frac{1}{3}\,(Da+Bb)x^{3}+Bax+\frac{1}{2}\,(Ca+Ab)x^{2}+Aa\log\left(x\right)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*
x^2 + A*a*log(x)
```

3.66.6 Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x}\,dx=Aa\log\left(x\right)+Bax+\frac{Cbx^{4}}{4}+\frac{Dbx^{5}}{5}+x^{3}\left(\frac{Bb}{3}+\frac{Da}{3}\right)+x^{2}\left(\frac{Ab}{2}+\frac{Ca}{2}\right)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a*log(x) + B*a*x + C*b*x**4/4 + D*b*x**5/5 + x**3*(B*b/3 + D*a/3) + x**2
*(A*b/2 + C*a/2)
```

3.66.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}(Da+Bb)x^3 + Bax + \frac{1}{2}(Ca+Ab)x^2 + Aa\log(x)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*(D*a + B*b)*x^3 + B*a*x + 1/2*(C*a + A*b)*
x^2 + A*a*log(x)
```

3.66.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{5}Dbx^5 + \frac{1}{4}Cbx^4 + \frac{1}{3}Dax^3 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/5*D*b*x^5 + 1/4*C*b*x^4 + 1/3*D*a*x^3 + 1/3*B*b*x^3 + 1/2*C*a*x^2 + 1/2*
A*b*x^2 + B*a*x + A*a*log(abs(x))
```

3.66.9 Mupad [B] (verification not implemented)

Time = 5.95 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{a\,x^3\,D}{3}+\frac{b\,x^5\,D}{5}+B\,a\,x+\frac{A\,b\,x^2}{2}+\frac{C\,a\,x^2}{2}+\frac{B\,b\,x^3}{3}+\frac{C\,b\,x^4}{4}+A\,a\,\ln(x)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(a*x^3*D)/3 + (b*x^5*D)/5 + B*a*x + (A*b*x^2)/2 + (C*a*x^2)/2 + (B*b*x^3)/
3 + (C*b*x^4)/4 + A*a*log(x)
```

3.67 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx$

3.67.1 Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

output
```
-a*A/x+(A*b+C*a)*x+1/2*(B*b+D*a)*x^2+1/3*b*C*x^3+1/4*b*D*x^4+a*B*ln(x)
```

3.67.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2} dx = -\frac{aA}{x}+(Ab+aC)x+\frac{1}{2}(bB+aD)x^2+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4+aB\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)
/4 + a*B*Log[x]
```

3.67.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(Ab\left(\frac{aC}{Ab}+1\right)+\frac{aA}{x^2}+x(aD+bB)+\frac{aB}{x}+bCx^2+bDx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$x(aC+Ab)-\frac{aA}{x}+\frac{1}{2}x^2(aD+bB)+aB\log(x)+\frac{1}{3}bCx^3+\frac{1}{4}bDx^4$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a*A)/x) + (A*b + a*C)*x + ((b*B + a*D)*x^2)/2 + (b*C*x^3)/3 + (b*D*x^4)
/4 + a*B*Log[x]
```

3.67.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.67.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

method	result	size
default	$\frac{Db\,x^4}{4}+\frac{bC\,x^3}{3}+\frac{bB\,x^2}{2}+\frac{Da\,x^2}{2}+Abx+Cax+aB\ln\left(x\right)-\frac{aA}{x}$	50
norman	$\frac{\left(\frac{Bb}{2}+\frac{Da}{2}\right)x^3+(Ab+Ca)x^2-Aa+\frac{bC\,x^4}{3}+\frac{bD\,x^5}{4}}{x}+aB\ln\left(x\right)$	54
parallelrisch	$\frac{3bD\,x^5+4bC\,x^4+6bB\,x^3+6Da\,x^3+12Ab\,x^2+12Ba\ln(x)x+12Ca\,x^2-12Aa}{12x}$	60

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2+1/2*D*a*x^2+A*b*x+C*a*x+a*B*ln(x)-a*A/
x
```

3.67.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int\frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx=\frac{3\,Dbx^5+4\,Cbx^4+6\,(Da+Bb)x^3+12\,Bax\log\left(x\right)+12\,(Ca+Ab)x^2-12\,Aa}{12\,x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/12*(3*D*b*x^5 + 4*C*b*x^4 + 6*(D*a + B*b)*x^3 + 12*B*a*x*log(x) + 12*(C*
a + A*b)*x^2 - 12*A*a)/x
```

3.67.6 Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int\frac{(a+bx^2)\left(A+Bx+Cx^2+Dx^3\right)}{x^2}\,dx=-\frac{Aa}{x}+Ba\log\left(x\right)+\frac{Cbx^3}{3}+\frac{Dbx^4}{4}+x^2\left(\frac{Bb}{2}+\frac{Da}{2}\right)+x(Ab+Ca)$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a/x + B*a*log(x) + C*b*x**3/3 + D*b*x**4/4 + x**2*(B*b/2 + D*a/2) + x*(
A*b + C*a)
```

3.67.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}Dbx^4 + \frac{1}{3}Cbx^3 + \frac{1}{2}(Da+Bb)x^2 + Ba\log(x) + (Ca+Ab)x - \frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*(D*a + B*b)*x^2 + B*a*log(x) + (C*a + A*b)
*x - A*a/x
```

3.67.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{4}Dbx^4 + \frac{1}{3}Cbx^3 + \frac{1}{2}Dax^2 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/4*D*b*x^4 + 1/3*C*b*x^3 + 1/2*D*a*x^2 + 1/2*B*b*x^2 + C*a*x + A*b*x + B*
a*log(abs(x)) - A*a/x
```

3.67.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{a\,x^2\,D}{2}+\frac{b\,x^4\,D}{4}+A\,b\,x+C\,a\,x-\frac{A\,a}{x}+\frac{B\,b\,x^2}{2}+\frac{C\,b\,x^3}{3}+B\,a\,\ln(x)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
(a*x^2*D)/2 + (b*x^4*D)/4 + A*b*x + C*a*x - (A*a)/x + (B*b*x^2)/2 + (C*b*x
^3)/3 + B*a*log(x)
```

3.68 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx$

3.68.1 Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{aA}{2x^2} - \frac{aB}{x} + (bB+aD)x + \frac{1}{2}bCx^2 + \frac{1}{3}bDx^3 + (Ab+aC)\log(x)$$

output
```
-1/2*a*A/x^2-a*B/x+(B*b+D*a)*x+1/2*b*C*x^2+1/3*b*D*x^3+(A*b+C*a)*ln(x)
```

3.68.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{6}bx(6B+3Cx+2Dx^2) - \frac{a(A+2Bx-2Dx^3)}{2x^2} + (Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
(b*x*(6*B + 3*C*x + 2*D*x^2))/6 - (a*(A + 2*B*x - 2*D*x^3))/(2*x^2) + (A*b
 + a*C)*Log[x]
```

3.68.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{aC+Ab}{x}+\frac{aA}{x^3}+bB\left(\frac{aD}{bB}+1\right)+\frac{aB}{x^2}+bCx+bDx^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}+x(aD+bB)-\frac{aB}{x}+\frac{1}{2}bCx^2+\frac{1}{3}bDx^3$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a*A)/x^2 - (a*B)/x + (b*B + a*D)*x + (b*C*x^2)/2 + (b*D*x^3)/3 + (A*
b + a*C)*Log[x]
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.68.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{D b \,x^{3}}{3}+\frac{b C \,x^{2}}{2}+b B x +D a x +\left(A b +C a \right) \ln \left(x \right)-\frac{a B}{x}-\frac{a A}{2 x^{2}}$	48
norman	$\frac{\left(B b +D a \right) x^{3}-\frac{A a}{2}-B a x +\frac{b C \,x^{4}}{2}+\frac{b D \,x^{5}}{3}}{x^{2}}+\left(A b +C a \right) \ln \left(x \right)$	51
parallelrisch	$\frac{2 b D \,x^{5}+3 b C \,x^{4}+6 A \ln \left(x \right) x^{2} b +6 b B \,x^{3}+6 C \ln \left(x \right) x^{2} a +6 D a \,x^{3}-6 B a x -3 A a}{6 x^{2}}$	62

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*D*b*x^3+1/2*b*C*x^2+b*B*x+D*a*x+(A*b+C*a)*ln(x)-a*B/x-1/2*a*A/x^2
```

3.68.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{2\,Dbx^5+3\,Cbx^4+6\,(Da+Bb)x^3+6\,(Ca+Ab)x^2\log(x)-6\,Bax-3\,Aa}{6\,x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/6*(2*D*b*x^5 + 3*C*b*x^4 + 6*(D*a + B*b)*x^3 + 6*(C*a + A*b)*x^2*log(x)
- 6*B*a*x - 3*A*a)/x^2
```

3.68.6 Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cbx^2}{2}+\frac{Dbx^3}{3}+x(Bb+Da)+(Ab+Ca)\log(x)+\frac{-Aa-2Bax}{2x^2}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b*x**2/2 + D*b*x**3/3 + x*(B*b + D*a) + (A*b + C*a)*log(x) + (-A*a - 2*B
*a*x)/(2*x**2)
```

3.68.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3 + \frac{1}{2}\,Cbx^2 + (Da+Bb)x + (Ca+Ab)\log(x) - \frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/3*D*b*x^3 + 1/2*C*b*x^2 + (D*a + B*b)*x + (C*a + A*b)*log(x) - 1/2*(2*B*
a*x + A*a)/x^2
```

3.68.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{3}\,Dbx^3 + \frac{1}{2}\,Cbx^2 + Dax + Bbx + (Ca+Ab)\log(|x|) - \frac{2\,Bax+Aa}{2\,x^2}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/3*D*b*x^3 + 1/2*C*b*x^2 + D*a*x + B*b*x + (C*a + A*b)*log(abs(x)) - 1/2*
(2*B*a*x + A*a)/x^2
```

3.68.9 Mupad [B] (verification not implemented)

Time = 6.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{b\,x^3\,D}{3}+B\,b\,x-\frac{A\,a}{2\,x^2}-\frac{B\,a}{x}+\frac{C\,b\,x^2}{2}+A\,b\,\ln\left(x\right)+C\,a\,\ln\left(x\right)+a\,x\,D$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(b*x^3*D)/3 + B*b*x - (A*a)/(2*x^2) - (B*a)/x + (C*b*x^2)/2 + A*b*log(x) +
 C*a*log(x) + a*x*D
```

3.69 $\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

3.69.1 Optimal result

Integrand size = 26, antiderivative size = 54

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

output
```
-1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+b*C*x+1/2*b*D*x^2+(B*b+D*a)*ln(x)
```

3.69.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{aA}{3x^3}-\frac{aB}{2x^2}+\frac{-Ab-aC}{x}+bCx+\frac{1}{2}bDx^2+(bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) + (-(A*b) - a*C)/x + b*C*x + (b*D*x^2)/2 +
(b*B + a*D)*Log[x]
```

3.69.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{aC+Ab}{x^2}+\frac{aA}{x^4}+\frac{aD+bB}{x}+\frac{aB}{x^3}+bC+bDx\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}+\log(x)(aD+bB)-\frac{aB}{2x^2}+bCx+\frac{1}{2}bDx^2$$

input
```
Int[((a + b*x^2)*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a*A)/x^3 - (a*B)/(2*x^2) - (A*b + a*C)/x + b*C*x + (b*D*x^2)/2 + (b*
B + a*D)*Log[x]
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.69.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
default	$\frac{Db\,x^2}{2}+bCx+(Bb+Da)\ln(x)-\frac{aA}{3x^3}-\frac{Ab+Ca}{x}-\frac{aB}{2x^2}$	49
norman	$\frac{(-Ab-Ca)x^2+bC\,x^4-\frac{Aa}{3}-\frac{Bax}{2}+\frac{bDx^5}{2}}{x^3}+(Bb+Da)\ln(x)$	52
parallelrisch	$-\frac{-3bDx^5-6B\ln(x)x^3b-6bC\,x^4-6D\ln(x)x^3a+6Ab\,x^2+6Ca\,x^2+3Bax+2Aa}{6x^3}$	62

input
```
int((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*D*b*x^2+b*C*x+(B*b+D*a)*ln(x)-1/3*a*A/x^3-(A*b+C*a)/x-1/2*a*B/x^2
```

3.69.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{3\,Dbx^5+6\,Cbx^4+6\,(Da+Bb)x^3\log(x)-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/6*(3*D*b*x^5 + 6*C*b*x^4 + 6*(D*a + B*b)*x^3*log(x) - 3*B*a*x - 6*(C*a +
 A*b)*x^2 - 2*A*a)/x^3
```

3.69.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = Cbx+\frac{Dbx^2}{2}+(Bb+Da)\log(x)+\frac{-2Aa-3Bax+x^2(-6Ab-6Ca)}{6x^3}$$

input
```
integrate((b*x**2+a)*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b*x + D*b*x**2/2 + (B*b + D*a)*log(x) + (-2*A*a - 3*B*a*x + x**2*(-6*A*b
 - 6*C*a))/(6*x**3)
```

3.69.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}\,Dbx^2 + Cbx + (Da+Bb)\log(x) - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(x) - 1/6*(3*B*a*x + 6*(C*a + A*b)*x^
2 + 2*A*a)/x^3
```

3.69.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{2}\,Dbx^2 + Cbx + (Da+Bb)\log(|x|) - \frac{3\,Bax + 6\,(Ca+Ab)x^2 + 2\,Aa}{6\,x^3}$$

input
```
integrate((b*x^2+a)*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/2*D*b*x^2 + C*b*x + (D*a + B*b)*log(abs(x)) - 1/6*(3*B*a*x + 6*(C*a + A*
b)*x^2 + 2*A*a)/x^3
```

3.69.9 Mupad [B] (verification not implemented)

Time = 6.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{b\,x^2\,D}{2}+a\,\ln(x)\,D+C\,b\,x-\frac{A\,a}{3\,x^3}-\frac{A\,b}{x}-\frac{B\,a}{2\,x^2}-\frac{C\,a}{x}+B\,b\,\ln(x)$$

input
```
int(((a + b*x^2)*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(b*x^2*D)/2 + a*log(x)*D + C*b*x - (A*a)/(3*x^3) - (A*b)/x - (B*a)/(2*x^2)
 - (C*a)/x + B*b*log(x)
```

3.70 $\int x^3(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx$

3.70.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int x^3(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx=\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{6}a(2Ab+aC)x^6+\frac{1}{7}a(2bB+aD)x^7+\frac{1}{8}b(Ab+2aC)x^8+\frac{1}{9}b(bB+2aD)x^9+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

output
```
1/4*a^2*A*x^4+1/5*a^2*B*x^5+1/6*a*(2*A*b+C*a)*x^6+1/7*a*(2*B*b+D*a)*x^7+1/
8*b*(A*b+2*C*a)*x^8+1/9*b*(B*b+2*D*a)*x^9+1/10*b^2*C*x^10+1/11*b^2*D*x^11
```

3.70.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.90

$$\int x^3(a+bx^2)^2(A+Bx+Cx^2+Dx^3)\,dx=a^2\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}+\frac{1}{42}x^6(7C+6Dx)\right)+\frac{b^2x^8(495A+4x(110B+99Cx+90Dx^2))}{3960}+\frac{1}{252}abx^6(84A+x(72B+7x(9C+8Dx)))$$

input
```
Integrate[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*((A*x^4)/4 + (B*x^5)/5 + (x^6*(7*C + 6*D*x))/42) + (b^2*x^8*(495*A + 4
*x*(110*B + 99*C*x + 90*D*x^2)))/3960 + (a*b*x^6*(84*A + x*(72*B + 7*x*(9*
C + 8*D*x))))/252
```

3.70.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(a^2Ax^3+a^2Bx^4+bx^7(2aC+Ab)+ax^5(aC+2Ab)+bx^8(2aD+bB)+ax^6(aD+2bB)+b^2Cx^9+b^2Dx^{10}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{4}a^2Ax^4+\frac{1}{5}a^2Bx^5+\frac{1}{8}bx^8(2aC+Ab)+\frac{1}{6}ax^6(aC+2Ab)+\frac{1}{9}bx^9(2aD+bB)+\frac{1}{7}ax^7(aD+2bB)+\frac{1}{10}b^2Cx^{10}+\frac{1}{11}b^2Dx^{11}$$

input
```
Int[x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*A*x^4)/4 + (a^2*B*x^5)/5 + (a*(2*A*b + a*C)*x^6)/6 + (a*(2*b*B + a*D)
*x^7)/7 + (b*(A*b + 2*a*C)*x^8)/8 + (b*(b*B + 2*a*D)*x^9)/9 + (b^2*C*x^10)
/10 + (b^2*D*x^11)/11
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.70.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2 D x^{11}}{11}+\frac{b^2 C x^{10}}{10}+\frac{(B b^2+2Dab)x^9}{9}+\frac{(b^2 A+2Cab)x^8}{8}+\frac{(2abB+Da^2)x^7}{7}+\frac{(2abA+C a^2)x^6}{6}+\frac{a^2 B x^5}{5}+\frac{a^2 A x}{4}$
norman	$\frac{b^2 D x^{11}}{11}+\frac{b^2 C x^{10}}{10}+\left(\frac{1}{9}B b^2+\frac{2}{9}Dab\right)x^9+\left(\frac{1}{8}b^2 A+\frac{1}{4}Cab\right)x^8+\left(\frac{2}{7}abB+\frac{1}{7}Da^2\right)x^7+\left(\frac{1}{3}abA+\right.$
gosper	$\frac{1}{11}b^2 D x^{11}+\frac{1}{10}b^2 C x^{10}+\frac{1}{9}b^2 B x^9+\frac{2}{9}x^9 Dab+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 Cab+\frac{2}{7}x^7 abB+\frac{1}{7}x^7 Da^2+\frac{1}{3}x$
parallelrisch	$\frac{1}{11}b^2 D x^{11}+\frac{1}{10}b^2 C x^{10}+\frac{1}{9}b^2 B x^9+\frac{2}{9}x^9 Dab+\frac{1}{8}x^8 b^2 A+\frac{1}{4}x^8 Cab+\frac{2}{7}x^7 abB+\frac{1}{7}x^7 Da^2+\frac{1}{3}x$

input
```
int(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^2*D*x^11+1/10*b^2*C*x^10+1/9*(B*b^2+2*D*a*b)*x^9+1/8*(A*b^2+2*C*a*b
)*x^8+1/7*(2*B*a*b+D*a^2)*x^7+1/6*(2*A*a*b+C*a^2)*x^6+1/5*a^2*B*x^5+1/4*a^
2*A*x^4
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a
*b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^
4 + 1/6*(C*a^2 + 2*A*a*b)*x^6
```

3.70.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aa^2x^4}{4}+\frac{Ba^2x^5}{5}+\frac{Cb^2x^{10}}{10}+\frac{Db^2x^{11}}{11}+x^9\left(\frac{Bb^2}{9}+\frac{2Dab}{9}\right)+x^8\left(\frac{Ab^2}{8}+\frac{Cab}{4}\right)+x^7\cdot\left(\frac{2Bab}{7}+\frac{Da^2}{7}\right)+x^6\left(\frac{Aab}{3}+\frac{Ca^2}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**4/4 + B*a**2*x**5/5 + C*b**2*x**10/10 + D*b**2*x**11/11 + x**9*(
B*b**2/9 + 2*D*a*b/9) + x**8*(A*b**2/8 + C*a*b/4) + x**7*(2*B*a*b/7 + D*a*
*2/7) + x**6*(A*a*b/3 + C*a**2/6)
```

3.70.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{1}{9}\,(2\,Dab+Bb^2)x^9+\frac{1}{8}\,(2\,Cab+Ab^2)x^8+\frac{1}{5}\,Ba^2x^5+\frac{1}{7}\,(Da^2+2\,Bab)x^7+\frac{1}{4}\,Aa^2x^4+\frac{1}{6}\,(Ca^2+2\,Aab)x^6$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 1/9*(2*D*a*b + B*b^2)*x^9 + 1/8*(2*C*a
*b + A*b^2)*x^8 + 1/5*B*a^2*x^5 + 1/7*(D*a^2 + 2*B*a*b)*x^7 + 1/4*A*a^2*x^
4 + 1/6*(C*a^2 + 2*A*a*b)*x^6
```

3.70.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{11}\,Db^2x^{11}+\frac{1}{10}\,Cb^2x^{10}+\frac{2}{9}\,Dabx^9+\frac{1}{9}\,Bb^2x^9+\frac{1}{4}\,Cabx^8+\frac{1}{8}\,Ab^2x^8+\frac{1}{7}\,Da^2x^7+\frac{2}{7}\,Babx^7+\frac{1}{6}\,Ca^2x^6+\frac{1}{3}\,Aabx^6+\frac{1}{5}\,Ba^2x^5+\frac{1}{4}\,Aa^2x^4$$

input
```
integrate(x^3*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^2*x^11 + 1/10*C*b^2*x^10 + 2/9*D*a*b*x^9 + 1/9*B*b^2*x^9 + 1/4*C*
a*b*x^8 + 1/8*A*b^2*x^8 + 1/7*D*a^2*x^7 + 2/7*B*a*b*x^7 + 1/6*C*a^2*x^6 +
1/3*A*a*b*x^6 + 1/5*B*a^2*x^5 + 1/4*A*a^2*x^4
```

3.70.9 Mupad [B] (verification not implemented)

Time = 5.80 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a^2\,x^7\,D}{7}+\frac{b^2\,x^{11}\,D}{11}+\frac{A\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{B\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{C\,x^6\,\left(10\,a^2+15\,a\,b\,x^2+6\,b^2\,x^4\right)}{60}+\frac{2\,a\,b\,x^9\,D}{9}$$

input
```
int(x^3*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^7*D)/7 + (b^2*x^11*D)/11 + (A*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/
24 + (B*x^5*(63*a^2 + 35*b^2*x^4 + 90*a*b*x^2))/315 + (C*x^6*(10*a^2 + 6*b
^2*x^4 + 15*a*b*x^2))/60 + (2*a*b*x^9*D)/9
```

3.70. $\int x^3(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.71 $\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.71.1 Optimal result

Integrand size = 28, antiderivative size = 109

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{6}a(2bB+aD)x^6+\frac{1}{7}b(Ab+2aC)x^7+\frac{1}{8}b(bB+2aD)x^8+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

output
```
1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/6*a*(2*B*b+D*a)*x^6+1/
7*b*(A*b+2*C*a)*x^7+1/8*b*(B*b+2*D*a)*x^8+1/9*b^2*C*x^9+1/10*b^2*D*x^10
```

3.71.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$\int x^2\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{42a^2x^3(20A+x(15B+2x(6C+5Dx)))+6abx^5(168A+5x(28B+3x(8C+7Dx)))+b^2x^7(360A+7x}{2520}$$

input
```
Integrate[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(42*a^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))) + 6*a*b*x^5*(168*A + 5*x
*(28*B + 3*x*(8*C + 7*D*x))) + b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*
D*x))))/2520
```

3.71.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+bx^2)^2 (A+Bx+Cx^2+Dx^3) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int (a^2Ax^2+a^2Bx^3+bx^6(2aC+Ab)+ax^4(aC+2Ab)+bx^7(2aD+bB)+ax^5(aD+2bB)+b^2Cx^8+b^2Dx^9) \, d$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{7}bx^7(2aC+Ab)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}bx^8(2aD+bB)+\frac{1}{6}ax^6(aD+2bB)+\frac{1}{9}b^2Cx^9+\frac{1}{10}b^2Dx^{10}$$

input
```
Int[x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*(2*b*B + a*D)
*x^6)/6 + (b*(A*b + 2*a*C)*x^7)/7 + (b*(b*B + 2*a*D)*x^8)/8 + (b^2*C*x^9)/
9 + (b^2*D*x^10)/10
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.71.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2 D x^{10}}{10}+\frac{b^2 C x^9}{9}+\frac{(B b^2+2Dab)x^8}{8}+\frac{(b^2 A+2Cab)x^7}{7}+\frac{(2abB+Da^2)x^6}{6}+\frac{(2abA+C a^2)x^5}{5}+\frac{a^2 B x^4}{4}+\frac{a^2 A x^3}{3}$
norman	$\frac{b^2 D x^{10}}{10}+\frac{b^2 C x^9}{9}+\left(\frac{1}{8}B b^2+\frac{1}{4}Dab\right)x^8+\left(\frac{1}{7}b^2 A+\frac{2}{7}Cab\right)x^7+\left(\frac{1}{3}abB+\frac{1}{6}Da^2\right)x^6+\left(\frac{2}{5}abA+\right.$
gosper	$\frac{1}{10}b^2 D x^{10}+\frac{1}{9}b^2 C x^9+\frac{1}{8}b^2 B x^8+\frac{1}{4}x^8 Dab+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 Cab+\frac{1}{3}x^6 abB+\frac{1}{6}x^6 Da^2+\frac{2}{5}x^5 a$
parallelrisch	$\frac{1}{10}b^2 D x^{10}+\frac{1}{9}b^2 C x^9+\frac{1}{8}b^2 B x^8+\frac{1}{4}x^8 Dab+\frac{1}{7}x^7 b^2 A+\frac{2}{7}x^7 Cab+\frac{1}{3}x^6 abB+\frac{1}{6}x^6 Da^2+\frac{2}{5}x^5 a$

input
```
int(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^2*D*x^10+1/9*b^2*C*x^9+1/8*(B*b^2+2*D*a*b)*x^8+1/7*(A*b^2+2*C*a*b)*
x^7+1/6*(2*B*a*b+D*a^2)*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*
A*x^3
```

3.71.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{10}Db^2x^{10}+\frac{1}{9}Cb^2x^9+\frac{1}{8}\left(2\,Dab+Bb^2\right)x^8+\frac{1}{7}\left(2\,Cab+Ab^2\right)x^7+\frac{1}{4}Ba^2x^4+\frac{1}{6}\left(Da^2+2\,Bab\right)x^6+\frac{1}{3}Aa^2x^3+\frac{1}{5}\left(Ca^2+2\,Aab\right)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
 + A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3
+ 1/5*(C*a^2 + 2*A*a*b)*x^5
```

3.71.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.01

$$\int x^2(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Cb^2x^9}{9}+\frac{Db^2x^{10}}{10} + x^8\left(\frac{Bb^2}{8}+\frac{Dab}{4}\right)+x^7\left(\frac{Ab^2}{7}+\frac{2Cab}{7}\right) + x^6\left(\frac{Bab}{3}+\frac{Da^2}{6}\right)+x^5\cdot\left(\frac{2Aab}{5}+\frac{Ca^2}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**3/3 + B*a**2*x**4/4 + C*b**2*x**9/9 + D*b**2*x**10/10 + x**8*(B*
b**2/8 + D*a*b/4) + x**7*(A*b**2/7 + 2*C*a*b/7) + x**6*(B*a*b/3 + D*a**2/6
) + x**5*(2*A*a*b/5 + C*a**2/5)
```

3.71.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.93

$$\int x^2(a+bx^2)^2\,(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}\,Db^2x^{10}+\frac{1}{9}\,Cb^2x^9+\frac{1}{8}\,(2\,Dab+Bb^2)x^8 + \frac{1}{7}\,(2\,Cab+Ab^2)x^7+\frac{1}{4}\,Ba^2x^4 + \frac{1}{6}\,(Da^2+2\,Bab)x^6 + \frac{1}{3}\,Aa^2x^3+\frac{1}{5}\,(Ca^2+2\,Aab)x^5$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/8*(2*D*a*b + B*b^2)*x^8 + 1/7*(2*C*a*b
 + A*b^2)*x^7 + 1/4*B*a^2*x^4 + 1/6*(D*a^2 + 2*B*a*b)*x^6 + 1/3*A*a^2*x^3
+ 1/5*(C*a^2 + 2*A*a*b)*x^5
```

3.71.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{10}Db^2x^{10}+\frac{1}{9}Cb^2x^9+\frac{1}{4}Dabx^8+\frac{1}{8}Bb^2x^8+\frac{2}{7}Cabx^7+\frac{1}{7}Ab^2x^7+\frac{1}{6}Da^2x^6+\frac{1}{3}Babx^6+\frac{1}{5}Ca^2x^5+\frac{2}{5}Aabx^5+\frac{1}{4}Ba^2x^4+\frac{1}{3}Aa^2x^3$$

input
```
integrate(x^2*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^2*x^10 + 1/9*C*b^2*x^9 + 1/4*D*a*b*x^8 + 1/8*B*b^2*x^8 + 2/7*C*a*
b*x^7 + 1/7*A*b^2*x^7 + 1/6*D*a^2*x^6 + 1/3*B*a*b*x^6 + 1/5*C*a^2*x^5 + 2/
5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

3.71.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{a^2\,x^6\,D}{6}+\frac{b^2\,x^{10}\,D}{10}+\frac{A\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105}+\frac{B\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{C\,x^5\,\left(63\,a^2+90\,a\,b\,x^2+35\,b^2\,x^4\right)}{315}+\frac{a\,b\,x^8\,D}{4}$$

input
```
int(x^2*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^6*D)/6 + (b^2*x^10*D)/10 + (A*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2
))/105 + (B*x^4*(6*a^2 + 3*b^2*x^4 + 8*a*b*x^2))/24 + (C*x^5*(63*a^2 + 35*
b^2*x^4 + 90*a*b*x^2))/315 + (a*b*x^8*D)/4
```

3.72 $\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$

3.72.1 Optimal result

Integrand size = 26, antiderivative size = 104

$$\begin{aligned}\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx &= \frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{1}{5}a(2bB+aD)x^5 \\ &\quad+\frac{1}{3}abCx^6+\frac{1}{7}b(bB+2aD)x^7 \\ &\quad+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9+\frac{A(a+bx^2)^3}{6b}\end{aligned}$$

output
```
1/3*a^2*B*x^3+1/4*a^2*C*x^4+1/5*a*(2*B*b+D*a)*x^5+1/3*a*b*C*x^6+1/7*b*(B*b
+2*D*a)*x^7+1/8*b^2*C*x^8+1/9*b^2*D*x^9+1/6*A*(b*x^2+a)^3/b
```

3.72.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$$
$$=\frac{42a^2x^2(30A+x(20B+3x(5C+4Dx)))+12abx^4(105A+2x(42B+5x(7C+6Dx)))+5b^2x^6(84A+x}{2520}$$

input
```
Integrate[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(42*a^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))) + 12*a*b*x^4*(105*A + 2*
x*(42*B + 5*x*(7*C + 6*D*x))) + 5*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D
*x))))/2520
```

3.72.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(bx^2+a\right)^2\left(x\left(Dx^3+Cx^2+Bx+A\right)-Ax\right)dx+\frac{A\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(b^2Dx^8+b^2Cx^7+b(bB+2aD)x^6+2abCx^5+a(2bB+aD)x^4+a^2Cx^3+a^2Bx^2\right)dx+\frac{A\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^2Bx^3+\frac{1}{4}a^2Cx^4+\frac{A\left(a+bx^2\right)^3}{6b}+\frac{1}{7}bx^7(2aD+bB)+\frac{1}{5}ax^5(aD+2bB)+\frac{1}{3}abCx^6+\frac{1}{8}b^2Cx^8+\frac{1}{9}b^2Dx^9$$

input
```
Int[x*(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^2*B*x^3)/3 + (a^2*C*x^4)/4 + (a*(2*b*B + a*D)*x^5)/5 + (a*b*C*x^6)/3 +
(b*(b*B + 2*a*D)*x^7)/7 + (b^2*C*x^8)/8 + (b^2*D*x^9)/9 + (A*(a + b*x^2)^3
)/(6*b)
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.72.4 Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.98

method	result
default	$\frac{b^2 D x^9}{9}+\frac{b^2 C\,x^8}{8}+\frac{(B\,b^2+2Dab)x^7}{7}+\frac{(b^2 A+2Cab)x^6}{6}+\frac{(2abB+Da^2)x^5}{5}+\frac{(2abA+C\,a^2)x^4}{4}+\frac{a^2 B\,x^3}{3}+\frac{a^2 A\,x^2}{2}$
norman	$\frac{b^2 D x^9}{9}+\frac{b^2 C\,x^8}{8}+\left(\frac{1}{7}B\,b^2+\frac{2}{7}Dab\right)x^7+\left(\frac{1}{6}b^2 A+\frac{1}{3}Cab\right)x^6+\left(\frac{2}{5}abB+\frac{1}{5}Da^2\right)x^5+\left(\frac{1}{2}abA+\right.$
gosper	$\frac{1}{9}b^2 D x^9+\frac{1}{8}b^2 C\,x^8+\frac{1}{7}b^2 B\,x^7+\frac{2}{7}x^7 Dab+\frac{1}{6}x^6 b^2 A+\frac{1}{3}abC\,x^6+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 Da^2+\frac{1}{2}x^4 ab$
parallelrisch	$\frac{1}{9}b^2 D x^9+\frac{1}{8}b^2 C\,x^8+\frac{1}{7}b^2 B\,x^7+\frac{2}{7}x^7 Dab+\frac{1}{6}x^6 b^2 A+\frac{1}{3}abC\,x^6+\frac{2}{5}x^5 abB+\frac{1}{5}x^5 Da^2+\frac{1}{2}x^4 ab$

input
```
int(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/9*b^2*D*x^9+1/8*b^2*C*x^8+1/7*(B*b^2+2*D*a*b)*x^7+1/6*(A*b^2+2*C*a*b)*x^
6+1/5*(2*B*a*b+D*a^2)*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*
x^2
```

3.72.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{9}\,Db^2x^9+\frac{1}{8}\,Cb^2x^8+\frac{1}{7}\,(2\,Dab+Bb^2)x^7+\frac{1}{6}\,(2\,Cab+Ab^2)x^6+\frac{1}{3}\,Ba^2x^3+\frac{1}{5}\,(Da^2+2\,Bab)x^5+\frac{1}{2}\,Aa^2x^2+\frac{1}{4}\,(Ca^2+2\,Aab)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
 A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 +
1/4*(C*a^2 + 2*A*a*b)*x^4
```

3.72.6 Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{Cb^2x^8}{8}+\frac{Db^2x^9}{9}+x^7\left(\frac{Bb^2}{7}+\frac{2Dab}{7}\right)+x^6\left(\frac{Ab^2}{6}+\frac{Cab}{3}\right)+x^5\cdot\left(\frac{2Bab}{5}+\frac{Da^2}{5}\right)+x^4\left(\frac{Aab}{2}+\frac{Ca^2}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x**2/2 + B*a**2*x**3/3 + C*b**2*x**8/8 + D*b**2*x**9/9 + x**7*(B*b*
*2/7 + 2*D*a*b/7) + x**6*(A*b**2/6 + C*a*b/3) + x**5*(2*B*a*b/5 + D*a**2/5
) + x**4*(A*a*b/2 + C*a**2/4)
```

3.72.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.97

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{9}Db^2x^9 + \frac{1}{8}Cb^2x^8 + \frac{1}{7}(2Dab+Bb^2)x^7 + \frac{1}{6}(2Cab+Ab^2)x^6 + \frac{1}{3}Ba^2x^3 + \frac{1}{5}(Da^2+2Bab)x^5 + \frac{1}{2}Aa^2x^2 + \frac{1}{4}(Ca^2+2Aab)x^4$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 1/7*(2*D*a*b + B*b^2)*x^7 + 1/6*(2*C*a*b +
 A*b^2)*x^6 + 1/3*B*a^2*x^3 + 1/5*(D*a^2 + 2*B*a*b)*x^5 + 1/2*A*a^2*x^2 +
1/4*(C*a^2 + 2*A*a*b)*x^4
```

3.72.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int x(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\ dx = \frac{1}{9}Db^2x^9 + \frac{1}{8}Cb^2x^8 + \frac{2}{7}Dabx^7 + \frac{1}{7}Bb^2x^7 + \frac{1}{3}Cabx^6 + \frac{1}{6}Ab^2x^6 + \frac{1}{5}Da^2x^5 + \frac{2}{5}Babx^5 + \frac{1}{4}Ca^2x^4 + \frac{1}{2}Aabx^4 + \frac{1}{3}Ba^2x^3 + \frac{1}{2}Aa^2x^2$$

input
```
integrate(x*(b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/9*D*b^2*x^9 + 1/8*C*b^2*x^8 + 2/7*D*a*b*x^7 + 1/7*B*b^2*x^7 + 1/3*C*a*b*
x^6 + 1/6*A*b^2*x^6 + 1/5*D*a^2*x^5 + 2/5*B*a*b*x^5 + 1/4*C*a^2*x^4 + 1/2*
A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

3.72.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03

$$\int x(a+bx^2)^2\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{a^2\,x^5\,D}{5}+\frac{b^2\,x^9\,D}{9}+\frac{A\,x^2\,\left(3\,a^2+3\,a\,b\,x^2+b^2\,x^4\right)}{6} + \frac{B\,x^3\,\left(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4\right)}{105} + \frac{C\,x^4\,\left(6\,a^2+8\,a\,b\,x^2+3\,b^2\,x^4\right)}{24}+\frac{2\,a\,b\,x^7\,D}{7}$$

input
```
int(x*(a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(a^2*x^5*D)/5 + (b^2*x^9*D)/9 + (A*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 +
(B*x^3*(35*a^2 + 15*b^2*x^4 + 42*a*b*x^2))/105 + (C*x^4*(6*a^2 + 3*b^2*x^4
 + 8*a*b*x^2))/24 + (2*a*b*x^7*D)/7
```

3.73 $\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx$

3.73.1 Optimal result

Integrand size = 25, antiderivative size = 99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = a^2Ax+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{4}a^2Dx^4+\frac{1}{5}b(Ab+2aC)x^5 + \frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8+\frac{B(a+bx^2)^3}{6b}$$

output
```
a^2*A*x+1/3*a*(2*A*b+C*a)*x^3+1/4*a^2*D*x^4+1/5*b*(A*b+2*C*a)*x^5+1/3*a*b*
D*x^6+1/7*b^2*C*x^7+1/8*b^2*D*x^8+1/6*B*(b*x^2+a)^3/b
```

3.73.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.89

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{840}(70a^2x(12A+x(6B+x(4C+3Dx))) + 28abx^3(20A+x(15B+2x(6C+5Dx))) + b^2x^5(168A+5x(28B+3x(8C+7Dx))))$$

input
```
Integrate[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(70*a^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 28*a*b*x^3*(20*A + x*(15*B
+ 2*x*(6*C + 5*D*x))) + b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/
840
```

3.73.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^2 \left(Dx^3+Cx^2+A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^2Dx^7+b^2Cx^6+2abDx^5+b(Ab+2aC)x^4+a^2Dx^3+a(2Ab+aC)x^2+a^2A\right)dx+\frac{B\left(a+bx^2\right)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2Ax+\frac{1}{4}a^2Dx^4+\frac{1}{5}bx^5(2aC+Ab)+\frac{1}{3}ax^3(aC+2Ab)+\frac{B\left(a+bx^2\right)^3}{6b}+\frac{1}{3}abDx^6+\frac{1}{7}b^2Cx^7+\frac{1}{8}b^2Dx^8$$

input
```
Int[(a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^2*A*x + (a*(2*A*b + a*C)*x^3)/3 + (a^2*D*x^4)/4 + (b*(A*b + 2*a*C)*x^5)/
5 + (a*b*D*x^6)/3 + (b^2*C*x^7)/7 + (b^2*D*x^8)/8 + (B*(a + b*x^2)^3)/(6*b
)
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.73.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

method	result
default	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\frac{(B\,b^2+2Dab)x^6}{6}+\frac{(b^2A+2Cab)x^5}{5}+\frac{(2abB+Da^2)x^4}{4}+\frac{(2abA+C\,a^2)x^3}{3}+\frac{a^2B\,x^2}{2}+a^2Ax$
norman	$\frac{b^2Dx^8}{8}+\frac{b^2C\,x^7}{7}+\left(\frac{1}{6}B\,b^2+\frac{1}{3}Dab\right)x^6+\left(\frac{1}{5}b^2A+\frac{2}{5}Cab\right)x^5+\left(\frac{1}{2}abB+\frac{1}{4}Da^2\right)x^4+\left(\frac{2}{3}abA+\right.$
gosper	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}x^3ab$
parallelrisch	$\frac{1}{8}b^2Dx^8+\frac{1}{7}b^2C\,x^7+\frac{1}{6}b^2B\,x^6+\frac{1}{3}abDx^6+\frac{1}{5}x^5b^2A+\frac{2}{5}x^5Cab+\frac{1}{2}Bab\,x^4+\frac{1}{4}a^2Dx^4+\frac{2}{3}x^3ab$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/8*b^2*D*x^8+1/7*b^2*C*x^7+1/6*(B*b^2+2*D*a*b)*x^6+1/5*(A*b^2+2*C*a*b)*x^
5+1/4*(2*B*a*b+D*a^2)*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

3.73.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,(2\,Dab+Bb^2)x^6 + \frac{1}{5}\,(2\,Cab+Ab^2)x^5+\frac{1}{2}\,Ba^2x^2 + \frac{1}{4}\,(Da^2+2\,Bab)x^4 + Aa^2x+\frac{1}{3}\,(Ca^2+2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

3.73.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.08

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = Aa^2x+\frac{Ba^2x^2}{2}+\frac{Cb^2x^7}{7}+\frac{Db^2x^8}{8} + x^6\left(\frac{Bb^2}{6}+\frac{Dab}{3}\right)+x^5\left(\frac{Ab^2}{5}+\frac{2Cab}{5}\right) + x^4\left(\frac{Bab}{2}+\frac{Da^2}{4}\right)+x^3\cdot\left(\frac{2Aab}{3}+\frac{Ca^2}{3}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**2*x + B*a**2*x**2/2 + C*b**2*x**7/7 + D*b**2*x**8/8 + x**6*(B*b**2/6
+ D*a*b/3) + x**5*(A*b**2/5 + 2*C*a*b/5) + x**4*(B*a*b/2 + D*a**2/4) + x**
3*(2*A*a*b/3 + C*a**2/3)
```

3.73.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{6}\,(2\,Dab+Bb^2)x^6 + \frac{1}{5}\,(2\,Cab+Ab^2)x^5+\frac{1}{2}\,Ba^2x^2 + \frac{1}{4}\,(Da^2+2\,Bab)x^4 + Aa^2x+\frac{1}{3}\,(Ca^2+2\,Aab)x^3$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/6*(2*D*a*b + B*b^2)*x^6 + 1/5*(2*C*a*b +
 A*b^2)*x^5 + 1/2*B*a^2*x^2 + 1/4*(D*a^2 + 2*B*a*b)*x^4 + A*a^2*x + 1/3*(C
*a^2 + 2*A*a*b)*x^3
```

3.73.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)^2 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{8}\,Db^2x^8+\frac{1}{7}\,Cb^2x^7+\frac{1}{3}\,Dabx^6+\frac{1}{6}\,Bb^2x^6 + \frac{2}{5}\,Cabx^5+\frac{1}{5}\,Ab^2x^5+\frac{1}{4}\,Da^2x^4+\frac{1}{2}\,Babx^4 + \frac{1}{3}\,Ca^2x^3+\frac{2}{3}\,Aabx^3+\frac{1}{2}\,Ba^2x^2+Aa^2x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/8*D*b^2*x^8 + 1/7*C*b^2*x^7 + 1/3*D*a*b*x^6 + 1/6*B*b^2*x^6 + 2/5*C*a*b*
x^5 + 1/5*A*b^2*x^5 + 1/4*D*a^2*x^4 + 1/2*B*a*b*x^4 + 1/3*C*a^2*x^3 + 2/3*
A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

3.73.9 Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06

$$\int (a+bx^2)^2 (A+Bx+Cx^2+Dx^3)\,dx = \frac{A\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15}+\frac{a^2\,x^4\,D}{4}+\frac{b^2\,x^8\,D}{8}+\frac{B\,x^2\,(3\,a^2+3\,a\,b\,x^2+b^2\,x^4)}{6}+\frac{C\,x^3\,(35\,a^2+42\,a\,b\,x^2+15\,b^2\,x^4)}{105}+\frac{a\,b\,x^6\,D}{3}$$

input
```
int((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (a^2*x^4*D)/4 + (b^2*x^8*D)/8
 + (B*x^2*(3*a^2 + b^2*x^4 + 3*a*b*x^2))/6 + (C*x^3*(35*a^2 + 15*b^2*x^4 +
 42*a*b*x^2))/105 + (a*b*x^6*D)/3
```

3.74 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx$

3.74.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = a^2Bx + aAbx^2 + \frac{1}{3}a(2bB+aD)x^3 + \frac{1}{4}Ab^2x^4 + \frac{1}{5}b(bB+2aD)x^5 + \frac{1}{7}b^2Dx^7 + \frac{C(a+bx^2)^3}{6b} + a^2A\log(x)$$

output
```
a^2*B*x+a*A*b*x^2+1/3*a*(2*B*b+D*a)*x^3+1/4*A*b^2*x^4+1/5*b*(B*b+2*D*a)*x^
5+1/7*b^2*D*x^7+1/6*C*(b*x^2+a)^3/b+a^2*A*ln(x)
```

3.74.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{420}x\left(70a^2(6B+x(3C+2Dx)) + 14abx(30A+x(20B+3x(5C+4Dx))) + b^2x^3(105A+2x(42B+5x(7C+6Dx)))\right) + a^2A\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(x*(70*a^2*(6*B + x*(3*C + 2*D*x)) + 14*a*b*x*(30*A + x*(20*B + 3*x*(5*C +
 4*D*x))) + b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x)))))/420 + a^2*A
*Log[x]
```

3.74.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} \, dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{(bx^2+a)^2 (Dx^3+Bx+A)}{x}dx + \frac{C(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2333}$$

$$\int \left(b^2Dx^6+b(bB+2aD)x^4+Ab^2x^3+a(2bB+aD)x^2+2aAbx+a^2B+\frac{a^2A}{x}\right)dx + \frac{C(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$a^2A\log(x)+a^2Bx+aAbx^2+\frac{1}{5}bx^5(2aD+bB)+\frac{1}{3}ax^3(aD+2bB)+\frac{C(a+bx^2)^3}{6b}+\frac{1}{4}Ab^2x^4+\frac{1}{7}b^2Dx^7$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a^2*B*x + a*A*b*x^2 + (a*(2*b*B + a*D)*x^3)/3 + (A*b^2*x^4)/4 + (b*(b*B +
2*a*D)*x^5)/5 + (b^2*D*x^7)/7 + (C*(a + b*x^2)^3)/(6*b) + a^2*A*Log[x]
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.74.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

method	result
norman	$\left(\frac{1}{5}B\,b^{2}+\frac{2}{5}Dab\right)x^{5}+\left(\frac{1}{4}b^{2}A+\frac{1}{2}Cab\right)x^{4}+\left(abA+\frac{1}{2}C\,a^{2}\right)x^{2}+\left(\frac{2}{3}abB+\frac{1}{3}Da^{2}\right)x^{3}+a^{2}Bx$
default	$\frac{b^{2}Dx^{7}}{7}+\frac{C\,b^{2}x^{6}}{6}+\frac{b^{2}B\,x^{5}}{5}+\frac{2Dab\,x^{5}}{5}+\frac{A\,b^{2}x^{4}}{4}+\frac{Cab\,x^{4}}{2}+\frac{2Bab\,x^{3}}{3}+\frac{Da^{2}x^{3}}{3}+aAb\,x^{2}+\frac{C\,a^{2}x^{2}}{2}+a^{2}B$
parallelrisch	$\frac{b^{2}Dx^{7}}{7}+\frac{C\,b^{2}x^{6}}{6}+\frac{b^{2}B\,x^{5}}{5}+\frac{2Dab\,x^{5}}{5}+\frac{A\,b^{2}x^{4}}{4}+\frac{Cab\,x^{4}}{2}+\frac{2Bab\,x^{3}}{3}+\frac{Da^{2}x^{3}}{3}+aAb\,x^{2}+\frac{C\,a^{2}x^{2}}{2}+a^{2}B$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/5*B*b^2+2/5*D*a*b)*x^5+(1/4*b^2*A+1/2*C*a*b)*x^4+(a*b*A+1/2*C*a^2)*x^2+
(2/3*a*b*B+1/3*D*a^2)*x^3+a^2*B*x+1/6*C*b^2*x^6+1/7*b^2*D*x^7+a^2*A*ln(x)
```

3.74.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} \, dx = \frac{1}{7} Db^2x^7 + \frac{1}{6} Cb^2x^6 + \frac{1}{5} (2\,Dab + Bb^2)x^5 + \frac{1}{4} (2\,Cab + Ab^2)x^4 + Ba^2x + \frac{1}{3} (Da^2 + 2\,Bab)x^3 + Aa^2 \log(x) + \frac{1}{2} (Ca^2 + 2\,Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
 A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*
a^2 + 2*A*a*b)*x^2
```

3.74.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} \, dx = Aa^2 \log(x) + Ba^2x + \frac{Cb^2x^6}{6} + \frac{Db^2x^7}{7} + x^5\left(\frac{Bb^2}{5} + \frac{2Dab}{5}\right) + x^4\left(\frac{Ab^2}{4} + \frac{Cab}{2}\right) + x^3 \cdot \left(\frac{2Bab}{3} + \frac{Da^2}{3}\right) + x^2\left(Aab + \frac{Ca^2}{2}\right)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a**2*log(x) + B*a**2*x + C*b**2*x**6/6 + D*b**2*x**7/7 + x**5*(B*b**2/5
+ 2*D*a*b/5) + x**4*(A*b**2/4 + C*a*b/2) + x**3*(2*B*a*b/3 + D*a**2/3) + x
**2*(A*a*b + C*a**2/2)
```

3.74.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{1}{5}(2Dab+Bb^2)x^5 + \frac{1}{4}(2Cab+Ab^2)x^4 + Ba^2x + \frac{1}{3}(Da^2+2Bab)x^3 + Aa^2\log(x) + \frac{1}{2}(Ca^2+2Aab)x^2$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 1/5*(2*D*a*b + B*b^2)*x^5 + 1/4*(2*C*a*b +
 A*b^2)*x^4 + B*a^2*x + 1/3*(D*a^2 + 2*B*a*b)*x^3 + A*a^2*log(x) + 1/2*(C*
a^2 + 2*A*a*b)*x^2
```

3.74.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{7}Db^2x^7 + \frac{1}{6}Cb^2x^6 + \frac{2}{5}Dabx^5 + \frac{1}{5}Bb^2x^5 + \frac{1}{2}Cabx^4 + \frac{1}{4}Ab^2x^4 + \frac{1}{3}Da^2x^3 + \frac{2}{3}Babx^3 + \frac{1}{2}Ca^2x^2 + Aabx^2 + Ba^2x + Aa^2\log(|x|)$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/7*D*b^2*x^7 + 1/6*C*b^2*x^6 + 2/5*D*a*b*x^5 + 1/5*B*b^2*x^5 + 1/2*C*a*b*
x^4 + 1/4*A*b^2*x^4 + 1/3*D*a^2*x^3 + 2/3*B*a*b*x^3 + 1/2*C*a^2*x^2 + A*a*
b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

3.74.9 Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x} \, dx = \frac{A (4 a^2 \ln(x) + b^2 x^4 + 4 a b x^2)}{4} + \frac{B x (15 a^2 + 10 a b x^2 + 3 b^2 x^4)}{15} + \frac{a^2 x^3 D}{3} + \frac{b^2 x^7 D}{7} + \frac{C x^2 (3 a^2 + 3 a b x^2 + b^2 x^4)}{6} + \frac{2 a b x^5 D}{5}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(A*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + (B*x*(15*a^2 + 3*b^2*x^4 + 10
*a*b*x^2))/15 + (a^2*x^3*D)/3 + (b^2*x^7*D)/7 + (C*x^2*(3*a^2 + b^2*x^4 +
3*a*b*x^2))/6 + (2*a*b*x^5*D)/5
```

3.75 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

3.75.1 Optimal result

Integrand size = 28, antiderivative size = 90

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}b(Ab+2aC)x^3+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5+\frac{D(a+bx^2)^3}{6b}+a^2B\log(x)$$

output
```
-a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*b*(A*b+2*C*a)*x^3+1/4*b^2*B*x^4+1/5
*b^2*C*x^5+1/6*D*(b*x^2+a)^3/b+a^2*B*ln(x)
```

3.75.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = a^2\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+\frac{1}{6}abx(12A+x(6B+x(4C+3Dx)))+\frac{1}{60}b^2x^3(20A+x(15B+2x(6C+5Dx)))+a^2B\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
a^2*(-(A/x) + C*x + (D*x^2)/2) + (a*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x)))
)/6 + (b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*B*Log[x]
```

3.75.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{(bx^2+a)^2 (Cx^2+Bx+A)}{x^2}dx+\frac{D(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2159}$$

$$\int \left(b^2Cx^4+b^2Bx^3+b(Ab+2aC)x^2+2abBx+a(2Ab+aC)+\frac{a^2B}{x}+\frac{a^2A}{x^2}\right)dx+\frac{D(a+bx^2)^3}{6b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{3}bx^3(2aC+Ab)+ax(aC+2Ab)+abBx^2+\frac{D(a+bx^2)^3}{6b}+\frac{1}{4}b^2Bx^4+\frac{1}{5}b^2Cx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + (b*(A*b + 2*a*C)*x^3)/3 + (
b^2*B*x^4)/4 + (b^2*C*x^5)/5 + (D*(a + b*x^2)^3)/(6*b) + a^2*B*Log[x]
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.75.4 Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

method	result
default	$\frac{b^2 D x^6}{6}+\frac{b^2 C x^5}{5}+\frac{b^2 B x^4}{4}+\frac{D a b x^4}{2}+\frac{A b^2 x^3}{3}+\frac{2 C a b x^3}{3}+B a b x^2+\frac{D a^2 x^2}{2}+2 a A b x+C a^2 x+a^2 B$
norman	$\frac{\left(\frac{1}{4} B b^2+\frac{1}{2} D a b\right) x^5+\left(\frac{1}{3} b^2 A+\frac{2}{3} C a b\right) x^4+\left(a b B+\frac{1}{2} D a^2\right) x^3+\left(2 a b A+C a^2\right) x^2-a^2 A+\frac{C b^2 x^6}{5}+\frac{b^2 D x^7}{6}}{x}+a^2 B \ln \left(x\right)$
parallelrisch	$\frac{10 b^2 D x^7+12 C b^2 x^6+15 b^2 B x^5+30 D a b x^5+20 A b^2 x^4+40 C a b x^4+60 B a b x^3+30 D a^2 x^3+120 a A b x^2+60 a^2 B \ln(x) x+60 C a^2 x^2}{60 x}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^2*D*x^6+1/5*b^2*C*x^5+1/4*b^2*B*x^4+1/2*D*a*b*x^4+1/3*A*b^2*x^3+2/3*
C*a*b*x^3+B*a*b*x^2+1/2*D*a^2*x^2+2*a*A*b*x+C*a^2*x+a^2*B*ln(x)-a^2*A/x
```

3.75.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx$$
$$= \frac{10\, Db^2x^7 + 12\, Cb^2x^6 + 15\, (2\, Dab + Bb^2)x^5 + 20\, (2\, Cab + Ab^2)x^4 + 60\, Ba^2x \log{(x)} + 30\, (Da^2 + 2\, Bab)x}{60\, x}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/60*(10*D*b^2*x^7 + 12*C*b^2*x^6 + 15*(2*D*a*b + B*b^2)*x^5 + 20*(2*C*a*b
 + A*b^2)*x^4 + 60*B*a^2*x*log(x) + 30*(D*a^2 + 2*B*a*b)*x^3 - 60*A*a^2 +
60*(C*a^2 + 2*A*a*b)*x^2)/x
```

3.75.6 Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = -\frac{Aa^2}{x} + Ba^2 \log{(x)} + \frac{Cb^2x^5}{5} + \frac{Db^2x^6}{6} + x^4 \left(\frac{Bb^2}{4} + \frac{Dab}{2}\right) + x^3 \left(\frac{Ab^2}{3} + \frac{2Cab}{3}\right) + x^2 \left(Bab + \frac{Da^2}{2}\right) + x(2Aab + Ca^2)$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**2/x + B*a**2*log(x) + C*b**2*x**5/5 + D*b**2*x**6/6 + x**4*(B*b**2/4
 + D*a*b/2) + x**3*(A*b**2/3 + 2*C*a*b/3) + x**2*(B*a*b + D*a**2/2) + x*(2
*A*a*b + C*a**2)
```

3.75.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6+\frac{1}{5}\,Cb^2x^5+\frac{1}{4}\,(2\,Dab+Bb^2)x^4+\frac{1}{3}\,(2\,Cab+Ab^2)x^3+Ba^2\log(x)+\frac{1}{2}\,(Da^2+2\,Bab)x^2-\frac{Aa^2}{x}+(Ca^2+2\,Aab)x$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/4*(2*D*a*b + B*b^2)*x^4 + 1/3*(2*C*a*b +
 A*b^2)*x^3 + B*a^2*log(x) + 1/2*(D*a^2 + 2*B*a*b)*x^2 - A*a^2/x + (C*a^2
+ 2*A*a*b)*x
```

3.75.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{6}\,Db^2x^6+\frac{1}{5}\,Cb^2x^5+\frac{1}{2}\,Dabx^4+\frac{1}{4}\,Bb^2x^4+\frac{2}{3}\,Cabx^3+\frac{1}{3}\,Ab^2x^3+\frac{1}{2}\,Da^2x^2+Babx^2+Ca^2x+2\,Aabx+Ba^2\log(|x|)-\frac{Aa^2}{x}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/6*D*b^2*x^6 + 1/5*C*b^2*x^5 + 1/2*D*a*b*x^4 + 1/4*B*b^2*x^4 + 2/3*C*a*b*
x^3 + 1/3*A*b^2*x^3 + 1/2*D*a^2*x^2 + B*a*b*x^2 + C*a^2*x + 2*A*a*b*x + B*
a^2*log(abs(x)) - A*a^2/x
```

3.75.9 Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{B\,(4\,a^2\,\ln(x)+b^2\,x^4+4\,a\,b\,x^2)}{4}+\frac{(b\,x^2+a)^3\,D}{6\,b} + \frac{C\,x\,(15\,a^2+10\,a\,b\,x^2+3\,b^2\,x^4)}{15} + \frac{A\,(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4)}{3\,x}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
(B*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + ((a + b*x^2)^3*D)/(6*b) + (C*
x*(15*a^2 + 3*b^2*x^4 + 10*a*b*x^2))/15 + (A*(b^2*x^4 - 3*a^2 + 6*a*b*x^2)
)/(3*x)
```

3.76 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

3.76.1 Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+a(2bB+aD)x+\frac{1}{2}b(Ab+2aC)x^2+\frac{1}{3}b(bB+2aD)x^3+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5+a(2Ab+aC)\log(x)$$

output
```
-1/2*a^2*A/x^2-a^2*B/x+a*(2*B*b+D*a)*x+1/2*b*(A*b+2*C*a)*x^2+1/3*b*(B*b+2*
D*a)*x^3+1/4*b^2*C*x^4+1/5*b^2*D*x^5+a*(2*A*b+C*a)*ln(x)
```

3.76.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = -\frac{a^2(A+2Bx-2Dx^3)}{2x^2}+\frac{1}{3}abx(6B+x(3C+2Dx))+\frac{1}{60}b^2x^2(30A+x(20B+3x(5C+4Dx)))+a(2Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^2*(A + 2*B*x - 2*D*x^3))/x^2 + (a*b*x*(6*B + x*(3*C + 2*D*x)))/3 +
 (b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/60 + a*(2*A*b + a*C)*Log[
x]
```

3.76.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+bx(2aC+Ab)+\frac{a(aC+2Ab)}{x}+bx^2(2aD+bB)+a(aD+2bB)+b^2Cx^3+b^2Dx^4\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{2}bx^2(2aC+Ab)+a\log(x)(aC+2Ab)+\frac{1}{3}bx^3(2aD+bB)+ax(aD+2bB)+\frac{1}{4}b^2Cx^4+\frac{1}{5}b^2Dx^5$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^2*A)/x^2 - (a^2*B)/x + a*(2*b*B + a*D)*x + (b*(A*b + 2*a*C)*x^2)/2
 + (b*(b*B + 2*a*D)*x^3)/3 + (b^2*C*x^4)/4 + (b^2*D*x^5)/5 + a*(2*A*b + a*
C)*Log[x]
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.76.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97

method	result
default	$\frac{b^2 D x^5}{5}+\frac{b^2 C x^4}{4}+\frac{b^2 B x^3}{3}+\frac{2 D a b x^3}{3}+\frac{A b^2 x^2}{2}+C a b x^2+2 B a b x+D a^2 x+a(2 A b+C a) \ln (x)-$
norman	$\frac{\left(\frac{1}{3} B b^2+\frac{2}{3} D a b\right) x^5+\left(\frac{1}{2} b^2 A+C a b\right) x^4+\left(2 a b B+D a^2\right) x^3-\frac{a^2 A}{2}+\frac{C b^2 x^6}{4}-a^2 B x+\frac{b^2 D x^7}{5}}{x^2}+\left(2 a b A+C a^2\right) \ln (x)$
parallelrisch	$\frac{12 b^2 D x^7+15 C b^2 x^6+20 b^2 B x^5+40 D a b x^5+30 A b^2 x^4+60 C a b x^4+120 A \ln (x) x^2 a b+120 B a b x^3+60 C \ln (x) x^2 a^2+60 D a^2 x^3-60}{60 x^2}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3+2/3*D*a*b*x^3+1/2*A*b^2*x^2+C*a*
b*x^2+2*B*a*b*x+D*a^2*x+a*(2*A*b+C*a)*ln(x)-a^2*B/x-1/2*a^2*A/x^2
```

3.76.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$=\frac{12\,Db^2x^7+15\,Cb^2x^6+20\,(2\,Dab+Bb^2)x^5+30\,(2\,Cab+Ab^2)x^4-60\,Ba^2x+60\,(Da^2+2\,Bab)x^3+60}{60\,x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/60*(12*D*b^2*x^7 + 15*C*b^2*x^6 + 20*(2*D*a*b + B*b^2)*x^5 + 30*(2*C*a*b
 + A*b^2)*x^4 - 60*B*a^2*x + 60*(D*a^2 + 2*B*a*b)*x^3 + 60*(C*a^2 + 2*A*a*
b)*x^2*log(x) - 30*A*a^2)/x^2
```

3.76.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cb^2x^4}{4} + \frac{Db^2x^5}{5} + a(2Ab+Ca)\log(x) + x^3\left(\frac{Bb^2}{3} + \frac{2Dab}{3}\right) + x^2\left(\frac{Ab^2}{2} + Cab\right) + x(2Bab+Da^2) + \frac{-Aa^2-2Ba^2x}{2x^2}$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b**2*x**4/4 + D*b**2*x**5/5 + a*(2*A*b + C*a)*log(x) + x**3*(B*b**2/3 +
2*D*a*b/3) + x**2*(A*b**2/2 + C*a*b) + x*(2*B*a*b + D*a**2) + (-A*a**2 - 2
*B*a**2*x)/(2*x**2)
```

3.76.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{1}{5}Db^2x^5 + \frac{1}{4}Cb^2x^4 + \frac{1}{3}(2Dab+Bb^2)x^3 + \frac{1}{2}(2Cab+Ab^2)x^2 + (Da^2+2Bab)x + (Ca^2+2Aab)\log(x) - \frac{2Ba^2x+Aa^2}{2x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 1/3*(2*D*a*b + B*b^2)*x^3 + 1/2*(2*C*a*b +
 A*b^2)*x^2 + (D*a^2 + 2*B*a*b)*x + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B*a^
2*x + A*a^2)/x^2
```

3.76.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{5} Db^2x^5 + \frac{1}{4} Cb^2x^4 + \frac{2}{3} Dabx^3 + \frac{1}{3} Bb^2x^3 + Cabx^2 + \frac{1}{2} Ab^2x^2 + Da^2x + 2 Babx + (Ca^2 + 2 Aab) \log(|x|) - \frac{2 Ba^2x + Aa^2}{2 x^2}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/5*D*b^2*x^5 + 1/4*C*b^2*x^4 + 2/3*D*a*b*x^3 + 1/3*B*b^2*x^3 + C*a*b*x^2
+ 1/2*A*b^2*x^2 + D*a^2*x + 2*B*a*b*x + (C*a^2 + 2*A*a*b)*log(abs(x)) - 1/
2*(2*B*a^2*x + A*a^2)/x^2
```

3.76.9 Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{C (4 a^2 \ln(x) + b^2 x^4 + 4 a b x^2)}{4} + a^2 x D + \frac{b^2 x^5 D}{5} + \frac{A (b^2 x^4 - a^2 + 4 a b x^2 \ln(x))}{2 x^2} + \frac{B (-3 a^2 + 6 a b x^2 + b^2 x^4)}{3 x} + \frac{2 a b x^3 D}{3}$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(C*(4*a^2*log(x) + b^2*x^4 + 4*a*b*x^2))/4 + a^2*x*D + (b^2*x^5*D)/5 + (A*
(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (B*(b^2*x^4 - 3*a^2 + 6*a*b*
x^2))/(3*x) + (2*a*b*x^3*D)/3
```

3.77 $\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

3.77.1 Optimal result

Integrand size = 28, antiderivative size = 98

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x} + b(Ab+2aC)x+\frac{1}{2}b(bB+2aD)x^2 + \frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4+a(2bB+aD)\log(x)$$

output
```
-1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+b*(A*b+2*C*a)*x+1/2*b*(B*b+2*
D*a)*x^2+1/3*b^2*C*x^3+1/4*b^2*D*x^4+a*(2*B*b+D*a)*ln(x)
```

3.77.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{2aAb}{x}+abx(2C+Dx) - \frac{a^2(2A+3x(B+2Cx))}{6x^3} + \frac{1}{12}b^2x\left(12A+x\left(6B+4Cx+3Dx^2\right)\right) + a(2bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
(-2*a*A*b)/x + a*b*x*(2*C + D*x) - (a^2*(2*A + 3*x*(B + 2*C*x)))/(6*x^3) +
 (b^2*x*(12*A + x*(6*B + 4*C*x + 3*D*x^2)))/12 + a*(2*b*B + a*D)*Log[x]
```

3.77.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4} dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(aC+2Ab)}{x^2}+b(2aC+Ab)+bx(2aD+bB)+\frac{a(aD+2bB)}{x}+b^2Cx^2+b^2Dx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+bx(2aC+Ab)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}bx^2(2aD+bB)+a\log(x)(aD+2bB)+\frac{1}{3}b^2Cx^3+\frac{1}{4}b^2Dx^4$$

input
```
Int[((a + b*x^2)^2*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + b*(A*b + 2*a*C)
*x + (b*(b*B + 2*a*D)*x^2)/2 + (b^2*C*x^3)/3 + (b^2*D*x^4)/4 + a*(2*b*B +
a*D)*Log[x]
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.77.4 Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.94

method	result
default	$\frac{b^2Dx^4}{4}+\frac{C\,b^2x^3}{3}+\frac{b^2B\,x^2}{2}+Dab\,x^2+A\,b^2x+2Cabx+a(2Bb+Da)\ln(x)-\frac{a^2A}{3x^3}-\frac{a(2Ab+Ca)}{x}$
norman	$\frac{(\frac{1}{2}B\,b^2+Dab)x^5+(b^2A+2Cab)x^4+(-2abA-C\,a^2)x^2-\frac{a^2A}{3}+\frac{C\,b^2x^6}{3}-\frac{a^2Bx}{2}+\frac{b^2Dx^7}{4}}{x^3}+(2abB+Da^2)\ln(x)$
parallelrisch	$\frac{3b^2Dx^7+4C\,b^2x^6+6b^2B\,x^5+12Dab\,x^5+12A\,b^2x^4+24B\ln(x)x^3ab+24Cab\,x^4+12D\ln(x)x^3a^2-24aAb\,x^2-12C\,a^2x^2-6a^2Bx}{12x^3}$

input
```
int((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^2*D*x^4+1/3*C*b^2*x^3+1/2*b^2*B*x^2+D*a*b*x^2+A*b^2*x+2*C*a*b*x+a*(2
*B*b+D*a)*ln(x)-1/3*a^2*A/x^3-a*(2*A*b+C*a)/x-1/2*a^2*B/x^2
```

3.77.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int\frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$=\frac{3\,Db^2x^7+4\,Cb^2x^6+6\,(2\,Dab+Bb^2)x^5+12\,(2\,Cab+Ab^2)x^4+12\,(Da^2+2\,Bab)x^3\log(x)-6\,Ba^2x-4}{12\,x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/12*(3*D*b^2*x^7 + 4*C*b^2*x^6 + 6*(2*D*a*b + B*b^2)*x^5 + 12*(2*C*a*b +
A*b^2)*x^4 + 12*(D*a^2 + 2*B*a*b)*x^3*log(x) - 6*B*a^2*x - 4*A*a^2 - 12*(C
*a^2 + 2*A*a*b)*x^2)/x^3
```

3.77.6 Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{Cb^2x^3}{3}+\frac{Db^2x^4}{4}+a(2Bb+Da)\log(x) + x^2\left(\frac{Bb^2}{2}+Dab\right)+x\left(Ab^2+2Cab\right) + \frac{-2Aa^2-3Ba^2x+x^2(-12Aab-6Ca^2)}{6x^3}$$

input
```
integrate((b*x**2+a)**2*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b**2*x**3/3 + D*b**2*x**4/4 + a*(2*B*b + D*a)*log(x) + x**2*(B*b**2/2 +
D*a*b) + x*(A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2*(-12*A*a*b
- 6*C*a**2))/(6*x**3)
```

3.77.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{4}Db^2x^4+\frac{1}{3}Cb^2x^3+\frac{1}{2}\left(2Dab+Bb^2\right)x^2 + \left(2Cab+Ab^2\right)x+\left(Da^2+2Bab\right)\log(x) - \frac{3Ba^2x+2Aa^2+6(Ca^2+2Aab)x^2}{6x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + 1/2*(2*D*a*b + B*b^2)*x^2 + (2*C*a*b + A*b
^2)*x + (D*a^2 + 2*B*a*b)*log(x) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2 + 2
*A*a*b)*x^2)/x^3
```

3.77.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{4}Db^2x^4 + \frac{1}{3}Cb^2x^3 + Dabx^2 + \frac{1}{2}Bb^2x^2 + 2\,Cabx + Ab^2x + \left(Da^2 + 2\,Bab\right)\log\left(|x|\right) - \frac{3\,Ba^2x + 2\,Aa^2 + 6\,(Ca^2 + 2\,Aab)x^2}{6\,x^3}$$

input
```
integrate((b*x^2+a)^2*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/4*D*b^2*x^4 + 1/3*C*b^2*x^3 + D*a*b*x^2 + 1/2*B*b^2*x^2 + 2*C*a*b*x + A*
b^2*x + (D*a^2 + 2*B*a*b)*log(abs(x)) - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^
2 + 2*A*a*b)*x^2)/x^3
```

3.77.9 Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)^2 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{b^2\,x^4\,D}{4} + \frac{a^2\,\ln\left(x^2\right)\,D}{2} - \frac{A\,\left(a^2+6\,a\,b\,x^2-3\,b^2\,x^4\right)}{3\,x^3} + \frac{B\,\left(b^2\,x^4-a^2+4\,a\,b\,x^2\,\ln\left(x\right)\right)}{2\,x^2} + \frac{C\,\left(-3\,a^2+6\,a\,b\,x^2+b^2\,x^4\right)}{3\,x} + a\,b\,x^2\,D$$

input
```
int(((a + b*x^2)^2*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(b^2*x^4*D)/4 + (a^2*log(x^2)*D)/2 - (A*(a^2 - 3*b^2*x^4 + 6*a*b*x^2))/(3*
x^3) + (B*(b^2*x^4 - a^2 + 4*a*b*x^2*log(x)))/(2*x^2) + (C*(b^2*x^4 - 3*a^
2 + 6*a*b*x^2))/(3*x) + a*b*x^2*D
```

3.78 $\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx$

3.78.1 Optimal result

Integrand size = 28, antiderivative size = 149

$$\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6 +\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC)x^8 +\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10} +\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

output
```
1/4*a^3*A*x^4+1/5*a^3*B*x^5+1/6*a^2*(3*A*b+C*a)*x^6+1/7*a^2*(3*B*b+D*a)*x^
7+3/8*a*b*(A*b+C*a)*x^8+1/3*a*b*(B*b+D*a)*x^9+1/10*b^2*(A*b+3*C*a)*x^10+1/
11*b^2*(B*b+3*D*a)*x^11+1/12*b^3*C*x^12+1/13*b^3*D*x^13
```

3.78.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00

$$\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{4}a^3Ax^4+\frac{1}{5}a^3Bx^5+\frac{1}{6}a^2(3Ab+aC)x^6 +\frac{1}{7}a^2(3bB+aD)x^7+\frac{3}{8}ab(Ab+aC)x^8 +\frac{1}{3}ab(bB+aD)x^9+\frac{1}{10}b^2(Ab+3aC)x^{10} +\frac{1}{11}b^2(bB+3aD)x^{11}+\frac{1}{12}b^3Cx^{12}+\frac{1}{13}b^3Dx^{13}$$

input
```
Integrate[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B +
a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(
A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b
^3*D*x^13)/13
```

3.78.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^3 A x^3+a^3 B x^4+a^2 x^5 (aC+3Ab)+a^2 x^6 (aD+3bB)+b^2 x^9 (3aC+Ab)+3abx^7 (aC+Ab)+b^2 x^{10} (3aD+b$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}a^3 A x^4+\frac{1}{5}a^3 B x^5+\frac{1}{6}a^2 x^6 (aC+3Ab)+\frac{1}{7}a^2 x^7 (aD+3bB)+\frac{1}{10}b^2 x^{10} (3aC+Ab)+\frac{3}{8}abx^8 (aC+Ab)+\frac{1}{11}b^2 x^{11} (3aD+bB)+\frac{1}{3}abx^9 (aD+bB)+\frac{1}{12}b^3 C x^{12}+\frac{1}{13}b^3 D x^{13}$$

input
```
Int[x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^4)/4 + (a^3*B*x^5)/5 + (a^2*(3*A*b + a*C)*x^6)/6 + (a^2*(3*b*B +
a*D)*x^7)/7 + (3*a*b*(A*b + a*C)*x^8)/8 + (a*b*(b*B + a*D)*x^9)/3 + (b^2*(
A*b + 3*a*C)*x^10)/10 + (b^2*(b*B + 3*a*D)*x^11)/11 + (b^3*C*x^12)/12 + (b
^3*D*x^13)/13
```

3.78.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.78.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^3 D x^{13}}{13}+\frac{b^3 C x^{12}}{12}+\left(\frac{1}{11} B b^3+\frac{3}{11} a b^2 D\right) x^{11}+\left(\frac{1}{10} b^3 A+\frac{3}{10} C b^2 a\right) x^{10}+\left(\frac{1}{3} a b^2 B+\frac{1}{3} D a^2 b\right) x^9$ -
default	$\frac{b^3 D x^{13}}{13}+\frac{b^3 C x^{12}}{12}+\frac{(B b^3+3 a b^2 D) x^{11}}{11}+\frac{(b^3 A+3 C b^2 a) x^{10}}{10}+\frac{(3 a b^2 B+3 D a^2 b) x^9}{9}+\frac{(3 a b^2 A+3 C a^2 b) x^8}{8}+$ (3a²
gosper	$\frac{1}{13} b^3 D x^{13}+\frac{1}{12} b^3 C x^{12}+\frac{1}{11} x^{11} B b^3+\frac{3}{11} x^{11} a b^2 D+\frac{1}{10} x^{10} b^3 A+\frac{3}{10} x^{10} C b^2 a+\frac{1}{3} x^9 a b^2 B+\frac{1}{3} x$
parallelrisch	$\frac{1}{13} b^3 D x^{13}+\frac{1}{12} b^3 C x^{12}+\frac{1}{11} x^{11} B b^3+\frac{3}{11} x^{11} a b^2 D+\frac{1}{10} x^{10} b^3 A+\frac{3}{10} x^{10} C b^2 a+\frac{1}{3} x^9 a b^2 B+\frac{1}{3} x$

input
```
int(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/13*b^3*D*x^13+1/12*b^3*C*x^12+(1/11*B*b^3+3/11*a*b^2*D)*x^11+(1/10*b^3*A
+3/10*C*b^2*a)*x^10+(1/3*a*b^2*B+1/3*D*a^2*b)*x^9+(3/8*a*b^2*A+3/8*C*a^2*b
)*x^8+(3/7*a^2*b*B+1/7*D*a^3)*x^7+(1/2*a^2*b*A+1/6*C*a^3)*x^6+1/5*a^3*B*x^
5+1/4*a^3*A*x^4
```

3.78.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^3 (a+bx^2)^3 \left(A+Bx+Cx^2+Dx^3\right) dx = \frac{1}{13} Db^3x^{13} + \frac{1}{12} Cb^3x^{12} + \frac{1}{11} \left(3 Dab^2 + Bb^3\right)x^{11} + \frac{1}{10} \left(3 Cab^2 + Ab^3\right)x^{10} + \frac{1}{3} \left(Da^2b + Bab^2\right)x^9 + \frac{1}{5} Ba^3x^5 + \frac{3}{8} \left(Ca^2b + Aab^2\right)x^8 + \frac{1}{4} Aa^3x^4 + \frac{1}{7} \left(Da^3 + 3 Ba^2b\right)x^7 + \frac{1}{6} \left(Ca^3 + 3 Aa^2b\right)x^6$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(
3*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/
8*(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 +
1/6*(C*a^3 + 3*A*a^2*b)*x^6
```

3.78.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{Aa^3x^4}{4}+\frac{Ba^3x^5}{5}+\frac{Cb^3x^{12}}{12}+\frac{Db^3x^{13}}{13} + x^{11}\left(\frac{Bb^3}{11}+\frac{3Dab^2}{11}\right)+x^{10}\left(\frac{Ab^3}{10}+\frac{3Cab^2}{10}\right) + x^9\left(\frac{Bab^2}{3}+\frac{Da^2b}{3}\right)+x^8\cdot\left(\frac{3Aab^2}{8}+\frac{3Ca^2b}{8}\right) + x^7\cdot\left(\frac{3Ba^2b}{7}+\frac{Da^3}{7}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ca^3}{6}\right)$$

input
```
integrate(x**3*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**4/4 + B*a**3*x**5/5 + C*b**3*x**12/12 + D*b**3*x**13/13 + x**11*
(B*b**3/11 + 3*D*a*b**2/11) + x**10*(A*b**3/10 + 3*C*a*b**2/10) + x**9*(B*
a*b**2/3 + D*a**2*b/3) + x**8*(3*A*a*b**2/8 + 3*C*a**2*b/8) + x**7*(3*B*a*
*2*b/7 + D*a**3/7) + x**6*(A*a**2*b/2 + C*a**3/6)
```

3.78.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^3(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{13}\,Db^3x^{13}+\frac{1}{12}\,Cb^3x^{12}+\frac{1}{11}\,(3\,Dab^2+Bb^3)x^{11} + \frac{1}{10}\,(3\,Cab^2+Ab^3)x^{10}+\frac{1}{3}\,(Da^2b+Bab^2)x^9 + \frac{1}{5}\,Ba^3x^5+\frac{3}{8}\,(Ca^2b+Aab^2)x^8+\frac{1}{4}\,Aa^3x^4 + \frac{1}{7}\,(Da^3+3\,Ba^2b)x^7+\frac{1}{6}\,(Ca^3+3\,Aa^2b)x^6$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 1/11*(3*D*a*b^2 + B*b^3)*x^11 + 1/10*(
3*C*a*b^2 + A*b^3)*x^10 + 1/3*(D*a^2*b + B*a*b^2)*x^9 + 1/5*B*a^3*x^5 + 3/
8*(C*a^2*b + A*a*b^2)*x^8 + 1/4*A*a^3*x^4 + 1/7*(D*a^3 + 3*B*a^2*b)*x^7 +
1/6*(C*a^3 + 3*A*a^2*b)*x^6
```

3.78.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{13}Db^3x^{13}+\frac{1}{12}Cb^3x^{12}+\frac{3}{11}Dab^2x^{11}+\frac{1}{11}Bb^3x^{11}+\frac{3}{10}Cab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{1}{3}Da^2bx^9+\frac{1}{3}Bab^2x^9+\frac{3}{8}Ca^2bx^8+\frac{3}{8}Aab^2x^8+\frac{1}{7}Da^3x^7+\frac{3}{7}Ba^2bx^7+\frac{1}{6}Ca^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{5}Ba^3x^5+\frac{1}{4}Aa^3x^4$$

input
```
integrate(x^3*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/13*D*b^3*x^13 + 1/12*C*b^3*x^12 + 3/11*D*a*b^2*x^11 + 1/11*B*b^3*x^11 +
3/10*C*a*b^2*x^10 + 1/10*A*b^3*x^10 + 1/3*D*a^2*b*x^9 + 1/3*B*a*b^2*x^9 +
3/8*C*a^2*b*x^8 + 3/8*A*a*b^2*x^8 + 1/7*D*a^3*x^7 + 3/7*B*a^2*b*x^7 + 1/6*
C*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/5*B*a^3*x^5 + 1/4*A*a^3*x^4
```

3.78.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^3(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{A\,a^3\,x^4}{4}+\frac{B\,a^3\,x^5}{5}+\frac{A\,b^3\,x^{10}}{10}+\frac{C\,a^3\,x^6}{6}+\frac{B\,b^3\,x^{11}}{11}+\frac{C\,b^3\,x^{12}}{12}+\frac{a^3\,x^7\,D}{7}+\frac{b^3\,x^{13}\,D}{13}+\frac{a^2\,b\,x^9\,D}{3}+\frac{3\,a\,b^2\,x^{11}\,D}{11}+\frac{A\,a^2\,b\,x^6}{2}+\frac{3\,A\,a\,b^2\,x^8}{8}+\frac{3\,B\,a^2\,b\,x^7}{7}+\frac{B\,a\,b^2\,x^9}{3}+\frac{3\,C\,a^2\,b\,x^8}{8}+\frac{3\,C\,a\,b^2\,x^{10}}{10}$$

input
```
int(x^3*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^4)/4 + (B*a^3*x^5)/5 + (A*b^3*x^10)/10 + (C*a^3*x^6)/6 + (B*b^3*x
^11)/11 + (C*b^3*x^12)/12 + (a^3*x^7*D)/7 + (b^3*x^13*D)/13 + (a^2*b*x^9*D
)/3 + (3*a*b^2*x^11*D)/11 + (A*a^2*b*x^6)/2 + (3*A*a*b^2*x^8)/8 + (3*B*a^2
*b*x^7)/7 + (B*a*b^2*x^9)/3 + (3*C*a^2*b*x^8)/8 + (3*C*a*b^2*x^10)/10
```

3.79 $\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.79.1 Optimal result

Integrand size = 28, antiderivative size = 149

$$\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2(3Ab+aC)x^5+\frac{1}{6}a^2(3bB+aD)x^6+\frac{3}{7}ab(Ab+aC)x^7+\frac{3}{8}ab(bB+aD)x^8+\frac{1}{9}b^2(Ab+3aC)x^9+\frac{1}{10}b^2(bB+3aD)x^{10}+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

output
```
1/3*a^3*A*x^3+1/4*a^3*B*x^4+1/5*a^2*(3*A*b+C*a)*x^5+1/6*a^2*(3*B*b+D*a)*x^
6+3/7*a*b*(A*b+C*a)*x^7+3/8*a*b*(B*b+D*a)*x^8+1/9*b^2*(A*b+3*C*a)*x^9+1/10
*b^2*(B*b+3*D*a)*x^10+1/11*b^3*C*x^11+1/12*b^3*D*x^12
```

3.79.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.84

$$\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$
$$=\frac{14b^3x^9(220A+3x(66B+60Cx+55Dx^2))+462a^3x^3(20A+x(15B+2x(6C+5Dx)))+99a^2bx^5(168A}{27720}$$

input
```
Integrate[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(14*b^3*x^9*(220*A + 3*x*(66*B + 60*C*x + 55*D*x^2)) + 462*a^3*x^3*(20*A +
 x*(15*B + 2*x*(6*C + 5*D*x))) + 99*a^2*b*x^5*(168*A + 5*x*(28*B + 3*x*(8*
C + 7*D*x))) + 33*a*b^2*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/277
20
```

3.79.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(a^3Ax^2+a^3Bx^3+a^2x^4(aC+3Ab)+a^2x^5(aD+3bB)+b^2x^8(3aC+Ab)+3abx^6(aC+Ab)+b^2x^9(3aD+b\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}a^3Ax^3+\frac{1}{4}a^3Bx^4+\frac{1}{5}a^2x^5(aC+3Ab)+\frac{1}{6}a^2x^6(aD+3bB)+\frac{1}{9}b^2x^9(3aC+Ab)+\frac{3}{7}abx^7(aC+Ab)+\frac{1}{10}b^2x^{10}(3aD+bB)+\frac{3}{8}abx^8(aD+bB)+\frac{1}{11}b^3Cx^{11}+\frac{1}{12}b^3Dx^{12}$$

input
```
Int[x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*A*x^3)/3 + (a^3*B*x^4)/4 + (a^2*(3*A*b + a*C)*x^5)/5 + (a^2*(3*b*B +
a*D)*x^6)/6 + (3*a*b*(A*b + a*C)*x^7)/7 + (3*a*b*(b*B + a*D)*x^8)/8 + (b^2
*(A*b + 3*a*C)*x^9)/9 + (b^2*(b*B + 3*a*D)*x^10)/10 + (b^3*C*x^11)/11 + (b
^3*D*x^12)/12
```

3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.79.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.99

method	result
norman	$\frac{b^3 D x^{12}}{12}+\frac{b^3 C x^{11}}{11}+\left(\frac{1}{10}B b^3+\frac{3}{10}a b^2 D\right)x^{10}+\left(\frac{1}{9}b^3 A+\frac{1}{3}C b^2 a\right)x^9+\left(\frac{3}{8}a b^2 B+\frac{3}{8}D a^2 b\right)x^8+($
default	$\frac{b^3 D x^{12}}{12}+\frac{b^3 C x^{11}}{11}+\frac{(B b^3+3a b^2 D)x^{10}}{10}+\frac{(b^3 A+3C b^2 a)x^9}{9}+\frac{(3a b^2 B+3D a^2 b)x^8}{8}+\frac{(3a b^2 A+3C a^2 b)x^7}{7}+\frac{(3a^2 b$
gosper	$\frac{1}{12}b^3 D x^{12}+\frac{1}{11}b^3 C x^{11}+\frac{1}{10}x^{10}B b^3+\frac{3}{10}x^{10}a b^2 D+\frac{1}{9}x^9 b^3 A+\frac{1}{3}x^9 C b^2 a+\frac{3}{8}x^8 a b^2 B+\frac{3}{8}x^8 D$
parallelrisch	$\frac{1}{12}b^3 D x^{12}+\frac{1}{11}b^3 C x^{11}+\frac{1}{10}x^{10}B b^3+\frac{3}{10}x^{10}a b^2 D+\frac{1}{9}x^9 b^3 A+\frac{1}{3}x^9 C b^2 a+\frac{3}{8}x^8 a b^2 B+\frac{3}{8}x^8 D$

input
```
int(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/12*b^3*D*x^12+1/11*b^3*C*x^11+(1/10*B*b^3+3/10*a*b^2*D)*x^10+(1/9*b^3*A+
1/3*C*b^2*a)*x^9+(3/8*a*b^2*B+3/8*D*a^2*b)*x^8+(3/7*a*b^2*A+3/7*C*a^2*b)*x
^7+(1/2*a^2*b*B+1/6*D*a^3)*x^6+(3/5*a^2*b*A+1/5*C*a^3)*x^5+1/4*a^3*B*x^4+1
/3*a^3*A*x^3
```

3.79.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\begin{aligned}\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx &= \frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}\left(3\,Dab^2+Bb^3\right)x^{10}\\ &+\frac{1}{9}\left(3\,Cab^2+Ab^3\right)x^9+\frac{3}{8}\left(Da^2b+Bab^2\right)x^8\\ &+\frac{1}{4}Ba^3x^4+\frac{3}{7}\left(Ca^2b+Aab^2\right)x^7+\frac{1}{3}Aa^3x^3\\ &+\frac{1}{6}\left(Da^3+3\,Ba^2b\right)x^6+\frac{1}{5}\left(Ca^3+3\,Aa^2b\right)x^5\end{aligned}$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3
*C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*
(C*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/
5*(C*a^3 + 3*A*a^2*b)*x^5
```

3.79.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.11

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^3x^3}{3}+\frac{Ba^3x^4}{4}+\frac{Cb^3x^{11}}{11}+\frac{Db^3x^{12}}{12}+x^{10}\left(\frac{Bb^3}{10}+\frac{3Dab^2}{10}\right)+x^9\left(\frac{Ab^3}{9}+\frac{Cab^2}{3}\right)+x^8\cdot\left(\frac{3Bab^2}{8}+\frac{3Da^2b}{8}\right)+x^7\cdot\left(\frac{3Aab^2}{7}+\frac{3Ca^2b}{7}\right)+x^6\left(\frac{Ba^2b}{2}+\frac{Da^3}{6}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ca^3}{5}\right)$$

input
```
integrate(x**2*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**3/3 + B*a**3*x**4/4 + C*b**3*x**11/11 + D*b**3*x**12/12 + x**10*
(B*b**3/10 + 3*D*a*b**2/10) + x**9*(A*b**3/9 + C*a*b**2/3) + x**8*(3*B*a*b
**2/8 + 3*D*a**2*b/8) + x**7*(3*A*a*b**2/7 + 3*C*a**2*b/7) + x**6*(B*a**2*
b/2 + D*a**3/6) + x**5*(3*A*a**2*b/5 + C*a**3/5)
```

3.79.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{1}{10}(3Dab^2+Bb^3)x^{10}+\frac{1}{9}(3Cab^2+Ab^3)x^9+\frac{3}{8}(Da^2b+Bab^2)x^8+\frac{1}{4}Ba^3x^4+\frac{3}{7}(Ca^2b+Aab^2)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{6}(Da^3+3Ba^2b)x^6+\frac{1}{5}(Ca^3+3Aa^2b)x^5$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 1/10*(3*D*a*b^2 + B*b^3)*x^10 + 1/9*(3
*C*a*b^2 + A*b^3)*x^9 + 3/8*(D*a^2*b + B*a*b^2)*x^8 + 1/4*B*a^3*x^4 + 3/7*
(C*a^2*b + A*a*b^2)*x^7 + 1/3*A*a^3*x^3 + 1/6*(D*a^3 + 3*B*a^2*b)*x^6 + 1/
5*(C*a^3 + 3*A*a^2*b)*x^5
```

3.79.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{12}Db^3x^{12}+\frac{1}{11}Cb^3x^{11}+\frac{3}{10}Dab^2x^{10}+\frac{1}{10}Bb^3x^{10}+\frac{1}{3}Cab^2x^9+\frac{1}{9}Ab^3x^9+\frac{3}{8}Da^2bx^8+\frac{3}{8}Bab^2x^8+\frac{3}{7}Ca^2bx^7+\frac{3}{7}Aab^2x^7+\frac{1}{6}Da^3x^6+\frac{1}{2}Ba^2bx^6+\frac{1}{5}Ca^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{4}Ba^3x^4+\frac{1}{3}Aa^3x^3$$

input
```
integrate(x^2*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/12*D*b^3*x^12 + 1/11*C*b^3*x^11 + 3/10*D*a*b^2*x^10 + 1/10*B*b^3*x^10 +
1/3*C*a*b^2*x^9 + 1/9*A*b^3*x^9 + 3/8*D*a^2*b*x^8 + 3/8*B*a*b^2*x^8 + 3/7*
C*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 1/6*D*a^3*x^6 + 1/2*B*a^2*b*x^6 + 1/5*C*a^
3*x^5 + 3/5*A*a^2*b*x^5 + 1/4*B*a^3*x^4 + 1/3*A*a^3*x^3
```

3.79.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.03

$$\int x^2(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{A\,a^3\,x^3}{3}+\frac{B\,a^3\,x^4}{4}+\frac{A\,b^3\,x^9}{9}+\frac{C\,a^3\,x^5}{5}+\frac{B\,b^3\,x^{10}}{10}+\frac{C\,b^3\,x^{11}}{11}+\frac{a^3\,x^6\,D}{6}+\frac{b^3\,x^{12}\,D}{12}+\frac{3\,a^2\,b\,x^8\,D}{8}+\frac{3\,a\,b^2\,x^{10}\,D}{10}+\frac{3\,A\,a^2\,b\,x^5}{5}+\frac{3\,A\,a\,b^2\,x^7}{7}+\frac{B\,a^2\,b\,x^6}{2}+\frac{3\,B\,a\,b^2\,x^8}{8}+\frac{3\,C\,a^2\,b\,x^7}{7}+\frac{C\,a\,b^2\,x^9}{3}$$

input
```
int(x^2*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^3)/3 + (B*a^3*x^4)/4 + (A*b^3*x^9)/9 + (C*a^3*x^5)/5 + (B*b^3*x^1
0)/10 + (C*b^3*x^11)/11 + (a^3*x^6*D)/6 + (b^3*x^12*D)/12 + (3*a^2*b*x^8*D
)/8 + (3*a*b^2*x^10*D)/10 + (3*A*a^2*b*x^5)/5 + (3*A*a*b^2*x^7)/7 + (B*a^2
*b*x^6)/2 + (3*B*a*b^2*x^8)/8 + (3*C*a^2*b*x^7)/7 + (C*a*b^2*x^9)/3
```

3.79. $\int x^2(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$

3.80 $\int x(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx$

3.80.1 Optimal result

Integrand size = 26, antiderivative size = 138

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2(3bB+aD)x^5+\frac{1}{2}a^2bCx^6+\frac{3}{7}ab(bB+aD)x^7+\frac{3}{8}ab^2Cx^8+\frac{1}{9}b^2(bB+3aD)x^9+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}+\frac{A(a+bx^2)^4}{8b}$$

output
```
1/3*a^3*B*x^3+1/4*a^3*C*x^4+1/5*a^2*(3*B*b+D*a)*x^5+1/2*a^2*b*C*x^6+3/7*a*
b*(B*b+D*a)*x^7+3/8*a*b^2*C*x^8+1/9*b^2*(B*b+3*D*a)*x^9+1/10*b^3*C*x^10+1/
11*b^3*D*x^11+1/8*A*(b*x^2+a)^4/b
```

3.80.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.90

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{7b^3x^8(495A+4x(110B+99Cx+90Dx^2))+462a^3x^2(30A+x(20B+3x(5C+4Dx)))+198a^2bx^4(105}{27720}$$

input
```
Integrate[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(7*b^3*x^8*(495*A + 4*x*(110*B + 99*C*x + 90*D*x^2)) + 462*a^3*x^2*(30*A +
 x*(20*B + 3*x*(5*C + 4*D*x))) + 198*a^2*b*x^4*(105*A + 2*x*(42*B + 5*x*(7
*C + 6*D*x))) + 165*a*b^2*x^6*(84*A + x*(72*B + 7*x*(9*C + 8*D*x))))/27720
```

3.80.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx$$

$$\Big\downarrow \text{ 2017}$$

$$\int \left(bx^2+a\right)^3\left(x\left(Dx^3+Cx^2+Bx+A\right)-Ax\right)dx+\frac{A\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{ 2341}$$

$$\int \left(b^3Dx^{10}+b^3Cx^9+b^2(bB+3aD)x^8+3ab^2Cx^7+3ab(bB+aD)x^6+3a^2bCx^5+a^2(3bB+aD)x^4+a^3Cx^3+\right.$$
$$\frac{A\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}a^3Bx^3+\frac{1}{4}a^3Cx^4+\frac{1}{5}a^2x^5(aD+3bB)+\frac{1}{2}a^2bCx^6+\frac{A\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^2x^9(3aD+bB)+$$
$$\frac{3}{8}ab^2Cx^8+\frac{3}{7}abx^7(aD+bB)+\frac{1}{10}b^3Cx^{10}+\frac{1}{11}b^3Dx^{11}$$

input
```
Int[x*(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(a^3*B*x^3)/3 + (a^3*C*x^4)/4 + (a^2*(3*b*B + a*D)*x^5)/5 + (a^2*b*C*x^6)/
2 + (3*a*b*(b*B + a*D)*x^7)/7 + (3*a*b^2*C*x^8)/8 + (b^2*(b*B + 3*a*D)*x^9
)/9 + (b^3*C*x^10)/10 + (b^3*D*x^11)/11 + (A*(a + b*x^2)^4)/(8*b)
```

3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.80.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.07

method	result
norman	$\frac{b^3 D x^{11}}{11}+\frac{b^3 C x^{10}}{10}+\left(\frac{1}{9} B b^3+\frac{1}{3} a b^2 D\right) x^9+\left(\frac{1}{8} b^3 A+\frac{3}{8} C b^2 a\right) x^8+\left(\frac{3}{7} a b^2 B+\frac{3}{7} D a^2 b\right) x^7+\left(\frac{1}{2} a\right.$
default	$\frac{b^3 D x^{11}}{11}+\frac{b^3 C x^{10}}{10}+\frac{(B b^3+3 a b^2 D) x^9}{9}+\frac{(b^3 A+3 C b^2 a) x^8}{8}+\frac{(3 a b^2 B+3 D a^2 b) x^7}{7}+\frac{(3 a b^2 A+3 C a^2 b) x^6}{6}+\frac{(3 a^2 b B}{}$
gosper	$\frac{1}{11} b^3 D x^{11}+\frac{1}{10} b^3 C x^{10}+\frac{1}{9} b^3 B x^9+\frac{1}{3} x^9 a b^2 D+\frac{1}{8} x^8 b^3 A+\frac{3}{8} a b^2 C x^8+\frac{3}{7} x^7 a b^2 B+\frac{3}{7} x^7 D a^2 b$
parallelrisch	$\frac{1}{11} b^3 D x^{11}+\frac{1}{10} b^3 C x^{10}+\frac{1}{9} b^3 B x^9+\frac{1}{3} x^9 a b^2 D+\frac{1}{8} x^8 b^3 A+\frac{3}{8} a b^2 C x^8+\frac{3}{7} x^7 a b^2 B+\frac{3}{7} x^7 D a^2 b$

input
```
int(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/11*b^3*D*x^11+1/10*b^3*C*x^10+(1/9*B*b^3+1/3*a*b^2*D)*x^9+(1/8*b^3*A+3/8
*C*b^2*a)*x^8+(3/7*a*b^2*B+3/7*D*a^2*b)*x^7+(1/2*a*b^2*A+1/2*C*a^2*b)*x^6+
(3/5*a^2*b*B+1/5*D*a^3)*x^5+(3/4*a^2*b*A+1/4*C*a^3)*x^4+1/3*a^3*B*x^3+1/2*
a^3*A*x^2
```

3.80.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{9}(3Dab^2+Bb^3)x^9+\frac{1}{8}(3Cab^2+Ab^3)x^8+\frac{3}{7}(Da^2b+Bab^2)x^7+\frac{1}{3}Ba^3x^3+\frac{1}{2}(Ca^2b+Aab^2)x^6+\frac{1}{2}Aa^3x^2+\frac{1}{5}(Da^3+3Ba^2b)x^5+\frac{1}{4}(Ca^3+3Aa^2b)x^4$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C
*a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C
*a^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*
(C*a^3 + 3*A*a^2*b)*x^4
```

3.80.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.18

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{Aa^3x^2}{2}+\frac{Ba^3x^3}{3}+\frac{Cb^3x^{10}}{10}+\frac{Db^3x^{11}}{11}+x^9\left(\frac{Bb^3}{9}+\frac{Dab^2}{3}\right)+x^8\left(\frac{Ab^3}{8}+\frac{3Cab^2}{8}\right)+x^7\cdot\left(\frac{3Bab^2}{7}+\frac{3Da^2b}{7}\right)+x^6\left(\frac{Aab^2}{2}+\frac{Ca^2b}{2}\right)+x^5\cdot\left(\frac{3Ba^2b}{5}+\frac{Da^3}{5}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ca^3}{4}\right)$$

input
```
integrate(x*(b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x**2/2 + B*a**3*x**3/3 + C*b**3*x**10/10 + D*b**3*x**11/11 + x**9*(
B*b**3/9 + D*a*b**2/3) + x**8*(A*b**3/8 + 3*C*a*b**2/8) + x**7*(3*B*a*b**2
/7 + 3*D*a**2*b/7) + x**6*(A*a*b**2/2 + C*a**2*b/2) + x**5*(3*B*a**2*b/5 +
 D*a**3/5) + x**4*(3*A*a**2*b/4 + C*a**3/4)
```

3.80.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.05

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{9}(3\,Dab^2+Bb^3)x^9+\frac{1}{8}(3\,Cab^2+Ab^3)x^8+\frac{3}{7}(Da^2b+Bab^2)x^7+\frac{1}{3}Ba^3x^3+\frac{1}{2}(Ca^2b+Aab^2)x^6+\frac{1}{2}Aa^3x^2+\frac{1}{5}(Da^3+3\,Ba^2b)x^5+\frac{1}{4}(Ca^3+3\,Aa^2b)x^4$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/9*(3*D*a*b^2 + B*b^3)*x^9 + 1/8*(3*C
*a*b^2 + A*b^3)*x^8 + 3/7*(D*a^2*b + B*a*b^2)*x^7 + 1/3*B*a^3*x^3 + 1/2*(C
*a^2*b + A*a*b^2)*x^6 + 1/2*A*a^3*x^2 + 1/5*(D*a^3 + 3*B*a^2*b)*x^5 + 1/4*
(C*a^3 + 3*A*a^2*b)*x^4
```

3.80.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x(a+bx^2)^3(A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{11}Db^3x^{11}+\frac{1}{10}Cb^3x^{10}+\frac{1}{3}Dab^2x^9+\frac{1}{9}Bb^3x^9+\frac{3}{8}Cab^2x^8+\frac{1}{8}Ab^3x^8+\frac{3}{7}Da^2bx^7+\frac{3}{7}Bab^2x^7+\frac{1}{2}Ca^2bx^6+\frac{1}{2}Aab^2x^6+\frac{1}{5}Da^3x^5+\frac{3}{5}Ba^2bx^5+\frac{1}{4}Ca^3x^4+\frac{3}{4}Aa^2bx^4+\frac{1}{3}Ba^3x^3+\frac{1}{2}Aa^3x^2$$

input
```
integrate(x*(b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/11*D*b^3*x^11 + 1/10*C*b^3*x^10 + 1/3*D*a*b^2*x^9 + 1/9*B*b^3*x^9 + 3/8*
C*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3/7*D*a^2*b*x^7 + 3/7*B*a*b^2*x^7 + 1/2*C*a^
2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/5*D*a^3*x^5 + 3/5*B*a^2*b*x^5 + 1/4*C*a^3*x^
4 + 3/4*A*a^2*b*x^4 + 1/3*B*a^3*x^3 + 1/2*A*a^3*x^2
```

3.80.9 Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.11

$$\int x(a+bx^2)^3\left(A+Bx+Cx^2+Dx^3\right)\,dx=\frac{A\,a^3\,x^2}{2}+\frac{B\,a^3\,x^3}{3}+\frac{A\,b^3\,x^8}{8}+\frac{C\,a^3\,x^4}{4}+\frac{B\,b^3\,x^9}{9}+\frac{C\,b^3\,x^{10}}{10}+\frac{a^3\,x^5\,D}{5}+\frac{b^3\,x^{11}\,D}{11}+\frac{3\,a^2\,b\,x^7\,D}{7}+\frac{a\,b^2\,x^9\,D}{3}+\frac{3\,A\,a^2\,b\,x^4}{4}+\frac{A\,a\,b^2\,x^6}{2}+\frac{3\,B\,a^2\,b\,x^5}{5}+\frac{3\,B\,a\,b^2\,x^7}{7}+\frac{C\,a^2\,b\,x^6}{2}+\frac{3\,C\,a\,b^2\,x^8}{8}$$

input
```
int(x*(a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output
```
(A*a^3*x^2)/2 + (B*a^3*x^3)/3 + (A*b^3*x^8)/8 + (C*a^3*x^4)/4 + (B*b^3*x^9
)/9 + (C*b^3*x^10)/10 + (a^3*x^5*D)/5 + (b^3*x^11*D)/11 + (3*a^2*b*x^7*D)/
7 + (a*b^2*x^9*D)/3 + (3*A*a^2*b*x^4)/4 + (A*a*b^2*x^6)/2 + (3*B*a^2*b*x^5
)/5 + (3*B*a*b^2*x^7)/7 + (C*a^2*b*x^6)/2 + (3*C*a*b^2*x^8)/8
```

3.81 $\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx$

3.81.1 Optimal result

Integrand size = 25, antiderivative size = 133

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = a^3Ax+\frac{1}{3}a^2(3Ab+aC)x^3+\frac{1}{4}a^3Dx^4 +\frac{3}{5}ab(Ab+aC)x^5+\frac{1}{2}a^2bDx^6 +\frac{1}{7}b^2(Ab+3aC)x^7+\frac{3}{8}ab^2Dx^8 +\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}+\frac{B(a+bx^2)^4}{8b}$$

output
```
a^3*A*x+1/3*a^2*(3*A*b+C*a)*x^3+1/4*a^3*D*x^4+3/5*a*b*(A*b+C*a)*x^5+1/2*a^
2*b*D*x^6+1/7*b^2*(A*b+3*C*a)*x^7+3/8*a*b^2*D*x^8+1/9*b^3*C*x^9+1/10*b^3*D
*x^10+1/8*B*(b*x^2+a)^4/b
```

3.81.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.91

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\, dx = \frac{210a^3x(12A+x(6B+x(4C+3Dx)))+126a^2bx^3(20A+x(15B+2x(6C+5Dx)))+9ab^2x^5(168A+5\ldots}{2520}$$

input
```
Integrate[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
(210*a^3*x*(12*A + x*(6*B + x*(4*C + 3*D*x))) + 126*a^2*b*x^3*(20*A + x*(1
5*B + 2*x*(6*C + 5*D*x))) + 9*a*b^2*x^5*(168*A + 5*x*(28*B + 3*x*(8*C + 7*
D*x))) + b^3*x^7*(360*A + 7*x*(45*B + 4*x*(10*C + 9*D*x))))/2520
```

3.81.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {2017, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right) \, dx$$

$$\Big\downarrow \text{2017}$$

$$\int \left(bx^2+a\right)^3 \left(Dx^3+Cx^2+A\right) dx+\frac{B\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{2341}$$

$$\int \left(b^3Dx^9+b^3Cx^8+3ab^2Dx^7+b^2(Ab+3aC)x^6+3a^2bDx^5+3ab(Ab+aC)x^4+a^3Dx^3+a^2(3Ab+aC)x^2+a\right.$$
$$\frac{B\left(a+bx^2\right)^4}{8b}$$

$$\Big\downarrow \text{2009}$$

$$a^3Ax+\frac{1}{4}a^3Dx^4+\frac{1}{3}a^2x^3(aC+3Ab)+\frac{1}{2}a^2bDx^6+\frac{1}{7}b^2x^7(3aC+Ab)+\frac{3}{5}abx^5(aC+Ab)+$$
$$\frac{3}{8}ab^2Dx^8+\frac{B\left(a+bx^2\right)^4}{8b}+\frac{1}{9}b^3Cx^9+\frac{1}{10}b^3Dx^{10}$$

input
```
Int[(a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3),x]
```

output
```
a^3*A*x + (a^2*(3*A*b + a*C)*x^3)/3 + (a^3*D*x^4)/4 + (3*a*b*(A*b + a*C)*x
^5)/5 + (a^2*b*D*x^6)/2 + (b^2*(A*b + 3*a*C)*x^7)/7 + (3*a*b^2*D*x^8)/8 +
(b^3*C*x^9)/9 + (b^3*D*x^10)/10 + (B*(a + b*x^2)^4)/(8*b)
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2017
```
Int[(Px_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[Coeff[Px, x, n -
 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coeff[Px, x, n - 1]
*x^(n - 1))*(a + b*x^n)^p, x] /; FreeQ[{a, b}, x] && PolyQ[Px, x] && IGtQ[p
, 1] && IGtQ[n, 1] && NeQ[Coeff[Px, x, n - 1], 0] && NeQ[Px, Coeff[Px, x, n
 - 1]*x^(n - 1)] &&  !MatchQ[Px, (Qx_.)*((c_) + (d_.)*x^(m_))^(q_) /; FreeQ
[{c, d}, x] && PolyQ[Qx, x] && IGtQ[q, 1] && IGtQ[m, 1] && NeQ[Coeff[Qx*(a
+ b*x^n)^p, x, m - 1], 0] && GtQ[m*q, n*p]]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.81.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.08

method	result
norman	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\left(\frac{1}{8}Bb^3+\frac{3}{8}ab^2D\right)x^8+\left(\frac{1}{7}b^3A+\frac{3}{7}Cb^2a\right)x^7+\left(\frac{1}{2}ab^2B+\frac{1}{2}Da^2b\right)x^6+(\frac{3}{5}a$
default	$\frac{b^3Dx^{10}}{10}+\frac{b^3Cx^9}{9}+\frac{(Bb^3+3ab^2D)x^8}{8}+\frac{(b^3A+3Cb^2a)x^7}{7}+\frac{(3ab^2B+3Da^2b)x^6}{6}+\frac{(3ab^2A+3Ca^2b)x^5}{5}+\frac{(3a^2bB}{}$
gosper	$\frac{1}{10}b^3Dx^{10}+\frac{1}{9}b^3Cx^9+\frac{1}{8}b^3Bx^8+\frac{3}{8}ab^2Dx^8+\frac{1}{7}x^7b^3A+\frac{3}{7}x^7Cb^2a+\frac{1}{2}x^6ab^2B+\frac{1}{2}a^2bDx^6+$
parallelrisch	$\frac{1}{10}b^3Dx^{10}+\frac{1}{9}b^3Cx^9+\frac{1}{8}b^3Bx^8+\frac{3}{8}ab^2Dx^8+\frac{1}{7}x^7b^3A+\frac{3}{7}x^7Cb^2a+\frac{1}{2}x^6ab^2B+\frac{1}{2}a^2bDx^6+$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x,method=_RETURNVERBOSE)
```

output
```
1/10*b^3*D*x^10+1/9*b^3*C*x^9+(1/8*B*b^3+3/8*a*b^2*D)*x^8+(1/7*b^3*A+3/7*C
*b^2*a)*x^7+(1/2*a*b^2*B+1/2*D*a^2*b)*x^6+(3/5*a*b^2*A+3/5*C*a^2*b)*x^5+(3
/4*a^2*b*B+1/4*D*a^3)*x^4+(a^2*b*A+1/3*C*a^3)*x^3+1/2*a^3*B*x^2+a^3*A*x
```

3.81.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = \frac{1}{10}Db^3x^{10} + \frac{1}{9}Cb^3x^9 + \frac{1}{8}(3\,Dab^2+Bb^3)x^8 + \frac{1}{7}(3\,Cab^2+Ab^3)x^7 + \frac{1}{2}(Da^2b+Bab^2)x^6 + \frac{1}{2}Ba^3x^2 + \frac{3}{5}(Ca^2b+Aab^2)x^5 + Aa^3x + \frac{1}{4}(Da^3+3\,Ba^2b)x^4 + \frac{1}{3}(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="fricas")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

3.81.6 Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.19

$$\int (a+bx^2)^3 (A+Bx+Cx^2+Dx^3)\,dx = Aa^3x + \frac{Ba^3x^2}{2} + \frac{Cb^3x^9}{9} + \frac{Db^3x^{10}}{10} + x^8\left(\frac{Bb^3}{8}+\frac{3Dab^2}{8}\right) + x^7\left(\frac{Ab^3}{7}+\frac{3Cab^2}{7}\right) + x^6\left(\frac{Bab^2}{2}+\frac{Da^2b}{2}\right) + x^5\cdot\left(\frac{3Aab^2}{5}+\frac{3Ca^2b}{5}\right) + x^4\cdot\left(\frac{3Ba^2b}{4}+\frac{Da^3}{4}\right) + x^3\left(Aa^2b+\frac{Ca^3}{3}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A),x)
```

output
```
A*a**3*x + B*a**3*x**2/2 + C*b**3*x**9/9 + D*b**3*x**10/10 + x**8*(B*b**3/
8 + 3*D*a*b**2/8) + x**7*(A*b**3/7 + 3*C*a*b**2/7) + x**6*(B*a*b**2/2 + D*
a**2*b/2) + x**5*(3*A*a*b**2/5 + 3*C*a**2*b/5) + x**4*(3*B*a**2*b/4 + D*a*
*3/4) + x**3*(A*a**2*b + C*a**3/3)
```

3.81.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{1}{8}\,(3\,Dab^2+Bb^3)x^8+\frac{1}{7}\,(3\,Cab^2+Ab^3)x^7+\frac{1}{2}\,(Da^2b+Bab^2)x^6+\frac{1}{2}\,Ba^3x^2+\frac{3}{5}\,(Ca^2b+Aab^2)x^5+Aa^3x+\frac{1}{4}\,(Da^3+3\,Ba^2b)x^4+\frac{1}{3}\,(Ca^3+3\,Aa^2b)x^3$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="maxima")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 1/8*(3*D*a*b^2 + B*b^3)*x^8 + 1/7*(3*C*a
*b^2 + A*b^3)*x^7 + 1/2*(D*a^2*b + B*a*b^2)*x^6 + 1/2*B*a^3*x^2 + 3/5*(C*a
^2*b + A*a*b^2)*x^5 + A*a^3*x + 1/4*(D*a^3 + 3*B*a^2*b)*x^4 + 1/3*(C*a^3 +
 3*A*a^2*b)*x^3
```

3.81.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{1}{10}\,Db^3x^{10}+\frac{1}{9}\,Cb^3x^9+\frac{3}{8}\,Dab^2x^8+\frac{1}{8}\,Bb^3x^8+\frac{3}{7}\,Cab^2x^7+\frac{1}{7}\,Ab^3x^7+\frac{1}{2}\,Da^2bx^6+\frac{1}{2}\,Bab^2x^6+\frac{3}{5}\,Ca^2bx^5+\frac{3}{5}\,Aab^2x^5+\frac{1}{4}\,Da^3x^4+\frac{3}{4}\,Ba^2bx^4+\frac{1}{3}\,Ca^3x^3+Aa^2bx^3+\frac{1}{2}\,Ba^3x^2+Aa^3x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A),x, algorithm="giac")
```

output
```
1/10*D*b^3*x^10 + 1/9*C*b^3*x^9 + 3/8*D*a*b^2*x^8 + 1/8*B*b^3*x^8 + 3/7*C*
a*b^2*x^7 + 1/7*A*b^3*x^7 + 1/2*D*a^2*b*x^6 + 1/2*B*a*b^2*x^6 + 3/5*C*a^2*
b*x^5 + 3/5*A*a*b^2*x^5 + 1/4*D*a^3*x^4 + 3/4*B*a^2*b*x^4 + 1/3*C*a^3*x^3
+ A*a^2*b*x^3 + 1/2*B*a^3*x^2 + A*a^3*x
```

3.81.9 Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)^3 \left(A+Bx+Cx^2+Dx^3\right)\,dx = \frac{B\,a^3\,x^2}{2}+\frac{A\,b^3\,x^7}{7}+\frac{C\,a^3\,x^3}{3}+\frac{B\,b^3\,x^8}{8}+\frac{C\,b^3\,x^9}{9}+\frac{a^3\,x^4\,D}{4}+\frac{b^3\,x^{10}\,D}{10}+A\,a^3\,x+\frac{a^2\,b\,x^6\,D}{2}+\frac{3\,a\,b^2\,x^8\,D}{8}+A\,a^2\,b\,x^3+\frac{3\,A\,a\,b^2\,x^5}{5}+\frac{3\,B\,a^2\,b\,x^4}{4}+\frac{B\,a\,b^2\,x^6}{2}+\frac{3\,C\,a^2\,b\,x^5}{5}+\frac{3\,C\,a\,b^2\,x^7}{7}$$

input

```
int((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D),x)
```

output

```
(B*a^3*x^2)/2 + (A*b^3*x^7)/7 + (C*a^3*x^3)/3 + (B*b^3*x^8)/8 + (C*b^3*x^9
)/9 + (a^3*x^4*D)/4 + (b^3*x^10*D)/10 + A*a^3*x + (a^2*b*x^6*D)/2 + (3*a*b
^2*x^8*D)/8 + A*a^2*b*x^3 + (3*A*a*b^2*x^5)/5 + (3*B*a^2*b*x^4)/4 + (B*a*b
^2*x^6)/2 + (3*C*a^2*b*x^5)/5 + (3*C*a*b^2*x^7)/7
```

3.82 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx$

3.82.1 Optimal result

Integrand size = 28, antiderivative size = 129

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx = a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2(3bB+aD)x^3+\frac{3}{4}aAb^2x^4 +\frac{3}{5}ab(bB+aD)x^5+\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2(bB+3aD)x^7 +\frac{1}{9}b^3Dx^9+\frac{C(a+bx^2)^4}{8b}+a^3A\log(x)$$

output
```
a^3*B*x+3/2*a^2*A*b*x^2+1/3*a^2*(3*B*b+D*a)*x^3+3/4*a*A*b^2*x^4+3/5*a*b*(B
*b+D*a)*x^5+1/6*A*b^3*x^6+1/7*b^2*(B*b+3*D*a)*x^7+1/9*b^3*D*x^9+1/8*C*(b*x
^2+a)^4/b+a^3*A*ln(x)
```

3.82.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{x(420a^3(6B+x(3C+2Dx))+126a^2bx(30A+x(20B+3x(5C+4Dx)))+18ab^2x^3(105A+2x(42B+}{2520} + a^3A\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
(x*(420*a^3*(6*B + x*(3*C + 2*D*x)) + 126*a^2*b*x*(30*A + x*(20*B + 3*x*(5
*C + 4*D*x))) + 18*a*b^2*x^3*(105*A + 2*x*(42*B + 5*x*(7*C + 6*D*x))) + 5*
b^3*x^5*(84*A + x*(72*B + 7*x*(9*C + 8*D*x)))))/2520 + a^3*A*Log[x]
```

3.82.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} \, dx$$

$$\Big\downarrow \text{2018}$$

$$\int \frac{(bx^2+a)^3 (Dx^3+Bx+A)}{x} dx + \frac{C(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{2333}$$

$$\int \left(b^3Dx^8+b^2(bB+3aD)x^6+Ab^3x^5+3ab(bB+aD)x^4+3aAb^2x^3+a^2(3bB+aD)x^2+3a^2Abx+a^3B+\frac{a^3A}{x}\right.$$

$$\frac{C(a+bx^2)^4}{8b}$$

$$\Big\downarrow \text{2009}$$

$$a^3A\log(x)+a^3Bx+\frac{3}{2}a^2Abx^2+\frac{1}{3}a^2x^3(aD+3bB)+\frac{3}{4}aAb^2x^4+\frac{1}{7}b^2x^7(3aD+bB)+\frac{3}{5}abx^5(aD+bB)+\frac{C(a+bx^2)^4}{8b}+\frac{1}{6}Ab^3x^6+\frac{1}{9}b^3Dx^9$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x,x]
```

output
```
a^3*B*x + (3*a^2*A*b*x^2)/2 + (a^2*(3*b*B + a*D)*x^3)/3 + (3*a*A*b^2*x^4)/
4 + (3*a*b*(b*B + a*D)*x^5)/5 + (A*b^3*x^6)/6 + (b^2*(b*B + 3*a*D)*x^7)/7
+ (b^3*D*x^9)/9 + (C*(a + b*x^2)^4)/(8*b) + a^3*A*Log[x]
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.82.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.10

method	result
norman	$\left(\frac{1}{7}B\,b^3+\frac{3}{7}a\,b^2D\right)x^7+\left(\frac{1}{6}b^3A+\frac{1}{2}C\,b^2a\right)x^6+\left(\frac{3}{4}a\,b^2A+\frac{3}{4}C\,a^2b\right)x^4+\left(\frac{3}{5}a\,b^2B+\frac{3}{5}Da^2b\right)x^5$
default	$\frac{b^3Dx^9}{9}+\frac{b^3C\,x^8}{8}+\frac{b^3B\,x^7}{7}+\frac{3Da\,b^2x^7}{7}+\frac{x^6b^3A}{6}+\frac{Ca\,b^2x^6}{2}+\frac{3Ba\,b^2x^5}{5}+\frac{3Da^2b\,x^5}{5}+\frac{3aA\,b^2x^4}{4}+\frac{3C\,a^2b\,x^4}{4}$
parallelrisch	$\frac{b^3Dx^9}{9}+\frac{b^3C\,x^8}{8}+\frac{b^3B\,x^7}{7}+\frac{3Da\,b^2x^7}{7}+\frac{x^6b^3A}{6}+\frac{Ca\,b^2x^6}{2}+\frac{3Ba\,b^2x^5}{5}+\frac{3Da^2b\,x^5}{5}+\frac{3aA\,b^2x^4}{4}+\frac{3C\,a^2b\,x^4}{4}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x,method=_RETURNVERBOSE)
```

output
```
(1/7*B*b^3+3/7*a*b^2*D)*x^7+(1/6*b^3*A+1/2*C*b^2*a)*x^6+(3/4*a*b^2*A+3/4*C
*a^2*b)*x^4+(3/5*a*b^2*B+3/5*D*a^2*b)*x^5+(3/2*a^2*b*A+1/2*C*a^3)*x^2+(a^2
*b*B+1/3*D*a^3)*x^3+a^3*B*x+1/8*b^3*C*x^8+1/9*b^3*D*x^9+a^3*A*ln(x)
```

3.82.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}\,Db^3x^9 + \frac{1}{8}\,Cb^3x^8 + \frac{1}{7}\,(3\,Dab^2+Bb^3)x^7 + \frac{1}{6}\,(3\,Cab^2+Ab^3)x^6 + \frac{3}{5}\,(Da^2b+Bab^2)x^5 + Ba^3x + \frac{3}{4}\,(Ca^2b+Aab^2)x^4 + Aa^3\log(x) + \frac{1}{3}\,(Da^3+3\,Ba^2b)x^3 + \frac{1}{2}\,(Ca^3+3\,Aa^2b)x^2$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="fricas")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b
^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A
*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*
A*a^2*b)*x^2
```

3.82.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x}\,dx = Aa^3\log(x) + Ba^3x + \frac{Cb^3x^8}{8} + \frac{Db^3x^9}{9} + x^7\left(\frac{Bb^3}{7} + \frac{3Dab^2}{7}\right) + x^6\left(\frac{Ab^3}{6} + \frac{Cab^2}{2}\right) + x^5 \cdot \left(\frac{3Bab^2}{5} + \frac{3Da^2b}{5}\right) + x^4 \cdot \left(\frac{3Aab^2}{4} + \frac{3Ca^2b}{4}\right) + x^3\left(Ba^2b + \frac{Da^3}{3}\right) + x^2 \cdot \left(\frac{3Aa^2b}{2} + \frac{Ca^3}{2}\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x,x)
```

output
```
A*a**3*log(x) + B*a**3*x + C*b**3*x**8/8 + D*b**3*x**9/9 + x**7*(B*b**3/7
+ 3*D*a*b**2/7) + x**6*(A*b**3/6 + C*a*b**2/2) + x**5*(3*B*a*b**2/5 + 3*D*
a**2*b/5) + x**4*(3*A*a*b**2/4 + 3*C*a**2*b/4) + x**3*(B*a**2*b + D*a**3/3
) + x**2*(3*A*a**2*b/2 + C*a**3/2)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{1}{7}\left(3Dab^2+Bb^3\right)x^7 + \frac{1}{6}\left(3Cab^2+Ab^3\right)x^6+\frac{3}{5}\left(Da^2b+Bab^2\right)x^5 + Ba^3x+\frac{3}{4}\left(Ca^2b+Aab^2\right)x^4+Aa^3\log(x) + \frac{1}{3}\left(Da^3+3Ba^2b\right)x^3+\frac{1}{2}\left(Ca^3+3Aa^2b\right)x^2$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="maxima")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 1/7*(3*D*a*b^2 + B*b^3)*x^7 + 1/6*(3*C*a*b
^2 + A*b^3)*x^6 + 3/5*(D*a^2*b + B*a*b^2)*x^5 + B*a^3*x + 3/4*(C*a^2*b + A
*a*b^2)*x^4 + A*a^3*log(x) + 1/3*(D*a^3 + 3*B*a^2*b)*x^3 + 1/2*(C*a^3 + 3*
A*a^2*b)*x^2
```

3.82.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.15

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x}\,dx = \frac{1}{9}Db^3x^9+\frac{1}{8}Cb^3x^8+\frac{3}{7}Dab^2x^7+\frac{1}{7}Bb^3x^7 + \frac{1}{2}Cab^2x^6+\frac{1}{6}Ab^3x^6+\frac{3}{5}Da^2bx^5+\frac{3}{5}Bab^2x^5 + \frac{3}{4}Ca^2bx^4+\frac{3}{4}Aab^2x^4+\frac{1}{3}Da^3x^3+Ba^2bx^3 + \frac{1}{2}Ca^3x^2+\frac{3}{2}Aa^2bx^2+Ba^3x+Aa^3\log(|x|)$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x,x, algorithm="giac")
```

output
```
1/9*D*b^3*x^9 + 1/8*C*b^3*x^8 + 3/7*D*a*b^2*x^7 + 1/7*B*b^3*x^7 + 1/2*C*a*
b^2*x^6 + 1/6*A*b^3*x^6 + 3/5*D*a^2*b*x^5 + 3/5*B*a*b^2*x^5 + 3/4*C*a^2*b*
x^4 + 3/4*A*a*b^2*x^4 + 1/3*D*a^3*x^3 + B*a^2*b*x^3 + 1/2*C*a^3*x^2 + 3/2*
A*a^2*b*x^2 + B*a^3*x + A*a^3*log(abs(x))
```

3.82.9 Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x} \, dx = \frac{A\,b^3\,x^6}{6}+\frac{C\,a^3\,x^2}{2}+\frac{B\,b^3\,x^7}{7}+\frac{C\,b^3\,x^8}{8}+A\,a^3\,\ln(x) + \frac{a^3\,x^3\,D}{3}+\frac{b^3\,x^9\,D}{9}+B\,a^3\,x+\frac{3\,a^2\,b\,x^5\,D}{5} + \frac{3\,a\,b^2\,x^7\,D}{7}+\frac{3\,A\,a^2\,b\,x^2}{2}+\frac{3\,A\,a\,b^2\,x^4}{4} + B\,a^2\,b\,x^3+\frac{3\,B\,a\,b^2\,x^5}{5}+\frac{3\,C\,a^2\,b\,x^4}{4}+\frac{C\,a\,b^2\,x^6}{2}$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x,x)
```

output
```
(A*b^3*x^6)/6 + (C*a^3*x^2)/2 + (B*b^3*x^7)/7 + (C*b^3*x^8)/8 + A*a^3*log(
x) + (a^3*x^3*D)/3 + (b^3*x^9*D)/9 + B*a^3*x + (3*a^2*b*x^5*D)/5 + (3*a*b^
2*x^7*D)/7 + (3*A*a^2*b*x^2)/2 + (3*A*a*b^2*x^4)/4 + B*a^2*b*x^3 + (3*B*a*
b^2*x^5)/5 + (3*C*a^2*b*x^4)/4 + (C*a*b^2*x^6)/2
```

3.83 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx$

3.83.1 Optimal result

Integrand size = 28, antiderivative size = 124

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = -\frac{a^3A}{x}+a^2(3Ab+aC)x+\frac{3}{2}a^2bBx^2+ab(Ab+aC)x^3+\frac{3}{4}ab^2Bx^4+\frac{1}{5}b^2(Ab+3aC)x^5+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7+\frac{D(a+bx^2)^4}{8b}+a^3B\log(x)$$

output
```
-a^3*A/x+a^2*(3*A*b+C*a)*x+3/2*a^2*b*B*x^2+a*b*(A*b+C*a)*x^3+3/4*a*b^2*B*x
^4+1/5*b^2*(A*b+3*C*a)*x^5+1/6*b^3*B*x^6+1/7*b^3*C*x^7+1/8*D*(b*x^2+a)^4/b
+a^3*B*ln(x)
```

3.83.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^2}\,dx = a^3\left(-\frac{A}{x}+Cx+\frac{Dx^2}{2}\right)+\frac{1}{4}a^2bx(12A+x(6B+x(4C+3Dx)))+\frac{1}{20}ab^2x^3(20A+x(15B+2x(6C+5Dx)))+\frac{1}{840}b^3x^5(168A+5x(28B+3x(8C+7Dx)))+a^3B\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
a^3*(-(A/x) + C*x + (D*x^2)/2) + (a^2*b*x*(12*A + x*(6*B + x*(4*C + 3*D*x)
)))/4 + (a*b^2*x^3*(20*A + x*(15*B + 2*x*(6*C + 5*D*x))))/20 + (b^3*x^5*(1
68*A + 5*x*(28*B + 3*x*(8*C + 7*D*x))))/840 + a^3*B*Log[x]
```

3.83.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {2018, 2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx$$

$$\Big\downarrow \ 2018$$

$$\int \frac{(bx^2+a)^3 (Cx^2+Bx+A)}{x^2}dx + \frac{D(a+bx^2)^4}{8b}$$

$$\Big\downarrow \ 2159$$

$$\int \left(b^3Cx^6+b^3Bx^5+b^2(Ab+3aC)x^4+3ab^2Bx^3+3ab(Ab+aC)x^2+3a^2bBx+a^2(3Ab+aC)+\frac{a^3B}{x}+\frac{a^3A}{x^2}\right)$$
$$\frac{D(a+bx^2)^4}{8b}$$

$$\Big\downarrow \ 2009$$

$$-\frac{a^3A}{x}+a^3B\log(x)+a^2x(aC+3Ab)+\frac{3}{2}a^2bBx^2+\frac{1}{5}b^2x^5(3aC+Ab)+abx^3(aC+Ab)+ \frac{3}{4}ab^2Bx^4+\frac{D(a+bx^2)^4}{8b}+\frac{1}{6}b^3Bx^6+\frac{1}{7}b^3Cx^7$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^2,x]
```

output
```
-((a^3*A)/x) + a^2*(3*A*b + a*C)*x + (3*a^2*b*B*x^2)/2 + a*b*(A*b + a*C)*x
^3 + (3*a*b^2*B*x^4)/4 + (b^2*(A*b + 3*a*C)*x^5)/5 + (b^3*B*x^6)/6 + (b^3*
C*x^7)/7 + (D*(a + b*x^2)^4)/(8*b) + a^3*B*Log[x]
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2018
```
Int[(Px_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[Coef
f[Px, x, n - m - 1]*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] + Int[(Px - Coe
ff[Px, x, n - m - 1]*x^(n - m - 1))*x^m*(a + b*x^n)^p, x] /; FreeQ[{a, b, m
, n}, x] && PolyQ[Px, x] && IGtQ[p, 1] && IGtQ[n - m, 0] && NeQ[Coeff[Px, x
, n - m - 1], 0]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.83.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

method	result
default	$\frac{b^3 D x^8}{8}+\frac{b^3 C x^7}{7}+\frac{b^3 B x^6}{6}+\frac{D a b^2 x^6}{2}+\frac{A b^3 x^5}{5}+\frac{3 C a b^2 x^5}{5}+\frac{3 B a b^2 x^4}{4}+\frac{3 D a^2 b x^4}{4}+a A b^2 x^3+C a^2 b x$
norman	$\frac{\left(\frac{1}{6} B b^3+\frac{1}{2} a b^2 D\right) x^7+\left(\frac{1}{5} b^3 A+\frac{3}{5} C b^2 a\right) x^6+\left(\frac{3}{4} a b^2 B+\frac{3}{4} D a^2 b\right) x^5+\left(\frac{3}{2} a^2 b B+\frac{1}{2} D a^3\right) x^3+\left(a b^2 A+C a^2 b\right) x^4+\left(3 a^2 b A+C a^3\right) x^2-}{x}$
parallelrisch	$\frac{105 b^3 D x^9+120 b^3 C x^8+140 b^3 B x^7+420 D a b^2 x^7+168 x^6 b^3 A+504 C a b^2 x^6+630 B a b^2 x^5+630 D a^2 b x^5+840 a A b^2 x^4+840 C a^2 b}{840 x}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*b^3*D*x^8+1/7*b^3*C*x^7+1/6*b^3*B*x^6+1/2*D*a*b^2*x^6+1/5*A*b^3*x^5+3/
5*C*a*b^2*x^5+3/4*B*a*b^2*x^4+3/4*D*a^2*b*x^4+a*A*b^2*x^3+C*a^2*b*x^3+3/2*
B*a^2*b*x^2+1/2*D*a^3*x^2+3*a^2*A*b*x+C*a^3*x+a^3*B*ln(x)-a^3*A/x
```

3.83.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.19

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = \frac{105 \, Db^3x^9 + 120 \, Cb^3x^8 + 140 \, (3 \, Dab^2 + Bb^3)x^7 + 168 \, (3 \, Cab^2 + Ab^3)x^6 + 630 \, (Da^2b + Bab^2)x^5 + 840 \, B}{840 \, x}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="fricas")
```

output
```
1/840*(105*D*b^3*x^9 + 120*C*b^3*x^8 + 140*(3*D*a*b^2 + B*b^3)*x^7 + 168*(
3*C*a*b^2 + A*b^3)*x^6 + 630*(D*a^2*b + B*a*b^2)*x^5 + 840*B*a^3*x*log(x)
+ 840*(C*a^2*b + A*a*b^2)*x^4 - 840*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 +
840*(C*a^3 + 3*A*a^2*b)*x^2)/x
```

3.83.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2} \, dx = -\frac{Aa^3}{x} + Ba^3 \log(x) + \frac{Cb^3x^7}{7} + \frac{Db^3x^8}{8} + x^6\left(\frac{Bb^3}{6} + \frac{Dab^2}{2}\right) + x^5\left(\frac{Ab^3}{5} + \frac{3Cab^2}{5}\right) + x^4 \cdot \left(\frac{3Bab^2}{4} + \frac{3Da^2b}{4}\right) + x^3\left(Aab^2 + Ca^2b\right) + x^2 \cdot \left(\frac{3Ba^2b}{2} + \frac{Da^3}{2}\right) + x\left(3Aa^2b + Ca^3\right)$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**2,x)
```

output
```
-A*a**3/x + B*a**3*log(x) + C*b**3*x**7/7 + D*b**3*x**8/8 + x**6*(B*b**3/6
 + D*a*b**2/2) + x**5*(A*b**3/5 + 3*C*a*b**2/5) + x**4*(3*B*a*b**2/4 + 3*D
*a**2*b/4) + x**3*(A*a*b**2 + C*a**2*b) + x**2*(3*B*a**2*b/2 + D*a**3/2) +
 x*(3*A*a**2*b + C*a**3)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{6}(3Dab^2+Bb^3)x^6 + \frac{1}{5}(3Cab^2+Ab^3)x^5 + \frac{3}{4}(Da^2b+Bab^2)x^4 + Ba^3\log(x) + (Ca^2b+Aab^2)x^3 - \frac{Aa^3}{x} + \frac{1}{2}(Da^3+3Ba^2b)x^2 + (Ca^3+3Aa^2b)x$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="maxima")
```

output
```
1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/6*(3*D*a*b^2 + B*b^3)*x^6 + 1/5*(3*C*a*b
^2 + A*b^3)*x^5 + 3/4*(D*a^2*b + B*a*b^2)*x^4 + B*a^3*log(x) + (C*a^2*b +
A*a*b^2)*x^3 - A*a^3/x + 1/2*(D*a^3 + 3*B*a^2*b)*x^2 + (C*a^3 + 3*A*a^2*b)
*x
```

3.83.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{1}{8}Db^3x^8 + \frac{1}{7}Cb^3x^7 + \frac{1}{2}Dab^2x^6 + \frac{1}{6}Bb^3x^6 + \frac{3}{5}Cab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{4}Da^2bx^4 + \frac{3}{4}Bab^2x^4 + Ca^2bx^3 + Aab^2x^3 + \frac{1}{2}Da^3x^2 + \frac{3}{2}Ba^2bx^2 + Ca^3x + 3Aa^2bx + Ba^3\log(|x|) - \frac{Aa^3}{x}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^2,x, algorithm="giac")
```

output
```
1/8*D*b^3*x^8 + 1/7*C*b^3*x^7 + 1/2*D*a*b^2*x^6 + 1/6*B*b^3*x^6 + 3/5*C*a*
b^2*x^5 + 1/5*A*b^3*x^5 + 3/4*D*a^2*b*x^4 + 3/4*B*a*b^2*x^4 + C*a^2*b*x^3
+ A*a*b^2*x^3 + 1/2*D*a^3*x^2 + 3/2*B*a^2*b*x^2 + C*a^3*x + 3*A*a^2*b*x +
B*a^3*log(abs(x)) - A*a^3/x
```

3.83.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^2}\,dx = \frac{(b\,x^2+a)^4\,D}{8\,b} - \frac{A\,a^3}{x} + \frac{A\,b^3\,x^5}{5} + \frac{B\,b^3\,x^6}{6} + \frac{C\,b^3\,x^7}{7} + B\,a^3\,\ln(x) + C\,a^3\,x + 3\,A\,a^2\,b\,x + A\,a\,b^2\,x^3 + \frac{3\,B\,a^2\,b\,x^2}{2} + \frac{3\,B\,a\,b^2\,x^4}{4} + C\,a^2\,b\,x^3 + \frac{3\,C\,a\,b^2\,x^5}{5}$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^2,x)
```

output
```
((a + b*x^2)^4*D)/(8*b) - (A*a^3)/x + (A*b^3*x^5)/5 + (B*b^3*x^6)/6 + (C*b
^3*x^7)/7 + B*a^3*log(x) + C*a^3*x + 3*A*a^2*b*x + A*a*b^2*x^3 + (3*B*a^2*
b*x^2)/2 + (3*B*a*b^2*x^4)/4 + C*a^2*b*x^3 + (3*C*a*b^2*x^5)/5
```

3.84 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx$

3.84.1 Optimal result

Integrand size = 28, antiderivative size = 135

$$\begin{aligned}\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = & -\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2(3bB+aD)x \\ & +\frac{3}{2}ab(Ab+aC)x^2+ab(bB+aD)x^3 \\ & +\frac{1}{4}b^2(Ab+3aC)x^4+\frac{1}{5}b^2(bB+3aD)x^5 \\ & +\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7+a^2(3Ab+aC)\log(x)\end{aligned}$$

output
```
-1/2*a^3*A/x^2-a^3*B/x+a^2*(3*B*b+D*a)*x+3/2*a*b*(A*b+C*a)*x^2+a*b*(B*b+D*
a)*x^3+1/4*b^2*(A*b+3*C*a)*x^4+1/5*b^2*(B*b+3*D*a)*x^5+1/6*b^3*C*x^6+1/7*b
^3*D*x^7+a^2*(3*A*b+C*a)*ln(x)
```

3.84.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = -\frac{a^3(A+2Bx-2Dx^3)}{2x^2} + \frac{1}{2}a^2bx(6B+x(3C+2Dx)) + \frac{1}{20}ab^2x^2(30A+x(20B+3x(5C+4Dx))) + \frac{1}{420}b^3x^4(105A+2x(42B+5x(7C+6Dx))) + a^2(3Ab+aC)\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^3*(A + 2*B*x - 2*D*x^3))/x^2 + (a^2*b*x*(6*B + x*(3*C + 2*D*x)))/2
 + (a*b^2*x^2*(30*A + x*(20*B + 3*x*(5*C + 4*D*x))))/20 + (b^3*x^4*(105*A
+ 2*x*(42*B + 5*x*(7*C + 6*D*x))))/420 + a^2*(3*A*b + a*C)*Log[x]
```

3.84.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^3A}{x^3}+\frac{a^3B}{x^2}+\frac{a^2(aC+3Ab)}{x}+a^2(aD+3bB)+b^2x^3(3aC+Ab)+3abx(aC+Ab)+b^2x^4(3aD+bB)+3ab\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{2x^2}-\frac{a^3B}{x}+a^2\log(x)(aC+3Ab)+a^2x(aD+3bB)+\frac{1}{4}b^2x^4(3aC+Ab)+\frac{3}{2}abx^2(aC+Ab)+\frac{1}{5}b^2x^5(3aD+bB)+abx^3(aD+bB)+\frac{1}{6}b^3Cx^6+\frac{1}{7}b^3Dx^7$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^3,x]
```

output
```
-1/2*(a^3*A)/x^2 - (a^3*B)/x + a^2*(3*b*B + a*D)*x + (3*a*b*(A*b + a*C)*x^
2)/2 + a*b*(b*B + a*D)*x^3 + (b^2*(A*b + 3*a*C)*x^4)/4 + (b^2*(b*B + 3*a*D
)*x^5)/5 + (b^3*C*x^6)/6 + (b^3*D*x^7)/7 + a^2*(3*A*b + a*C)*Log[x]
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.84.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.05

method	result
default	$\frac{b^3 D x^7}{7}+\frac{b^3 C x^6}{6}+\frac{b^3 B x^5}{5}+\frac{3 D a b^2 x^5}{5}+\frac{A b^3 x^4}{4}+\frac{3 C a b^2 x^4}{4}+B a b^2 x^3+D a^2 b x^3+\frac{3 a A b^2 x^2}{2}+\frac{3 C a^2 b x}{2}$
norman	$\frac{\left(\frac{1}{5} B b^3+\frac{3}{5} a b^2 D\right) x^7+\left(\frac{1}{4} b^3 A+\frac{3}{4} C b^2 a\right) x^6+\left(\frac{3}{2} a b^2 A+\frac{3}{2} C a^2 b\right) x^4+\left(a b^2 B+D a^2 b\right) x^5+\left(3 a^2 b B+D a^3\right) x^3-\frac{a^3 A}{2}-a^3 B x+\frac{b^3 C x^8}{6}}{x^2}$
parallelrisch	$\frac{60 b^3 D x^9+70 b^3 C x^8+84 b^3 B x^7+252 D a b^2 x^7+105 x^6 b^3 A+315 C a b^2 x^6+420 B a b^2 x^5+420 D a^2 b x^5+630 a A b^2 x^4+630 C a^2 b x^4+}{420 x^2}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/7*b^3*D*x^7+1/6*b^3*C*x^6+1/5*b^3*B*x^5+3/5*D*a*b^2*x^5+1/4*A*b^3*x^4+3/
4*C*a*b^2*x^4+B*a*b^2*x^3+D*a^2*b*x^3+3/2*a*A*b^2*x^2+3/2*C*a^2*b*x^2+3*a^
2*b*B*x+D*a^3*x+a^2*(3*A*b+C*a)*ln(x)-a^3*B/x-1/2*a^3*A/x^2
```

3.84.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.09

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx$$
$$= \frac{60\,Db^3x^9 + 70\,Cb^3x^8 + 84\,(3\,Dab^2 + Bb^3)x^7 + 105\,(3\,Cab^2 + Ab^3)x^6 + 420\,(Da^2b + Bab^2)x^5 - 420\,Ba^3x}{420\,x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="fricas")
```

output
```
1/420*(60*D*b^3*x^9 + 70*C*b^3*x^8 + 84*(3*D*a*b^2 + B*b^3)*x^7 + 105*(3*C
*a*b^2 + A*b^3)*x^6 + 420*(D*a^2*b + B*a*b^2)*x^5 - 420*B*a^3*x + 630*(C*a
^2*b + A*a*b^2)*x^4 - 210*A*a^3 + 420*(D*a^3 + 3*B*a^2*b)*x^3 + 420*(C*a^3
 + 3*A*a^2*b)*x^2*log(x))/x^2
```

3.84.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{Cb^3x^6}{6} + \frac{Db^3x^7}{7} + a^2\cdot(3Ab+Ca)\log(x) + x^5\left(\frac{Bb^3}{5} + \frac{3Dab^2}{5}\right) + x^4\left(\frac{Ab^3}{4} + \frac{3Cab^2}{4}\right) + x^3\left(Bab^2 + Da^2b\right) + x^2\cdot\left(\frac{3Aab^2}{2} + \frac{3Ca^2b}{2}\right) + x\left(3Ba^2b + Da^3\right) + \frac{-Aa^3 - 2Ba^3x}{2x^2}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**3,x)
```

output
```
C*b**3*x**6/6 + D*b**3*x**7/7 + a**2*(3*A*b + C*a)*log(x) + x**5*(B*b**3/5
 + 3*D*a*b**2/5) + x**4*(A*b**3/4 + 3*C*a*b**2/4) + x**3*(B*a*b**2 + D*a**
2*b) + x**2*(3*A*a*b**2/2 + 3*C*a**2*b/2) + x*(3*B*a**2*b + D*a**3) + (-A*
a**3 - 2*B*a**3*x)/(2*x**2)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{7} Db^3x^7 + \frac{1}{6} Cb^3x^6 + \frac{1}{5} (3 Dab^2 + Bb^3)x^5 + \frac{1}{4} (3 Cab^2 + Ab^3)x^4 + (Da^2b + Bab^2)x^3 + \frac{3}{2} (Ca^2b + Aab^2)x^2 + (Da^3 + 3 Ba^2b)x + (Ca^3 + 3 Aa^2b) \log(x) - \frac{2 Ba^3x + Aa^3}{2 x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="maxima")
```

output
```
1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 1/5*(3*D*a*b^2 + B*b^3)*x^5 + 1/4*(3*C*a*b
^2 + A*b^3)*x^4 + (D*a^2*b + B*a*b^2)*x^3 + 3/2*(C*a^2*b + A*a*b^2)*x^2 +
(D*a^3 + 3*B*a^2*b)*x + (C*a^3 + 3*A*a^2*b)*log(x) - 1/2*(2*B*a^3*x + A*a^
3)/x^2
```

3.84.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.07

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^3} dx = \frac{1}{7} Db^3x^7 + \frac{1}{6} Cb^3x^6 + \frac{3}{5} Dab^2x^5 + \frac{1}{5} Bb^3x^5 + \frac{3}{4} Cab^2x^4 + \frac{1}{4} Ab^3x^4 + Da^2bx^3 + Bab^2x^3 + \frac{3}{2} Ca^2bx^2 + \frac{3}{2} Aab^2x^2 + Da^3x + 3 Ba^2bx + (Ca^3 + 3 Aa^2b) \log(|x|) - \frac{2 Ba^3x + Aa^3}{2 x^2}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^3,x, algorithm="giac")
```

output
```
1/7*D*b^3*x^7 + 1/6*C*b^3*x^6 + 3/5*D*a*b^2*x^5 + 1/5*B*b^3*x^5 + 3/4*C*a*
b^2*x^4 + 1/4*A*b^3*x^4 + D*a^2*b*x^3 + B*a*b^2*x^3 + 3/2*C*a^2*b*x^2 + 3/
2*A*a*b^2*x^2 + D*a^3*x + 3*B*a^2*b*x + (C*a^3 + 3*A*a^2*b)*log(abs(x)) -
1/2*(2*B*a^3*x + A*a^3)/x^2
```

3.84.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^3}\,dx = \frac{A\,b^3\,x^4}{4}-\frac{B\,a^3}{x}-\frac{A\,a^3}{2\,x^2}+\frac{B\,b^3\,x^5}{5}+\frac{C\,b^3\,x^6}{6}+C\,a^3\,\ln(x)+a^3\,x\,D+\frac{b^3\,x^7\,D}{7}+a^2\,b\,x^3\,D+\frac{3\,a\,b^2\,x^5\,D}{5}+3\,B\,a^2\,b\,x+\frac{3\,A\,a\,b^2\,x^2}{2}+B\,a\,b^2\,x^3+\frac{3\,C\,a^2\,b\,x^2}{2}+\frac{3\,C\,a\,b^2\,x^4}{4}+3\,A\,a^2\,b\,\ln(x)$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^3,x)
```

output
```
(A*b^3*x^4)/4 - (B*a^3)/x - (A*a^3)/(2*x^2) + (B*b^3*x^5)/5 + (C*b^3*x^6)/
6 + C*a^3*log(x) + a^3*x*D + (b^3*x^7*D)/7 + a^2*b*x^3*D + (3*a*b^2*x^5*D)
/5 + 3*B*a^2*b*x + (3*A*a*b^2*x^2)/2 + B*a*b^2*x^3 + (3*C*a^2*b*x^2)/2 + (
3*C*a*b^2*x^4)/4 + 3*A*a^2*b*log(x)
```

3.85 $\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$

3.85.1 Optimal result

Integrand size = 28, antiderivative size = 139

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(3Ab+aC)}{x}+3ab(Ab+aC)x+\frac{3}{2}ab(bB+aD)x^2+\frac{1}{3}b^2(Ab+3aC)x^3+\frac{1}{4}b^2(bB+3aD)x^4+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6+a^2(3bB+aD)\log(x)$$

output
```
-1/3*a^3*A/x^3-1/2*a^3*B/x^2-a^2*(3*A*b+C*a)/x+3*a*b*(A*b+C*a)*x+3/2*a*b*(
B*b+D*a)*x^2+1/3*b^2*(A*b+3*C*a)*x^3+1/4*b^2*(B*b+3*D*a)*x^4+1/5*b^3*C*x^5
+1/6*b^3*D*x^6+a^2*(3*B*b+D*a)*ln(x)
```

3.85.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = -\frac{a^3(2A+3x(B+2Cx))}{6x^3} + \frac{3a^2b(-2A+x^2(2C+Dx))}{2x} + \frac{1}{4}ab^2x(12A+x(6B+x(4C+3Dx))) + \frac{1}{60}b^3x^3(20A+x(15B+2x(6C+5Dx))) + a^2(3bB+aD)\log(x)$$

input
```
Integrate[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/6*(a^3*(2*A + 3*x*(B + 2*C*x)))/x^3 + (3*a^2*b*(-2*A + x^2*(2*C + D*x))
)/(2*x) + (a*b^2*x*(12*A + x*(6*B + x*(4*C + 3*D*x))))/4 + (b^3*x^3*(20*A
+ x*(15*B + 2*x*(6*C + 5*D*x))))/60 + a^2*(3*b*B + a*D)*Log[x]
```

3.85.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^3\,(A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^3A}{x^4}+\frac{a^3B}{x^3}+\frac{a^2(aC+3Ab)}{x^2}+\frac{a^2(aD+3bB)}{x}+b^2x^2(3aC+Ab)+3ab(aC+Ab)+b^2x^3(3aD+bB)+3ab$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3A}{3x^3}-\frac{a^3B}{2x^2}-\frac{a^2(aC+3Ab)}{x}+a^2\log(x)(aD+3bB)+\frac{1}{3}b^2x^3(3aC+Ab)+3abx(aC+Ab)+\frac{1}{4}b^2x^4(3aD+bB)+\frac{3}{2}abx^2(aD+bB)+\frac{1}{5}b^3Cx^5+\frac{1}{6}b^3Dx^6$$

input
```
Int[((a + b*x^2)^3*(A + B*x + C*x^2 + D*x^3))/x^4,x]
```

output
```
-1/3*(a^3*A)/x^3 - (a^3*B)/(2*x^2) - (a^2*(3*A*b + a*C))/x + 3*a*b*(A*b +
a*C)*x + (3*a*b*(b*B + a*D)*x^2)/2 + (b^2*(A*b + 3*a*C)*x^3)/3 + (b^2*(b*B
 + 3*a*D)*x^4)/4 + (b^3*C*x^5)/5 + (b^3*D*x^6)/6 + a^2*(3*b*B + a*D)*Log[x
]
```

3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.85.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{b^3 D x^6}{6}+\frac{b^3 C x^5}{5}+\frac{b^3 B x^4}{4}+\frac{3 D a b^2 x^4}{4}+\frac{A b^3 x^3}{3}+C a b^2 x^3+\frac{3 B a b^2 x^2}{2}+\frac{3 D a^2 b x^2}{2}+3 a b^2 A x+3 C a^2$
norman	$\frac{\left(\frac{1}{4} B b^3+\frac{3}{4} a b^2 D\right) x^7+\left(\frac{1}{3} b^3 A+C b^2 a\right) x^6+\left(\frac{3}{2} a b^2 B+\frac{3}{2} D a^2 b\right) x^5+\left(3 a b^2 A+3 C a^2 b\right) x^4+\left(-3 a^2 b A-C a^3\right) x^2-\frac{a^3 A}{3}-\frac{a^3 B x}{2}+\frac{b^3 C}{5}}{x^3}$
parallelrisch	$\frac{10 b^3 D x^9+12 b^3 C x^8+15 b^3 B x^7+45 D a b^2 x^7+20 x^6 b^3 A+60 C a b^2 x^6+90 B a b^2 x^5+90 D a^2 b x^5+180 a A b^2 x^4+180 B \ln(x) x^3 a^2 b+}{60 x^3}$

input
```
int((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/6*b^3*D*x^6+1/5*b^3*C*x^5+1/4*b^3*B*x^4+3/4*D*a*b^2*x^4+1/3*A*b^3*x^3+C*
a*b^2*x^3+3/2*B*a*b^2*x^2+3/2*D*a^2*b*x^2+3*a*b^2*A*x+3*C*a^2*b*x+a^2*(3*B
*b+D*a)*ln(x)-1/3*a^3*A/x^3-a^2*(3*A*b+C*a)/x-1/2*a^3*B/x^2
```

3.85.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx$$
$$= \frac{10\,Db^3x^9 + 12\,Cb^3x^8 + 15\,(3\,Dab^2 + Bb^3)x^7 + 20\,(3\,Cab^2 + Ab^3)x^6 + 90\,(Da^2b + Bab^2)x^5 - 30\,Ba^3x + 1}{60\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="fricas")
```

output
```
1/60*(10*D*b^3*x^9 + 12*C*b^3*x^8 + 15*(3*D*a*b^2 + B*b^3)*x^7 + 20*(3*C*a
*b^2 + A*b^3)*x^6 + 90*(D*a^2*b + B*a*b^2)*x^5 - 30*B*a^3*x + 180*(C*a^2*b
 + A*a*b^2)*x^4 + 60*(D*a^3 + 3*B*a^2*b)*x^3*log(x) - 20*A*a^3 - 60*(C*a^3
 + 3*A*a^2*b)*x^2)/x^3
```

3.85.6 Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{Cb^3x^5}{5} + \frac{Db^3x^6}{6} + a^2 \cdot (3Bb + Da)\log(x) + x^4\left(\frac{Bb^3}{4} + \frac{3Dab^2}{4}\right) + x^3\left(\frac{Ab^3}{3} + Cab^2\right) + x^2 \cdot \left(\frac{3Bab^2}{2} + \frac{3Da^2b}{2}\right) + x\left(3Aab^2 + 3Ca^2b\right) + \frac{-2Aa^3 - 3Ba^3x + x^2(-18Aa^2b - 6Ca^3)}{6x^3}$$

input
```
integrate((b*x**2+a)**3*(D*x**3+C*x**2+B*x+A)/x**4,x)
```

output
```
C*b**3*x**5/5 + D*b**3*x**6/6 + a**2*(3*B*b + D*a)*log(x) + x**4*(B*b**3/4
 + 3*D*a*b**2/4) + x**3*(A*b**3/3 + C*a*b**2) + x**2*(3*B*a*b**2/2 + 3*D*a
**2*b/2) + x*(3*A*a*b**2 + 3*C*a**2*b) + (-2*A*a**3 - 3*B*a**3*x + x**2*(-
18*A*a**2*b - 6*C*a**3))/(6*x**3)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{1}{4}\,(3\,Dab^2+Bb^3)x^4 + \frac{1}{3}\,(3\,Cab^2+Ab^3)x^3 + \frac{3}{2}\,(Da^2b+Bab^2)x^2 + 3\,(Ca^2b+Aab^2)x + \left(Da^3+3\,Ba^2b\right)\log(x) - \frac{3\,Ba^3x+2\,Aa^3+6\,(Ca^3+3\,Aa^2b)x^2}{6\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="maxima")
```

output
```
1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 1/4*(3*D*a*b^2 + B*b^3)*x^4 + 1/3*(3*C*a*b
^2 + A*b^3)*x^3 + 3/2*(D*a^2*b + B*a*b^2)*x^2 + 3*(C*a^2*b + A*a*b^2)*x +
(D*a^3 + 3*B*a^2*b)*log(x) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6*(C*a^3 + 3*A*a^2
*b)*x^2)/x^3
```

3.85.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)^3 (A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{1}{6}\,Db^3x^6 + \frac{1}{5}\,Cb^3x^5 + \frac{3}{4}\,Dab^2x^4 + \frac{1}{4}\,Bb^3x^4 + Cab^2x^3 + \frac{1}{3}\,Ab^3x^3 + \frac{3}{2}\,Da^2bx^2 + \frac{3}{2}\,Bab^2x^2 + 3\,Ca^2bx + 3\,Aab^2x + \left(Da^3+3\,Ba^2b\right)\log(|x|) - \frac{3\,Ba^3x+2\,Aa^3+6\,(Ca^3+3\,Aa^2b)x^2}{6\,x^3}$$

input
```
integrate((b*x^2+a)^3*(D*x^3+C*x^2+B*x+A)/x^4,x, algorithm="giac")
```

output
```
1/6*D*b^3*x^6 + 1/5*C*b^3*x^5 + 3/4*D*a*b^2*x^4 + 1/4*B*b^3*x^4 + C*a*b^2*
x^3 + 1/3*A*b^3*x^3 + 3/2*D*a^2*b*x^2 + 3/2*B*a*b^2*x^2 + 3*C*a^2*b*x + 3*
A*a*b^2*x + (D*a^3 + 3*B*a^2*b)*log(abs(x)) - 1/6*(3*B*a^3*x + 2*A*a^3 + 6
*(C*a^3 + 3*A*a^2*b)*x^2)/x^3
```

3.85.9 Mupad [B] (verification not implemented)

Time = 6.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^3(A+Bx+Cx^2+Dx^3)}{x^4}\,dx = \frac{B\,b^3\,x^4}{4} - \frac{C\,a^3}{x} - \frac{B\,a^3}{2\,x^2} + \frac{C\,b^3\,x^5}{5} + \frac{b^3\,x^6\,D}{6} - \frac{A\,(a^3+9\,a^2\,b\,x^2-9\,a\,b^2\,x^4-b^3\,x^6)}{3\,x^3} + \frac{a^3\,\ln(x^2)\,D}{2} + \frac{3\,a^2\,b\,x^2\,D}{2} + 3\,C\,a^2\,b\,x + \frac{3\,a\,b^2\,x^4\,D}{4} + \frac{3\,B\,a\,b^2\,x^2}{2} + C\,a\,b^2\,x^3 + 3\,B\,a^2\,b\,\ln(x)$$

input
```
int(((a + b*x^2)^3*(A + B*x + C*x^2 + x^3*D))/x^4,x)
```

output
```
(B*b^3*x^4)/4 - (C*a^3)/x - (B*a^3)/(2*x^2) + (C*b^3*x^5)/5 + (b^3*x^6*D)/
6 - (A*(a^3 - b^3*x^6 + 9*a^2*b*x^2 - 9*a*b^2*x^4))/(3*x^3) + (a^3*log(x^2
)*D)/2 + (3*a^2*b*x^2*D)/2 + 3*C*a^2*b*x + (3*a*b^2*x^4*D)/4 + (3*B*a*b^2*
x^2)/2 + C*a*b^2*x^3 + 3*B*a^2*b*log(x)
```

3.86 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

3.86.1 Optimal result

Integrand size = 28, antiderivative size = 151

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a(Ab-aC)x}{b^3}-\frac{a(bB-aD)x^2}{2b^3}+\frac{(Ab-aC)x^3}{3b^2}+\frac{(bB-aD)x^4}{4b^2}+\frac{Cx^5}{5b}+\frac{Dx^6}{6b}+\frac{a^{3/2}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{a^2(bB-aD)\log(a+bx^2)}{2b^4}$$

output
```
-a*(A*b-C*a)*x/b^3-1/2*a*(B*b-D*a)*x^2/b^3+1/3*(A*b-C*a)*x^3/b^2+1/4*(B*b-
D*a)*x^4/b^2+1/5*C*x^5/b+1/6*D*x^6/b+a^(3/2)*(A*b-C*a)*arctan(x*b^(1/2)/a^
(1/2))/b^(7/2)+1/2*a^2*(B*b-D*a)*ln(b*x^2+a)/b^4
```

3.86.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.86

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$= \frac{bx(30a^2(2C+Dx)-5ab(12A+x(6B+x(4C+3Dx)))+b^2x^2(20A+x(15B+2x(6C+5Dx))))-60a}{60b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(b*x*(30*a^2*(2*C + D*x) - 5*a*b*(12*A + x*(6*B + x*(4*C + 3*D*x))) + b^2*
x^2*(20*A + x*(15*B + 2*x*(6*C + 5*D*x)))) - 60*a^(3/2)*Sqrt[b]*(-(A*b) +
a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] - 30*a^2*(-(b*B) + a*D)*Log[a + b*x^2])/(
60*b^4)
```

3.86.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 (A+Bx+Cx^2+Dx^3)}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^2(Ab-aC)+a^2x(bB-aD)}{b^3(a+bx^2)} - \frac{a(Ab-aC)}{b^3} + \frac{x^2(Ab-aC)}{b^2} - \frac{ax(bB-aD)}{b^3} + \frac{x^3(bB-aD)}{b^2} + \frac{Cx^4}{b} + \frac{Dx}{b}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}} + \frac{a^2(bB-aD)\log(a+bx^2)}{2b^4} - \frac{ax(Ab-aC)}{b^3} + \frac{x^3(Ab-aC)}{3b^2} - \frac{ax^2(bB-aD)}{2b^3} + \frac{x^4(bB-aD)}{4b^2} + \frac{Cx^5}{5b} + \frac{Dx^6}{6b}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
-((a*(A*b - a*C)*x)/b^3) - (a*(b*B - a*D)*x^2)/(2*b^3) + ((A*b - a*C)*x^3)
/(3*b^2) + ((b*B - a*D)*x^4)/(4*b^2) + (C*x^5)/(5*b) + (D*x^6)/(6*b) + (a^
(3/2)*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2) + (a^2*(b*B - a*D)*
Log[a + b*x^2])/(2*b^4)
```

3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.86.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.93

method	result
default	$-\frac{-\frac{1}{6}b^2Dx^6-\frac{1}{5}b^2Cx^5-\frac{1}{4}b^2Bx^4+\frac{1}{4}Dabx^4-\frac{1}{3}Ab^2x^3+\frac{1}{3}Cabx^3+\frac{1}{2}Babx^2-\frac{1}{2}Da^2x^2+aAbx-Ca^2x}{b^3}+\frac{a^2\left(\frac{(Bb-Da)\ln\left(bx^2+a\right)}{2b}+\right.}{b^3}$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/6*b^2*D*x^6-1/5*b^2*C*x^5-1/4*b^2*B*x^4+1/4*D*a*b*x^4-1/3*A*b^2
*x^3+1/3*C*a*b*x^3+1/2*B*a*b*x^2-1/2*D*a^2*x^2+a*A*b*x-C*a^2*x)+a^2/b^3*(1
/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.86.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.20

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\left[\frac{10\,Db^3x^6+12\,Cb^3x^5-15\,(Dab^2-Bb^3)x^4-20\,(Cab^2-Ab^3)x^3+30\,(Da^2b-Bab^2)x^2-30\,(Ca^2b-A}{60\,b^4}\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/60*(10*D*b^3*x^6 + 12*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^
2 - A*b^3)*x^3 + 30*(D*a^2*b - B*a*b^2)*x^2 - 30*(C*a^2*b - A*a*b^2)*sqrt(
-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 60*(C*a^2*b - A*a*
b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4, 1/60*(10*D*b^3*x^6 + 12
*C*b^3*x^5 - 15*(D*a*b^2 - B*b^3)*x^4 - 20*(C*a*b^2 - A*b^3)*x^3 + 30*(D*a
^2*b - B*a*b^2)*x^2 - 60*(C*a^2*b - A*a*b^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b
)/a) + 60*(C*a^2*b - A*a*b^2)*x - 30*(D*a^3 - B*a^2*b)*log(b*x^2 + a))/b^4
]
```

3.86.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(134) = 268$.

Time = 0.55 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.09

$$\begin{aligned}
&\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx \\
&= \frac{Cx^5}{5b}+\frac{Dx^6}{6b}+x^4\left(\frac{B}{4b}-\frac{Da}{4b^2}\right)+x^3\left(\frac{A}{3b}-\frac{Ca}{3b^2}\right) \\
&\quad +x^2\left(-\frac{Ba}{2b^2}+\frac{Da^2}{2b^3}\right)+x\left(-\frac{Aa}{b^2}+\frac{Ca^2}{b^3}\right)+\left(-\frac{a^2(-Bb+Da)}{2b^4}\right. \\
&\quad \left.-\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x+\frac{Ba^2b-Da^3-2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4}-\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2+Ca^2b}\right) \\
&\quad +\left(-\frac{a^2(-Bb+Da)}{2b^4}\right. \\
&\quad \left.+\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)\log\left(x+\frac{Ba^2b-Da^3-2b^4\left(-\frac{a^2(-Bb+Da)}{2b^4}+\frac{\sqrt{-a^3b^9}(-Ab+Ca)}{2b^8}\right)}{-Aab^2+Ca^2b}\right)
\end{aligned}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**5/(5*b) + D*x**6/(6*b) + x**4*(B/(4*b) - D*a/(4*b**2)) + x**3*(A/(3*b
) - C*a/(3*b**2)) + x**2*(-B*a/(2*b**2) + D*a**2/(2*b**3)) + x*(-A*a/b**2
+ C*a**2/b**3) + (-a**2*(-B*b + D*a)/(2*b**4) - sqrt(-a**3*b**9)*(-A*b + C
*a)/(2*b**8))*log(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b
**4) - sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b)) +
(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3*b**9)*(-A*b + C*a)/(2*b**8))*log
(x + (B*a**2*b - D*a**3 - 2*b**4*(-a**2*(-B*b + D*a)/(2*b**4) + sqrt(-a**3
*b**9)*(-A*b + C*a)/(2*b**8)))/(-A*a*b**2 + C*a**2*b))
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.96

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca^3-Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{10\,Db^2x^6+12\,Cb^2x^5-15\,(Dab-Bb^2)x^4-20\,(Cab-Ab^2)x^3+30\,(Da^2-Bab)x^2+60\,(Ca^2-Aab)x}{60\,b^3} - \frac{(Da^3-Ba^2b)\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/60*(10*D*b^2*
x^6 + 12*C*b^2*x^5 - 15*(D*a*b - B*b^2)*x^4 - 20*(C*a*b - A*b^2)*x^3 + 30*
(D*a^2 - B*a*b)*x^2 + 60*(C*a^2 - A*a*b)*x)/b^3 - 1/2*(D*a^3 - B*a^2*b)*lo
g(b*x^2 + a)/b^4
```

3.86.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca^3-Aa^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} - \frac{(Da^3-Ba^2b)\log\left(bx^2+a\right)}{2\,b^4} + \frac{10\,Db^5x^6+12\,Cb^5x^5-15\,Dab^4x^4+15\,Bb^5x^4-20\,Cab^4x^3+20\,Ab^5x^3+30\,Da^2b^3x^2-30\,Bab^4x^2+60}{60\,b^6}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a^3 - A*a^2*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(D*a^3 - B*
a^2*b)*log(b*x^2 + a)/b^4 + 1/60*(10*D*b^5*x^6 + 12*C*b^5*x^5 - 15*D*a*b^4
*x^4 + 15*B*b^5*x^4 - 20*C*a*b^4*x^3 + 20*A*b^5*x^3 + 30*D*a^2*b^3*x^2 - 3
0*B*a*b^4*x^2 + 60*C*a^2*b^3*x - 60*A*a*b^4*x)/b^6
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{b\,x^2+a}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.87 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

3.87.1 Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a(bB-aD)x}{b^3}+\frac{(Ab-aC)x^2}{2b^2}+\frac{(bB-aD)x^3}{3b^2}+\frac{Cx^4}{4b}+\frac{Dx^5}{5b}+\frac{a^{3/2}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}-\frac{a(Ab-aC)\log\left(a+bx^2\right)}{2b^3}$$

output
```
-a*(B*b-D*a)*x/b^3+1/2*(A*b-C*a)*x^2/b^2+1/3*(B*b-D*a)*x^3/b^2+1/4*C*x^4/b
+1/5*D*x^5/b+a^(3/2)*(B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)-1/2*a*(A*
b-C*a)*ln(b*x^2+a)/b^3
```

3.87.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{a^{3/2}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{7/2}}+\frac{x(60a^2D-10ab(6B+x(3C+2Dx))+b^2x(30A+x(20B+3x(5C+4Dx))))+30a(-Ab+aC)\log(}{60b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
-((a^(3/2)*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2)) + (x*(60*a
^2*D - 10*a*b*(6*B + x*(3*C + 2*D*x)) + b^2*x*(30*A + x*(20*B + 3*x*(5*C +
 4*D*x)))) + 30*a*(-(A*b) + a*C)*Log[a + b*x^2])/(60*b^3)
```

3.87.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+B x+C x^2+D x^3\right)}{a+b x^2} \, dx$$

$$\Big\downarrow \text{ 2333}$$

$$\int \left(\frac{a^2 (b B-a D)-a b x (A b-a C)}{b^3 \left(a+b x^2\right)}+\frac{x (A b-a C)}{b^2}-\frac{a (b B-a D)}{b^3}+\frac{x^2 (b B-a D)}{b^2}+\frac{C x^3}{b}+\frac{D x^4}{b}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^{3/2} \arctan \left(\frac{\sqrt{b} x}{\sqrt{a}}\right) (b B-a D)}{b^{7/2}}-\frac{a (A b-a C) \log \left(a+b x^2\right)}{2 b^3}+\frac{x^2 (A b-a C)}{2 b^2}-\frac{a x (b B-a D)}{b^3}+\frac{x^3 (b B-a D)}{3 b^2}+\frac{C x^4}{4 b}+\frac{D x^5}{5 b}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
-((a*(b*B - a*D)*x)/b^3) + ((A*b - a*C)*x^2)/(2*b^2) + ((b*B - a*D)*x^3)/(
3*b^2) + (C*x^4)/(4*b) + (D*x^5)/(5*b) + (a^(3/2)*(b*B - a*D)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/b^(7/2) - (a*(A*b - a*C)*Log[a + b*x^2])/(2*b^3)
```

3.87.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.87.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result
default	$\frac{\frac{1}{5}b^2Dx^5+\frac{1}{4}b^2Cx^4+\frac{1}{3}b^2Bx^3-\frac{1}{3}Dabx^3+\frac{1}{2}Ab^2x^2-\frac{1}{2}Cabx^2-Babx+Da^2x}{b^3}-\frac{a\left(\frac{\left(b^2A-Cab\right)\ln\left(bx^2+a\right)}{2b}+\frac{\left(-abB+Da^2\right)\arctan\left(\frac{bx}{\sqrt{a\ldots}}\right.}{\sqrt{ab}}\right.}{b^3}$

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*b^2*D*x^5+1/4*b^2*C*x^4+1/3*b^2*B*x^3-1/3*D*a*b*x^3+1/2*A*b^2*x
^2-1/2*C*a*b*x^2-B*a*b*x+D*a^2*x)-a/b^3*(1/2*(A*b^2-C*a*b)/b*ln(b*x^2+a)+(
-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.87.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 270, normalized size of antiderivative = 2.08

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\left[\frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2+30\,(Da^2-Bab)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}\ldots}{bx^2+a}\right.}{60\,b^3}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C*a*b -
A*b^2)*x^2 + 30*(D*a^2 - B*a*b)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) -
 a)/(b*x^2 + a)) + 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a
))/b^3, 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B*b^2)*x^3 - 30*(C
*a*b - A*b^2)*x^2 - 60*(D*a^2 - B*a*b)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) +
 60*(D*a^2 - B*a*b)*x + 30*(C*a^2 - A*a*b)*log(b*x^2 + a))/b^3]
```

3.87.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(116) = 232$.

Time = 0.53 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.11

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$

$$= \frac{Cx^4}{4b} + \frac{Dx^5}{5b} + x^3\left(\frac{B}{3b} - \frac{Da}{3b^2}\right) + x^2\left(\frac{A}{2b} - \frac{Ca}{2b^2}\right) + x\left(-\frac{Ba}{b^2} + \frac{Da^2}{b^3}\right) + \left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x + \frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3} - \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right) + \left(\frac{a(-Ab+Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)\log\left(x + \frac{-Aab+Ca^2-2b^3\left(\frac{a(-Ab+Ca)}{2b^3} + \frac{\sqrt{-a^3b^7}(-Bb+Da)}{2b^7}\right)}{-Bab+Da^2}\right)$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**4/(4*b) + D*x**5/(5*b) + x**3*(B/(3*b) - D*a/(3*b**2)) + x**2*(A/(2*b
) - C*a/(2*b**2)) + x*(-B*a/b**2 + D*a**2/b**3) + (a*(-A*b + C*a)/(2*b**3)
 - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b*
*3*(a*(-A*b + C*a)/(2*b**3) - sqrt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B
*a*b + D*a**2)) + (a*(-A*b + C*a)/(2*b**3) + sqrt(-a**3*b**7)*(-B*b + D*a)
/(2*b**7))*log(x + (-A*a*b + C*a**2 - 2*b**3*(a*(-A*b + C*a)/(2*b**3) + sq
rt(-a**3*b**7)*(-B*b + D*a)/(2*b**7)))/(-B*a*b + D*a**2))
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.98

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^2x^5+15\,Cb^2x^4-20\,(Dab-Bb^2)x^3-30\,(Cab-Ab^2)x^2+60\,(Da^2-Bab)x}{60\,b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^2*x^5 + 15*C*b^2*x^4 - 20*(D*a*b - B
*b^2)*x^3 - 30*(C*a*b - A*b^2)*x^2 + 60*(D*a^2 - B*a*b)*x)/b^3
```

3.87.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\log(bx^2+a)}{2\,b^3} - \frac{(Da^3-Ba^2b)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{12\,Db^4x^5+15\,Cb^4x^4-20\,Dab^3x^3+20\,Bb^4x^3-30\,Cab^3x^2+30\,Ab^4x^2+60\,Da^2b^2x-60\,Bab^3x}{60\,b^5}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
1/2*(C*a^2 - A*a*b)*log(b*x^2 + a)/b^3 - (D*a^3 - B*a^2*b)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^3) + 1/60*(12*D*b^4*x^5 + 15*C*b^4*x^4 - 20*D*a*b^3*x^
3 + 20*B*b^4*x^3 - 30*C*a*b^3*x^2 + 30*A*b^4*x^2 + 60*D*a^2*b^2*x - 60*B*a
*b^3*x)/b^5
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x^3\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.88 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$

3.88.1 Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ab-aC)x}{b^2}+\frac{(bB-aD)x^2}{2b^2}+\frac{Cx^3}{3b}+\frac{Dx^4}{4b}-\frac{\sqrt{a}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a(bB-aD)\log\left(a+bx^2\right)}{2b^3}$$

output
```
(A*b-C*a)*x/b^2+1/2*(B*b-D*a)*x^2/b^2+1/3*C*x^3/b+1/4*D*x^4/b-1/2*a*(B*b-D
*a)*ln(b*x^2+a)/b^3-(A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{bx(12Ab-6a(2C+Dx)+bx(6B+4Cx+3Dx^2))+12\sqrt{a}\sqrt{b}(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)+6a(-bB+a}{12b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(b*x*(12*A*b - 6*a*(2*C + D*x) + b*x*(6*B + 4*C*x + 3*D*x^2)) + 12*Sqrt[a]
*Sqrt[b]*(-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-(b*B) + a*D)*L
og[a + b*x^2])/(12*b^3)
```

3.88.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (A + Bx + Cx^2 + Dx^3)}{a + bx^2} \, dx$$

$$\Big\downarrow \; 2333$$

$$\int \left(-\frac{a(Ab - aC) + ax(bB - aD)}{b^2 (a + bx^2)} + \frac{Ab - aC}{b^2} + \frac{x(bB - aD)}{b^2} + \frac{Cx^2}{b} + \frac{Dx^3}{b} \right) dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{a}(Ab - aC) \arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x(Ab - aC)}{b^2} - \frac{a(bB - aD) \log(a + bx^2)}{2b^3} + \frac{x^2(bB - aD)}{2b^2} + \frac{Cx^3}{3b} + \frac{Dx^4}{4b}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((A*b - a*C)*x)/b^2 + ((b*B - a*D)*x^2)/(2*b^2) + (C*x^3)/(3*b) + (D*x^4)/
(4*b) - (Sqrt[a]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) - (a*(b*
B - a*D)*Log[a + b*x^2])/(2*b^3)
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.88.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

method	result	size
default	$\frac{\frac{1}{4}Db\,x^4+\frac{1}{3}bC\,x^3+\frac{1}{2}bB\,x^2-\frac{1}{2}Da\,x^2+Abx-Cax}{b^2}-\frac{a\left(\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}\right)}{b^2}$	95

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2-1/2*D*a*x^2+A*b*x-C*a*x)-a/b^2*
(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
)
```

3.88.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.14

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\left[\frac{3\,Db^2x^4+4\,Cb^2x^3-6\,(Dab-Bb^2)x^2-6\,(Cab-Ab^2)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2-2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-12\,(Cab-Ab^2)x+}{12\,b^3}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/12*(3*D*b^2*x^4 + 4*C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 - 6*(C*a*b - A*b^
2)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 12*(C*a*b
- A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/b^3, 1/12*(3*D*b^2*x^4 + 4*
C*b^2*x^3 - 6*(D*a*b - B*b^2)*x^2 + 12*(C*a*b - A*b^2)*sqrt(a/b)*arctan(b*
x*sqrt(a/b)/a) - 12*(C*a*b - A*b^2)*x + 6*(D*a^2 - B*a*b)*log(b*x^2 + a))/
b^3]
```

3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(97) = 194$.

Time = 0.48 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.21

$$\begin{aligned}
&\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx \\
&= \frac{Cx^3}{3b}+\frac{Dx^4}{4b}+x^2\left(\frac{B}{2b}-\frac{Da}{2b^2}\right)+x\left(\frac{A}{b}-\frac{Ca}{b^2}\right)+\left(\frac{a(-Bb+Da)}{2b^3}\right. \\
&\quad\left.-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right) \\
&\quad+\left(\frac{a(-Bb+Da)}{2b^3}\right. \\
&\quad\left.+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)\log\left(x+\frac{Bab-Da^2+2b^3\left(\frac{a(-Bb+Da)}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+Ca)}{2b^6}\right)}{-Ab^2+Cab}\right)
\end{aligned}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**3/(3*b) + D*x**4/(4*b) + x**2*(B/(2*b) - D*a/(2*b**2)) + x*(A/b - C*a
/b**2) + (a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6))*l
og(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*b + D*a)/(2*b**3) - sqrt(-a*b**7)*(
-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b)) + (a*(-B*b + D*a)/(2*b**3) + sqr
t(-a*b**7)*(-A*b + C*a)/(2*b**6))*log(x + (B*a*b - D*a**2 + 2*b**3*(a*(-B*
b + D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + C*a)/(2*b**6)))/(-A*b**2 + C*a*b
))
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{3\,Dbx^4+4\,Cbx^3-6\,(Da-Bb)x^2-12\,(Ca-Ab)x}{12\,b^2} + \frac{(Da^2-Bab)\log(bx^2+a)}{2\,b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/12*(3*D*b*x^4 +
4*C*b*x^3 - 6*(D*a - B*b)*x^2 - 12*(C*a - A*b)*x)/b^2 + 1/2*(D*a^2 - B*a*b
)*log(b*x^2 + a)/b^3
```

3.88.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.01

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{(Ca^2-Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{(Da^2-Bab)\log(bx^2+a)}{2\,b^3} + \frac{3\,Db^3x^4+4\,Cb^3x^3-6\,Dab^2x^2+6\,Bb^3x^2-12\,Cab^2x+12\,Ab^3x}{12\,b^4}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
(C*a^2 - A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*a^2 - B*a*b
)*log(b*x^2 + a)/b^3 + 1/12*(3*D*b^3*x^4 + 4*C*b^3*x^3 - 6*D*a*b^2*x^2 + 6
*B*b^3*x^2 - 12*C*a*b^2*x + 12*A*b^3*x)/b^4
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx=\int \frac{x^2\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.89 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx$

3.89.1 Optimal result

Integrand size = 26, antiderivative size = 92

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{(bB-aD)x}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b} - \frac{\sqrt{a}(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}+\frac{(Ab-aC)\log(a+bx^2)}{2b^2}$$

output
```
(B*b-D*a)*x/b^2+1/2*C*x^2/b+1/3*D*x^3/b+1/2*(A*b-C*a)*ln(b*x^2+a)/b^2-(B*b
-D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2} dx = \frac{\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}} + \frac{x(6bB-6aD+bx(3C+2Dx))+3(Ab-aC)\log(a+bx^2)}{6b^2}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
(Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (x*(6*b*B -
 6*a*D + b*x*(3*C + 2*D*x)) + 3*(A*b - a*C)*Log[a + b*x^2])/(6*b^2)
```

3.89.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(-\frac{a(bB-aD)-bx(Ab-aC)}{b^2(a+bx^2)}+\frac{bB-aD}{b^2}+\frac{Cx}{b}+\frac{Dx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(Ab-aC)\log(a+bx^2)}{2b^2}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{b^{5/2}}+\frac{x(bB-aD)}{b^2}+\frac{Cx^2}{2b}+\frac{Dx^3}{3b}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2),x]
```

output
```
((b*B - a*D)*x)/b^2 + (C*x^2)/(2*b) + (D*x^3)/(3*b) - (Sqrt[a]*(b*B - a*D)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + ((A*b - a*C)*Log[a + b*x^2])/(2*b^
2)
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.89.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\frac{1}{3}Db\,x^{3}+\frac{1}{2}bC\,x^{2}+bBx-Dax}{b^{2}}+\frac{\frac{\left(b^{2}A-Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-abB+Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b^{2}}$	85

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/3*D*b*x^3+1/2*b*C*x^2+b*B*x-D*a*x)+1/b^2*(1/2*(A*b^2-C*a*b)/b*ln(
b*x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.89.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx$$
$$=\left[\frac{2\,Dbx^3+3\,Cbx^2+3\,(Da-Bb)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)-6\,(Da-Bb)x-3\,(Ca-Ab)\log\left(bx^2+a\right)}{6\,b^2}\right.$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*D*b*x^3 + 3*C*b*x^2 + 3*(D*a - B*b)*sqrt(-a/b)*log((b*x^2 + 2*b*x*
sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 +
 a))/b^2, 1/6*(2*D*b*x^3 + 3*C*b*x^2 + 6*(D*a - B*b)*sqrt(a/b)*arctan(b*x*
sqrt(a/b)/a) - 6*(D*a - B*b)*x - 3*(C*a - A*b)*log(b*x^2 + a))/b^2]
```

3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(80) = 160.

Time = 0.45 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.29

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \frac{Cx^2}{2b}+\frac{Dx^3}{3b}+x\left(\frac{B}{b}-\frac{Da}{b^2}\right)+\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}-\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)+\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)\log\left(x+\frac{-Ab+Ca+2b^2\left(-\frac{-Ab+Ca}{2b^2}+\frac{\sqrt{-ab^5}(-Bb+Da)}{2b^5}\right)}{-Bb+Da}\right)$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x**2/(2*b) + D*x**3/(3*b) + x*(B/b - D*a/b**2) + (-(-A*b + C*a)/(2*b**2)
 - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-A*b + C*a + 2*b**2*(-(-A
*b + C*a)/(2*b**2) - sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5)))/(-B*b + D*a)) +
 (-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2*b**5))*log(x + (-
A*b + C*a + 2*b**2*(-(-A*b + C*a)/(2*b**2) + sqrt(-a*b**5)*(-B*b + D*a)/(2
*b**5)))/(-B*b + D*a))
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log\left(bx^2+a\right)}{2\,b^2}+\frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{2\,Dbx^3+3\,Cbx^2-6\,(Da-Bb)x}{6\,b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(D*a - B*b)*x)/b^2
```

3.89.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.96

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = -\frac{(Ca-Ab)\log(bx^2+a)}{2\,b^2} + \frac{(Da^2-Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\,Db^2x^3+3\,Cb^2x^2-6\,Dabx+6\,Bb^2x}{6\,b^3}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-1/2*(C*a - A*b)*log(b*x^2 + a)/b^2 + (D*a^2 - B*a*b)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^2) + 1/6*(2*D*b^2*x^3 + 3*C*b^2*x^2 - 6*D*a*b*x + 6*B*b^2*x
)/b^3
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{a+bx^2}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{b\,x^2+a}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2),x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2), x)
```

3.90 $\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$

3.90.1 Optimal result

Integrand size = 25, antiderivative size = 73

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx=\frac{Cx}{b}+\frac{Dx^2}{2b}+\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log(a+bx^2)}{2b^2}$$

output
```
C*x/b+1/2*D*x^2/b+1/2*(B*b-D*a)*ln(b*x^2+a)/b^2+(A*b-C*a)*arctan(x*b^(1/2)
/a^(1/2))/b^(3/2)/a^(1/2)
```

3.90.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx=\frac{bx(2C+Dx)+\frac{2\sqrt{b}(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-aD)\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(b*x*(2*C + D*x) + (2*Sqrt[b]*(A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqr
t[a] + (b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

3.90.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}dx$$
$$\Big\downarrow \text{2341}$$
$$\int \left(\frac{x(bB-aD)-aC+Ab}{b\left(a+bx^2\right)}+\frac{C}{b}+\frac{Dx}{b}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{(bB-aD)\log\left(a+bx^2\right)}{2b^2}+\frac{Cx}{b}+\frac{Dx^2}{2b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2),x]
```

output
```
(C*x)/b + (D*x^2)/(2*b) + ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[
a]*b^(3/2)) + ((b*B - a*D)*Log[a + b*x^2])/(2*b^2)
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.90.4 Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b}+\frac{\frac{(Bb-Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{b}$	65

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/2*D*x^2+C*x)+1/b*(1/2*(B*b-D*a)/b*ln(b*x^2+a)+(A*b-C*a)/(a*b)^(1/2)
*arctan(b*x/(a*b)^(1/2)))
```

3.90.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx$$
$$=\left[\frac{Dabx^2+2\,Cabx+(Ca-Ab)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-(Da^2-Bab)\log\left(bx^2+a\right)}{2\,ab^2}, \frac{Dabx^2+2\,Cabx}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(D*a*b*x^2 + 2*C*a*b*x + (C*a - A*b)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) - (D*a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2), 1/2*(D
*a*b*x^2 + 2*C*a*b*x - 2*(C*a - A*b)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*
a^2 - B*a*b)*log(b*x^2 + a))/(a*b^2)]
```

3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(65) = 130$.

Time = 0.42 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{Cx}{b}+\frac{Dx^2}{2b}+\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}-\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)+\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)\log\left(x+\frac{Bab-Da^2-2ab^2\left(-\frac{-Bb+Da}{2b^2}+\frac{\sqrt{-ab^5}(-Ab+Ca)}{2ab^4}\right)}{-Ab^2+Cab}\right)$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a),x)
```

output
```
C*x/b + D*x**2/(2*b) + (-(-B*b + D*a)/(2*b**2) - sqrt(-a*b**5)*(-A*b + C*a
)/(2*a*b**4))*log(x + (B*a*b - D*a**2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) -
 sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4)))/(-A*b**2 + C*a*b)) + (-(-B*b + D*
a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b**4))*log(x + (B*a*b - D*a*
*2 - 2*a*b**2*(-(-B*b + D*a)/(2*b**2) + sqrt(-a*b**5)*(-A*b + C*a)/(2*a*b*
*4)))/(-A*b**2 + C*a*b))
```

3.90.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{Dx^2+2\,Cx}{2\,b}-\frac{(Da-Bb)\log\left(bx^2+a\right)}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2*(D*x^2 + 2*C*x)/b -
 1/2*(D*a - B*b)*log(b*x^2 + a)/b^2
```

3.90.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = -\frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b} - \frac{(Da-Bb)\log(bx^2+a)}{2\,b^2} + \frac{Dbx^2+2\,Cbx}{2\,b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) - 1/2*(D*a - B*b)*log(b*x
^2 + a)/b^2 + 1/2*(D*b*x^2 + 2*C*b*x)/b^2
```

3.90.9 Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx+Cx^2+Dx^3}{a+bx^2}\,dx = \frac{B\,\ln(b\,x^2+a)}{2\,b} - \frac{\left(a\,\ln(b\,x^2+a)-b\,x^2\right)\,D}{2\,b^2} + \frac{C\,x}{b} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}} - \frac{C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2),x)
```

output
```
(B*log(a + b*x^2))/(2*b) - ((a*log(a + b*x^2) - b*x^2)*D)/(2*b^2) + (C*x)/
b + (A*atan((b^(1/2)*x)/a^(1/2)))/(a^(1/2)*b^(1/2)) - (C*a^(1/2)*atan((b^(
1/2)*x)/a^(1/2)))/b^(3/2)
```

3.91 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx$

3.91.1 Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}+\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}-\frac{(Ab-aC)\log(a+bx^2)}{2ab}$$

output
```
D*x/b+A*ln(x)/a-1/2*(A*b-C*a)*ln(b*x^2+a)/a/b+(B*b-D*a)*arctan(x*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}-\frac{(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{A\log(x)}{a}+\frac{(-Ab+aC)\log(a+bx^2)}{2ab}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

output
```
(D*x)/b - ((-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) +
 (A*Log[x])/a + ((-(A*b) + a*C)*Log[a + b*x^2])/(2*a*b)
```

3.91.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}dx$$

$$\Big\downarrow \ 2333$$

$$\int \left(\frac{a(bB-aD)-bx(Ab-aC)}{ab\left(a+bx^2\right)}+\frac{A}{ax}+\frac{D}{b}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(Ab-aC)\log\left(a+bx^2\right)}{2ab}+\frac{A\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{\sqrt{a}b^{3/2}}+\frac{Dx}{b}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)),x]
```

output
```
(D*x)/b + ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (A
*Log[x])/a - ((A*b - a*C)*Log[a + b*x^2])/(2*a*b)
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.91.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

method	result	size
default	$\frac{Dx}{b}+\frac{A\ln\left(x\right)}{a}+\frac{\frac{\left(-b^{2}A+Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(abB-Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{ba}$	73

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
D*x/b+A*ln(x)/a+1/b/a*(1/2*(-A*b^2+C*a*b)/b*ln(b*x^2+a)+(B*a*b-D*a^2)/(a*b
)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.19

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{2\,Dabx+2\,Ab^2\log\left(x\right)-\left(Da-Bb\right)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+\left(Cab-Ab^2\right)\log\left(bx^2+a\right)}{2\,ab^2},\ \frac{2\,Dabx+2}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*D*a*b*x + 2*A*b^2*log(x) - (D*a - B*b)*sqrt(-a*b)*log((b*x^2 + 2*s
qrt(-a*b)*x - a)/(b*x^2 + a)) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2), 1
/2*(2*D*a*b*x + 2*A*b^2*log(x) - 2*(D*a - B*b)*sqrt(a*b)*arctan(sqrt(a*b)*
x/a) + (C*a*b - A*b^2)*log(b*x^2 + a))/(a*b^2)]
```

3.91.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a),x)
```

output
```
Timed out
```

3.91.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}+\frac{A\log(x)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{(Ca-Ab)\log(bx^2+a)}{2\,ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
D*x/b + A*log(x)/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b) + 1/2
*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

3.91.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)}\,dx=\frac{Dx}{b}+\frac{A\log(|x|)}{a}-\frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b}+\frac{(Ca-Ab)\log(bx^2+a)}{2\,ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
D*x/b + A*log(abs(x))/a - (D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
+ 1/2*(C*a - A*b)*log(b*x^2 + a)/(a*b)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\,(a+bx^2)}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,\left(b\,x^2+a\right)}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)), x)
```

3.92 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx$

3.92.1 Optimal result

Integrand size = 28, antiderivative size = 76

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = -\frac{A}{ax}-\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}$$

output
```
-A/a/x+B*ln(x)/a-1/2*(B*b-D*a)*ln(b*x^2+a)/a/b-(A*b-C*a)*arctan(x*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)
```

3.92.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = -\frac{A}{ax}+\frac{(-Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}+\frac{B\log(x)}{a}+\frac{(-bB+aD)\log\left(a+bx^2\right)}{2ab}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) + ((-(A*b) + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]
) + (B*Log[x])/a + ((-(b*B) + a*D)*Log[a + b*x^2])/(2*a*b)
```

3.92.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{-x(bB-aD)+aC-Ab}{a\left(a+bx^2\right)}+\frac{A}{ax^2}+\frac{B}{ax}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{A}{ax}-\frac{(bB-aD)\log\left(a+bx^2\right)}{2ab}+\frac{B\log(x)}{a}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)),x]
```

output
```
-(A/(a*x)) - ((A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) +
 (B*Log[x])/a - ((b*B - a*D)*Log[a + b*x^2])/(2*a*b)
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.92.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{A}{ax}+\frac{B\ln(x)}{a}+\frac{\frac{(-Bb+Da)\ln\left(b\,x^2+a\right)}{2b}+\frac{(-Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a}$	67

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-A/a/x+B*ln(x)/a+1/a*(1/2*(-B*b+D*a)/b*ln(b*x^2+a)+(-A*b+C*a)/(a*b)^(1/2)*
arctan(b*x/(a*b)^(1/2)))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{2\,Babx\log\left(x\right)+(Ca-Ab)\sqrt{-ab}x\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)-2\,Aab+(Da^2-Bab)x\log\left(bx^2+a\right)}{2\,a^2bx},\ \frac{2\,Babx\,\mathrm{l}}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/2*(2*B*a*b*x*log(x) + (C*a - A*b)*sqrt(-a*b)*x*log((b*x^2 + 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b
*x), 1/2*(2*B*a*b*x*log(x) + 2*(C*a - A*b)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/
a) - 2*A*a*b + (D*a^2 - B*a*b)*x*log(b*x^2 + a))/(a^2*b*x)]
```

3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a),x)
```

output
```
Timed out
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = \frac{B\log(x)}{a} + \frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{(Da-Bb)\log(bx^2+a)}{2\,ab} - \frac{A}{ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
B*log(x)/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D*a -
B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)}\,dx = \frac{B\log(|x|)}{a} + \frac{(Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{(Da-Bb)\log(bx^2+a)}{2\,ab} - \frac{A}{ax}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
B*log(abs(x))/a + (C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*(D
*a - B*b)*log(b*x^2 + a)/(a*b) - A/(a*x)
```

3.92.9 Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)}\,dx=\frac{\ln\left(b\,x^2+a\right)\,D}{2\,b}-\frac{A}{a\,x}-\frac{B\left(\ln\left(b\,x^2+a\right)-2\,\ln\left(x\right)\right)}{2\,a}-\frac{A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)),x)
```

output
```
(log(a + b*x^2)*D)/(2*b) - A/(a*x) - (B*(log(a + b*x^2) - 2*log(x)))/(2*a)
 - (A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2) + (C*atan((b^(1/2)*x)/a^(
1/2)))/(a^(1/2)*b^(1/2))
```

3.93 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$

3.93.1 Optimal result

Integrand size = 28, antiderivative size = 92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = -\frac{A}{2ax^2}-\frac{B}{ax}-\frac{(bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{(Ab-aC)\log(x)}{a^2}+\frac{(Ab-aC)\log(a+bx^2)}{2a^2}$$

output
```
-1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/2*(A*b-C*a)*ln(b*x^2+a)/a^2-(B*b-
D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^(1/2)
```

3.93.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{2\sqrt{a}(-bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-Ab+aC)\log(x)+(Ab-aC)\log(a+bx^2)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

output
```
(-((a*A)/x^2) - (2*a*B)/x + (2*Sqrt[a]*(-(b*B) + a*D)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/Sqrt[b] + 2*(-(A*b) + a*C)*Log[x] + (A*b - a*C)*Log[a + b*x^2])/(
2*a^2)
```

3.93.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{bx(Ab-aC)-a(bB-aD)}{a^2\left(a+bx^2\right)}+\frac{aC-Ab}{a^2x}+\frac{A}{ax^3}+\frac{B}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(Ab-aC)\log\left(a+bx^2\right)}{2a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B}{ax}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*A/(a*x^2) - B/(a*x) - ((b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3
/2)*Sqrt[b]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2])/(2*
a^2)
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.93.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{A}{2a\,x^{2}}-\frac{B}{ax}+\frac{(-Ab+Ca)\ln(x)}{a^{2}}+\frac{\frac{\left(b^{2}A-Cab\right)\ln\left(b\,x^{2}+a\right)}{2b}+\frac{\left(-abB+Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{a^{2}}$	89

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a/x^2-B/a/x+1/a^2*(-A*b+C*a)*ln(x)+1/a^2*(1/2*(A*b^2-C*a*b)/b*ln(b*
x^2+a)+(-B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.93.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx$$
$$=\left[-\frac{(Da-Bb)\sqrt{-ab}x^2\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,Babx+(Cab-Ab^2)x^2\log\left(bx^2+a\right)-2\,(Cab-Ab^2)x^2\log}{2\,a^2bx^2}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/2*((D*a - B*b)*sqrt(-a*b)*x^2*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2
+ a)) + 2*B*a*b*x + (C*a*b - A*b^2)*x^2*log(b*x^2 + a) - 2*(C*a*b - A*b^2)
*x^2*log(x) + A*a*b)/(a^2*b*x^2), 1/2*(2*(D*a - B*b)*sqrt(a*b)*x^2*arctan(
sqrt(a*b)*x/a) - 2*B*a*b*x - (C*a*b - A*b^2)*x^2*log(b*x^2 + a) + 2*(C*a*b
 - A*b^2)*x^2*log(x) - A*a*b)/(a^2*b*x^2)]
```

3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a),x)
```

output
```
Timed out
```

3.93.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2a^2} + \frac{(Ca-Ab)\log(x)}{a^2} - \frac{2Bx+A}{2ax^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
(D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^
2 + a)/a^2 + (C*a - A*b)*log(x)/a^2 - 1/2*(2*B*x + A)/(a*x^2)
```

3.93.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)}\,dx = \frac{(Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{(Ca-Ab)\log(bx^2+a)}{2a^2} + \frac{(Ca-Ab)\log(|x|)}{a^2} - \frac{2Bax+Aa}{2a^2x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
(D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - 1/2*(C*a - A*b)*log(b*x^
2 + a)/a^2 + (C*a - A*b)*log(abs(x))/a^2 - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

3.93.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\,(a+bx^2)}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,D}{\sqrt{a}\,\sqrt{b}} - \frac{B}{a\,x} - \frac{C\,(\ln\left(b\,x^2+a\right) - 2\,\ln\left(x\right))}{2\,a} - \frac{A}{2\,a\,x^2} + \frac{A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^2} - \frac{A\,b\,\ln\left(x\right)}{a^2} - \frac{B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*D)/(a^(1/2)*b^(1/2)) - B/(a*x) - (C*(log(a + b*
x^2) - 2*log(x)))/(2*a) - A/(2*a*x^2) + (A*b*log(a + b*x^2))/(2*a^2) - (A*
b*log(x))/a^2 - (B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/a^(3/2)
```

3.94 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

3.94.1 Optimal result

Integrand size = 28, antiderivative size = 176

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(3Ab-5aC)x}{2b^3}+\frac{(2bB-3aD)x^2}{2b^3}-\frac{(3Ab-5aC)x^3}{6ab^2} + \frac{Dx^4}{4b^2}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} - \frac{\sqrt{a}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}} - \frac{a(2bB-3aD)\log(a+bx^2)}{2b^4}$$

output
```
1/2*(3*A*b-5*C*a)*x/b^3+1/2*(2*B*b-3*D*a)*x^2/b^3-1/6*(3*A*b-5*C*a)*x^3/a/
b^2+1/4*D*x^4/b^2-1/2*x^4*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)-1/2*a*(2
*B*b-3*D*a)*ln(b*x^2+a)/b^4-1/2*(3*A*b-5*C*a)*arctan(x*b^(1/2)/a^(1/2))*a^
(1/2)/b^(7/2)
```

3.94.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{12b(Ab-2aC)x+6b(bB-2aD)x^2+4b^2Cx^3+3b^2Dx^4+\frac{6a(a^2D+Ab^2x-ab(B+Cx))}{a+bx^2}+6\sqrt{a}\sqrt{b}(-3Ab+5aC}{12b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(12*b*(A*b - 2*a*C)*x + 6*b*(b*B - 2*a*D)*x^2 + 4*b^2*C*x^3 + 3*b^2*D*x^4
+ (6*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2) + 6*Sqrt[a]*Sqrt[b]*
(-3*A*b + 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]] + 6*a*(-2*b*B + 3*a*D)*Log[a
+ b*x^2])/(12*b^4)
```

3.94.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$\Big\downarrow \text{2335}$$
$$-\frac{\int -\frac{x^3\left(2aDx^2-(3Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)}$$
$$\Big\downarrow \text{25}$$
$$\frac{\int \frac{x^3\left(2aDx^2-(3Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)}$$
$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2aDx^3}{b}-\frac{(3Ab-5aC)x^2}{b}+\frac{2a(2bB-3aD)x}{b^2}+\frac{a(3Ab-5aC)}{b^2}-\frac{(3Ab-5aC)a^2+2(2bB-3aD)xa^2}{b^2(bx^2+a)}\right)dx}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$\big\downarrow$ 2009

$$\frac{-\frac{a^{3/2}(3Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{5/2}}-\frac{a^2(2bB-3aD)\log(a+bx^2)}{b^3}+\frac{ax(3Ab-5aC)}{b^2}-\frac{x^3(3Ab-5aC)}{3b}+\frac{ax^2(2bB-3aD)}{b^2}+\frac{aDx^4}{2b}}{2ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((a*(3*A*
b - 5*a*C)*x)/b^2 + (a*(2*b*B - 3*a*D)*x^2)/b^2 - ((3*A*b - 5*a*C)*x^3)/(3
*b) + (a*D*x^4)/(2*b) - (a^(3/2)*(3*A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/b^(5/2) - (a^2*(2*b*B - 3*a*D)*Log[a + b*x^2])/b^3)/(2*a*b)
```

3.94.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.94.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{1}{4} D b\,x^4+\frac{1}{3} b C\,x^3+\frac{1}{2} b B\,x^2-D a\,x^2+A b x-2 C a x}{b^3}-\frac{a\left(\frac{\left(-\frac{A b}{2}+\frac{C a}{2}\right) x+\frac{a(B b-D a)}{2 b}}{b\,x^2+a}+\frac{(4 B b-6 D a) \ln\left(b\,x^2+a\right)}{4 b}+\frac{(3 A b-5 C a) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}\right)}{b^3}$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/4*D*b*x^4+1/3*b*C*x^3+1/2*b*B*x^2-D*a*x^2+A*b*x-2*C*a*x)-a/b^3*((
(-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x^2+a)+1/4*(4*B*b-6*D*a)/b*ln(b
*x^2+a)+1/2*(3*A*b-5*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.94.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 468, normalized size of antiderivative = 2.66

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{3\,Db^3x^6+4\,Cb^3x^5-3\,(3\,Dab^2-2\,Bb^3)x^4+6\,Da^3-6\,Ba^2b-4\,(5\,Cab^2-3\,Ab^3)x^3-6\,(2\,Da^2b-Bab\ldots}{}\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 + 6*D*a^3 -
 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^2)*x^2 - 3
*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(-a/b)*log((b*x^2
 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3
*D*a^3 - 2*B*a^2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2
 + a*b^4), 1/12*(3*D*b^3*x^6 + 4*C*b^3*x^5 - 3*(3*D*a*b^2 - 2*B*b^3)*x^4 +
 6*D*a^3 - 6*B*a^2*b - 4*(5*C*a*b^2 - 3*A*b^3)*x^3 - 6*(2*D*a^2*b - B*a*b^
2)*x^2 + 6*(5*C*a^2*b - 3*A*a*b^2 + (5*C*a*b^2 - 3*A*b^3)*x^2)*sqrt(a/b)*a
rctan(b*x*sqrt(a/b)/a) - 6*(5*C*a^2*b - 3*A*a*b^2)*x + 6*(3*D*a^3 - 2*B*a^
2*b + (3*D*a^2*b - 2*B*a*b^2)*x^2)*log(b*x^2 + a))/(b^5*x^2 + a*b^4)]
```

3.94.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(160) = 320$.

Time = 1.87 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.90

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Cx^3}{3b^2}+\frac{Dx^4}{4b^2}+x^2\left(\frac{B}{2b^2}-\frac{Da}{b^3}\right)+x\left(\frac{A}{b^2}-\frac{2Ca}{b^3}\right)+\left(\frac{a(-2Bb+3Da)}{2b^4}-\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}-\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)+\left(\frac{a(-2Bb+3Da)}{2b^4}+\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)\log\left(x+\frac{4Bab-6Da^2+4b^4\left(\frac{a(-2Bb+3Da)}{2b^4}+\frac{\sqrt{-ab^9}(-3Ab+5Ca)}{4b^8}\right)}{-3Ab^2+5Cab}\right)+\frac{-Ba^2b+Da^3+x(Aab^2-Ca^2b)}{2ab^4+2b^5x^2}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x**3/(3*b**2) + D*x**4/(4*b**2) + x**2*(B/(2*b**2) - D*a/b**3) + x*(A/b*
*2 - 2*C*a/b**3) + (a*(-2*B*b + 3*D*a)/(2*b**4) - sqrt(-a*b**9)*(-3*A*b +
5*C*a)/(4*b**8))*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/
(2*b**4) - sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)
) + (a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)
)*log(x + (4*B*a*b - 6*D*a**2 + 4*b**4*(a*(-2*B*b + 3*D*a)/(2*b**4) + sqrt
(-a*b**9)*(-3*A*b + 5*C*a)/(4*b**8)))/(-3*A*b**2 + 5*C*a*b)) + (-B*a**2*b
+ D*a**3 + x*(A*a*b**2 - C*a**2*b))/(2*a*b**4 + 2*b**5*x**2)
```

3.94.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(b^5x^2+ab^4)} + \frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{3\,Dbx^4+4\,Cbx^3-6\,(2\,Da-Bb)x^2-12\,(2\,Ca-Ab)x}{12\,b^3} + \frac{(3\,Da^2-2\,Bab)\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b^2)*x)/(b^5*x^2 + a*b^4) + 1/2*(5*C
*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/12*(3*D*b*x^4 +
4*C*b*x^3 - 6*(2*D*a - B*b)*x^2 - 12*(2*C*a - A*b)*x)/b^3 + 1/2*(3*D*a^2 -
 2*B*a*b)*log(b*x^2 + a)/b^4
```

3.94.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.90

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(5\,Ca^2-3\,Aab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{(3\,Da^2-2\,Bab)\log\left(bx^2+a\right)}{2\,b^4} + \frac{Da^3-Ba^2b-(Ca^2b-Aab^2)x}{2\,(bx^2+a)b^4} + \frac{3\,Db^6x^4+4\,Cb^6x^3-12\,Dab^5x^2+6\,Bb^6x^2-24\,Cab^5x+12\,Ab^6x}{12\,b^8}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*C*a^2 - 3*A*a*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(3*D*a
^2 - 2*B*a*b)*log(b*x^2 + a)/b^4 + 1/2*(D*a^3 - B*a^2*b - (C*a^2*b - A*a*b
^2)*x)/((b*x^2 + a)*b^4) + 1/12*(3*D*b^6*x^4 + 4*C*b^6*x^3 - 12*D*a*b^5*x^
2 + 6*B*b^6*x^2 - 24*C*a*b^5*x + 12*A*b^6*x)/b^8
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{x^4\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.95 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

3.95.1 Optimal result

Integrand size = 28, antiderivative size = 154

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(3bB-5aD)x}{2b^3}-\frac{(Ab-2aC)x^2}{2ab^2}+\frac{Dx^3}{3b^2}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab(a+bx^2)}-\frac{\sqrt{a}(3bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log(a+bx^2)}{2b^3}$$

output
```
1/2*(3*B*b-5*D*a)*x/b^3-1/2*(A*b-2*C*a)*x^2/a/b^2+1/3*D*x^3/b^2-1/2*x^3*(a
*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b-2*C*a)*ln(b*x^2+a)/b^3-1/2*
(3*B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(7/2)
```

3.95.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{(bB-2aD)x}{b^3}+\frac{Cx^2}{2b^2}+\frac{Dx^3}{3b^2}+\frac{a(Ab+bBx-a(C+Dx))}{2b^3(a+bx^2)}+\frac{\sqrt{a}(-3bB+5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{7/2}}+\frac{(Ab-2aC)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
((b*B - 2*a*D)*x)/b^3 + (C*x^2)/(2*b^2) + (D*x^3)/(3*b^2) + (a*(A*b + b*B*
x - a*(C + D*x)))/(2*b^3*(a + b*x^2)) + (Sqrt[a]*(-3*b*B + 5*a*D)*ArcTan[(
Sqrt[b]*x)/Sqrt[a]])/(2*b^(7/2)) + ((A*b - 2*a*C)*Log[a + b*x^2])/(2*b^3)
```

3.95.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(A+Bx+Cx^2+Dx^3\right)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^2\left(2aDx^2-2(Ab-2aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^2\left(2aDx^2-2(Ab-2aC)x+3a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2333}$$

$$\frac{\int \left(\frac{2aDx^2}{b} - \frac{2(Ab-2aC)x}{b} + \frac{a(3bB-5aD)}{b^2} - \frac{a^2(3bB-5aD)-2ab(Ab-2aC)x}{b^2(bx^2+a)}\right) dx}{2ab} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-5aD)}{b^{5/2}} + \frac{a(Ab-2aC)\log(a+bx^2)}{b^2} - \frac{x^2(Ab-2aC)}{b} + \frac{ax(3bB-5aD)}{b^2} + \frac{2aDx^3}{3b}}{2ab} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\,(a+bx^2)}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((a*(3*b*
B - 5*a*D)*x)/b^2 - ((A*b - 2*a*C)*x^2)/b + (2*a*D*x^3)/(3*b) - (a^(3/2)*(
3*b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(5/2) + (a*(A*b - 2*a*C)*Log
[a + b*x^2])/b^2)/(2*a*b)
```

3.95.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.95.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.81

method	result	size
default	$\frac{\frac{1}{3}Db\,x^3+\frac{1}{2}bC\,x^2+bBx-2Dax}{b^3}+\frac{\frac{\left(\frac{1}{2}abB-\frac{1}{2}Da^2\right)x+\frac{a(Ab-Ca)}{2}}{b\,x^2+a}+\frac{\left(2b^2A-4Cab\right)\ln\left(b\,x^2+a\right)}{4b}+\frac{\left(-3abB+5Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^3}$	124

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/3*D*b*x^3+1/2*b*C*x^2+b*B*x-2*D*a*x)+1/b^3*(((1/2*a*b*B-1/2*D*a^2
)*x+1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(2*A*b^2-4*C*a*b)/b*ln(b*x^2+a)+1/2*(-3
*B*a*b+5*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.95.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.42

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$

$$=\left[\frac{4\,Db^2x^5+6\,Cb^2x^4+6\,Cabx^2-4\,(5\,Dab-3\,Bb^2)x^3-6\,Ca^2+6\,Aab+3\,(5\,Da^2-3\,Bab+(5\,Dab-3}{12\,(b}\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*D*b^2*x^5 + 6*C*b^2*x^4 + 6*C*a*b*x^2 - 4*(5*D*a*b - 3*B*b^2)*x^3
 - 6*C*a^2 + 6*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*b^2)*x^2)*sqr
t(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 6*(5*D*a^2 - 3*B
*a*b)*x - 6*(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4
*x^2 + a*b^3), 1/6*(2*D*b^2*x^5 + 3*C*b^2*x^4 + 3*C*a*b*x^2 - 2*(5*D*a*b -
 3*B*b^2)*x^3 - 3*C*a^2 + 3*A*a*b + 3*(5*D*a^2 - 3*B*a*b + (5*D*a*b - 3*B*
b^2)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 3*(5*D*a^2 - 3*B*a*b)*x - 3*
(2*C*a^2 - A*a*b + (2*C*a*b - A*b^2)*x^2)*log(b*x^2 + a))/(b^4*x^2 + a*b^3
)]
```

3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(134) = 268$.

Time = 1.71 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx^2}{2b^2} + \frac{Dx^3}{3b^2} + x\left(\frac{B}{b^2} - \frac{2Da}{b^3}\right) + \left(-\frac{-Ab+2Ca}{2b^3} - \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x + \frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3} - \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right) + \left(-\frac{-Ab+2Ca}{2b^3} + \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)\log\left(x + \frac{-2Ab+4Ca+4b^3\left(-\frac{-Ab+2Ca}{2b^3} + \frac{\sqrt{-ab^7}(-3Bb+5Da)}{4b^7}\right)}{-3Bb+5Da}\right) + \frac{Aab-Ca^2+x(Bab-Da^2)}{2ab^3+2b^4x^2}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x**2/(2*b**2) + D*x**3/(3*b**2) + x*(B/b**2 - 2*D*a/b**3) + (-(-A*b + 2*
C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b +
 4*C*a + 4*b**3*(-(-A*b + 2*C*a)/(2*b**3) - sqrt(-a*b**7)*(-3*B*b + 5*D*a)
/(4*b**7)))/(-3*B*b + 5*D*a)) + (-(-A*b + 2*C*a)/(2*b**3) + sqrt(-a*b**7)*
(-3*B*b + 5*D*a)/(4*b**7))*log(x + (-2*A*b + 4*C*a + 4*b**3*(-(-A*b + 2*C*
a)/(2*b**3) + sqrt(-a*b**7)*(-3*B*b + 5*D*a)/(4*b**7)))/(-3*B*b + 5*D*a))
+ (A*a*b - C*a**2 + x*(B*a*b - D*a**2))/(2*a*b**3 + 2*b**4*x**2)
```

3.95.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.82

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(b^4x^2+ab^3)} - \frac{(2\,Ca-Ab)\log(bx^2+a)}{2\,b^3} + \frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} + \frac{2\,Dbx^3+3\,Cbx^2-6\,(2\,Da-Bb)x}{6\,b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/(b^4*x^2 + a*b^3) - 1/2*(2*C*a -
A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^3) + 1/6*(2*D*b*x^3 + 3*C*b*x^2 - 6*(2*D*a - B*b)*x)/b^3
```

3.95.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.85

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(2\,Ca-Ab)\log\left(bx^2+a\right)}{2\,b^3} + \frac{(5\,Da^2-3\,Bab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^3} - \frac{Ca^2-Aab+(Da^2-Bab)x}{2\,(bx^2+a)b^3} + \frac{2\,Db^4x^3+3\,Cb^4x^2-12\,Dab^3x+6\,Bb^4x}{6\,b^6}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(2*C*a - A*b)*log(b*x^2 + a)/b^3 + 1/2*(5*D*a^2 - 3*B*a*b)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*
x^2 + a)*b^3) + 1/6*(2*D*b^4*x^3 + 3*C*b^4*x^2 - 12*D*a*b^3*x + 6*B*b^4*x)
/b^6
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \int \frac{x^3\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.96 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

3.96.1 Optimal result

Integrand size = 28, antiderivative size = 134

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(Ab-3aC)x}{2ab^2}+\frac{Dx^2}{2b^2}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)}+\frac{(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}}+\frac{(bB-2aD)\log(a+bx^2)}{2b^3}$$

output
```
-1/2*(A*b-3*C*a)*x/a/b^2+1/2*D*x^2/b^2-1/2*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a
/b/(b*x^2+a)+1/2*(B*b-2*D*a)*ln(b*x^2+a)/b^3+1/2*(A*b-3*C*a)*arctan(x*b^(1
/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

3.96.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.75

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{2bCx+bDx^2+\frac{-a^2D-Ab^2x+ab(B+Cx)}{a+bx^2}+\frac{\sqrt{b}(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+(bB-2aD)\log(a+bx^2)}{2b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(2*b*C*x + b*D*x^2 + (-(a^2*D) - A*b^2*x + a*b*(B + C*x))/(a + b*x^2) + (S
qrt[b]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + (b*B - 2*a*D)*
Log[a + b*x^2])/(2*b^3)
```

3.96.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \; 2335$$

$$-\frac{\int -\frac{x\left(2aDx^2-(Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x\left(2aDx^2-(Ab-3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{bx^2+a}dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2333$$

$$\frac{\int \left(-A+\frac{3aC}{b}+\frac{2aDx}{b}+\frac{a(Ab-3aC)+2a(bB-2aD)x}{b\left(bx^2+a\right)}\right)dx}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\sqrt{a}(Ab-3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}-x\left(A-\frac{3aC}{b}\right)+\frac{a(bB-2aD)\log\left(a+bx^2\right)}{b^2}+\frac{aDx^2}{b}}{2ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + (-((A - (
3*a*C)/b)*x) + (a*D*x^2)/b + (Sqrt[a]*(A*b - 3*a*C)*ArcTan[(Sqrt[b]*x)/Sqr
t[a]])/b^(3/2) + (a*(b*B - 2*a*D)*Log[a + b*x^2])/b^2)/(2*a*b)
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.96.4 Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{\frac{1}{2}Dx^2+Cx}{b^2}+\frac{\frac{\left(-\frac{Ab}{2}+\frac{Ca}{2}\right)x+\frac{a(Bb-Da)}{2b}}{b\,x^2+a}+\frac{(2Bb-4Da)\ln\left(b\,x^2+a\right)}{4b}+\frac{(Ab-3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	103

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*(1/2*D*x^2+C*x)+1/b^2*(((-1/2*A*b+1/2*C*a)*x+1/2*a*(B*b-D*a)/b)/(b*x
^2+a)+1/4*(2*B*b-4*D*a)/b*ln(b*x^2+a)+1/2*(A*b-3*C*a)/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2)))
```

3.96.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.66

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{2\,Dab^2x^4+4\,Cab^2x^3+2\,Da^2bx^2-2\,Da^3+2\,Ba^2b+(3\,Ca^2-Aab+(3\,Cab-Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-}{}\right.}{4\,(ab^4x^2+a^2b^3)}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a*b^2*x^4 + 4*C*a*b^2*x^3 + 2*D*a^2*b*x^2 - 2*D*a^3 + 2*B*a^2*b
+ (3*C*a^2 - A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)) + 2*(3*C*a^2*b - A*a*b^2)*x - 2*(2*D*a^3 - B*a^
2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3), 1/
2*(D*a*b^2*x^4 + 2*C*a*b^2*x^3 + D*a^2*b*x^2 - D*a^3 + B*a^2*b - (3*C*a^2
- A*a*b + (3*C*a*b - A*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*C*a^
2*b - A*a*b^2)*x - (2*D*a^3 - B*a^2*b + (2*D*a^2*b - B*a*b^2)*x^2)*log(b*x
^2 + a))/(a*b^4*x^2 + a^2*b^3)]
```

3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(116) = 232$.

Time = 1.60 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.12

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Cx}{b^2}+\frac{Dx^2}{2b^2}+\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}-\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right)+\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)\log\left(x+\frac{2Bab-4Da^2-4ab^3\left(-\frac{-Bb+2Da}{2b^3}+\frac{\sqrt{-ab^7}(-Ab+3Ca)}{4ab^6}\right)}{-Ab^2+3Cab}\right)+\frac{Bab-Da^2+x(-Ab^2+Cab)}{2ab^3+2b^4x^2}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
C*x/b**2 + D*x**2/(2*b**2) + (-(-B*b + 2*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A
*b + 3*C*a)/(4*a*b**6))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2
*D*a)/(2*b**3) - sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*
a*b)) + (-(-B*b + 2*D*a)/(2*b**3) + sqrt(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6
))*log(x + (2*B*a*b - 4*D*a**2 - 4*a*b**3*(-(-B*b + 2*D*a)/(2*b**3) + sqrt
(-a*b**7)*(-A*b + 3*C*a)/(4*a*b**6)))/(-A*b**2 + 3*C*a*b)) + (B*a*b - D*a*
*2 + x*(-A*b**2 + C*a*b))/(2*a*b**3 + 2*b**4*x**2)
```

3.96.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(b^4x^2+ab^3)} - \frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Dx^2+2\,Cx}{2\,b^2} - \frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(b^4*x^2 + a*b^3) - 1/2*(3*C*a -
A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/2*(D*x^2 + 2*C*x)/b^2 - 1/2
*(2*D*a - B*b)*log(b*x^2 + a)/b^3
```

3.96.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = -\frac{(3\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} - \frac{(2\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^3} + \frac{Db^2x^2+2\,Cb^2x}{2\,b^4} - \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(bx^2+a)b^3}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) - 1/2*(2*D*a - B*
b)*log(b*x^2 + a)/b^3 + 1/2*(D*b^2*x^2 + 2*C*b^2*x)/b^4 - 1/2*(D*a^2 - B*a
*b - (C*a*b - A*b^2)*x)/((b*x^2 + a)*b^3)
```

3.96.9 Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.13

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^2}+\frac{x^2\,D}{2\,b^2}+\frac{C\,x}{b^2}-\frac{a^2\,D}{2\,b^3\,\left(b\,x^2+a\right)} + \frac{B\,a}{2\,b^2\,\left(b\,x^2+a\right)}-\frac{A\,x}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,a\,x}{2\,\left(b^3\,x^2+a\,b^2\right)} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}-\frac{3\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,b^{5/2}}-\frac{a\,\ln\left(b\,x^2+a\right)\,D}{b^3}$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
(B*log(a + b*x^2))/(2*b^2) + (x^2*D)/(2*b^2) + (C*x)/b^2 - (a^2*D)/(2*b^3*
(a + b*x^2)) + (B*a)/(2*b^2*(a + b*x^2)) - (A*x)/(2*b*(a + b*x^2)) + (C*a*
x)/(2*(a*b^2 + b^3*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2
)) - (3*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*b^(5/2)) - (a*log(a + b*x^
2)*D)/b^3
```

3.97 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$

3.97.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2} - \frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{2ab\,(a+bx^2)} + \frac{(bB-3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{C\log(a+bx^2)}{2b^2}$$

output
```
D*x/b^2-1/2*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^
2+1/2*(B*b-3*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(5/2)/a^(1/2)
```

3.97.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.91

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2} + \frac{-Ab+aC-bBx+aDx}{2b^2\,(a+bx^2)} - \frac{(-bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{5/2}} + \frac{C\log(a+bx^2)}{2b^2}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
(D*x)/b^2 + (-(A*b) + a*C - b*B*x + a*D*x)/(2*b^2*(a + b*x^2)) - ((-(b*B)
+ 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(5/2)) + (C*Log[a + b*x
^2])/(2*b^2)
```

3.97.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2335, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{2aDx^2+2aCx+\frac{a(bB-aD)}{b}}{bx^2+a}dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2aDx^2+2aCx+\frac{a(bB-aD)}{b}}{bx^2+a}dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(\frac{2aD}{b}+\frac{a(bB-3aD)+2abCx}{b(bx^2+a)}\right)dx}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-3aD)}{b^{3/2}}+\frac{aC\log\left(a+bx^2\right)}{b}+\frac{2aDx}{b}}{2ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{2ab\left(a+bx^2\right)}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)) + ((2*a*D*x)/
b + (Sqrt[a]*(b*B - 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (a*C*Log
[a + b*x^2])/b)/(2*a*b)
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.97.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{Dx}{b^2}+\frac{\frac{\left(-\frac{Bb}{2}+\frac{Da}{2}\right)x-\frac{Ab}{2}+\frac{Ca}{2}}{b\,x^2+a}+\frac{C\ln\left(b\,x^2+a\right)}{2}+\frac{(Bb-3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{b^2}$	78

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
D*x/b^2+1/b^2*(((-1/2*B*b+1/2*D*a)*x-1/2*A*b+1/2*C*a)/(b*x^2+a)+1/2*C*ln(b
*x^2+a)+1/2*(B*b-3*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.97.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 287, normalized size of antiderivative = 2.84

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,Dab^2x^3 + 2\,Ca^2b - 2\,Aab^2 - (3\,Da^2 - Bab + (3\,Dab - Bb^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right) + 2\,(3\,Da^2b}{4\,(ab^4x^2 + a^2b^3)}\right.$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*D*a*b^2*x^3 + 2*C*a^2*b - 2*A*a*b^2 - (3*D*a^2 - B*a*b + (3*D*a*b
- B*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2
*(3*D*a^2*b - B*a*b^2)*x + 2*(C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^
4*x^2 + a^2*b^3), 1/2*(2*D*a*b^2*x^3 + C*a^2*b - A*a*b^2 - (3*D*a^2 - B*a*
b + (3*D*a*b - B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^2*b -
B*a*b^2)*x + (C*a*b^2*x^2 + C*a^2*b)*log(b*x^2 + a))/(a*b^4*x^2 + a^2*b^3)
]
```

3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(87) = 174.

Time = 1.25 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.10

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx$$
$$= \frac{Dx}{b^2} + \left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} - \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right)$$
$$+ \left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)\log\left(x + \frac{2Ca - 4ab^2\left(\frac{C}{2b^2} + \frac{\sqrt{-ab^5}(-Bb+3Da)}{4ab^5}\right)}{-Bb+3Da}\right)$$
$$+ \frac{-Ab + Ca + x(-Bb+Da)}{2ab^2 + 2b^3x^2}$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
D*x/b**2 + (C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*log(x +
(2*C*a - 4*a*b**2*(C/(2*b**2) - sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5)))/
(-B*b + 3*D*a)) + (C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b**5))*l
og(x + (2*C*a - 4*a*b**2*(C/(2*b**2) + sqrt(-a*b**5)*(-B*b + 3*D*a)/(4*a*b
**5)))/(-B*b + 3*D*a)) + (-A*b + C*a + x*(-B*b + D*a))/(2*a*b**2 + 2*b**3*
x**2)
```

3.97.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Ca-Ab+(Da-Bb)x}{2\,(b^3x^2+ab^2)} + \frac{Dx}{b^2} + \frac{C\log(bx^2+a)}{2\,b^2} - \frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(C*a - A*b + (D*a - B*b)*x)/(b^3*x^2 + a*b^2) + D*x/b^2 + 1/2*C*log(b*
x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2)
```

3.97.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx = \frac{Dx}{b^2} + \frac{C\log(bx^2+a)}{2\,b^2} - \frac{(3\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^2} + \frac{Ca-Ab+(Da-Bb)x}{2\,(bx^2+a)b^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
D*x/b^2 + 1/2*C*log(b*x^2 + a)/b^2 - 1/2*(3*D*a - B*b)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^2) + 1/2*(C*a - A*b + (D*a - B*b)*x)/((b*x^2 + a)*b^2)
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^2}\,dx=\int \frac{x\,\left(A+B\,x+C\,x^2+x^3\,D\right)}{{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2,x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^2, x)
```

3.98 $\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$

3.98.1 Optimal result

Integrand size = 25, antiderivative size = 93

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx=\frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{2ab\left(a+bx^2\right)}+\frac{(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{D\log\left(a+bx^2\right)}{2b^2}$$

output
```
1/2*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)+1/2*(A*b+C*a)*arctan(x*b^(1/2
)/a^(1/2))/a^(3/2)/b^(3/2)+1/2*D*ln(b*x^2+a)/b^2
```

3.98.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx=\frac{\frac{a^2D+Ab^2x-ab(B+Cx)}{a(a+bx^2)}+\frac{\sqrt{b}(Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}+D\log\left(a+bx^2\right)}{2b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
((a^2*D + A*b^2*x - a*b*(B + C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + a*C)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(3/2) + D*Log[a + b*x^2])/(2*b^2)
```

3.98.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {2345, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{2345}$$

$$-\frac{\int -\frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{Ab+aC+2aDx}{b(bx^2+a)}dx}{2a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{Ab+aC+2aDx}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{452}$$

$$\frac{(aC+Ab)\int \frac{1}{bx^2+a}dx+2aD\int \frac{x}{bx^2+a}dx}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2aD\int \frac{x}{bx^2+a}dx+\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{(aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{aD\log(a+bx^2)}{b}}{2ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{2ab\,(a+bx^2)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^2,x]
```

output
```
-1/2*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)) + (((A*b + a*C)*A
rcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (a*D*Log[a + b*x^2])/b)/(2
*a*b)
```

3.98.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.98.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{\frac{(Ab-Ca)x}{2ab}-\frac{Bb-Da}{2b^2}}{b\,x^2+a}+\frac{\frac{Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2ba}$	88

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
(1/2*(A*b-C*a)/a/b*x-1/2*(B*b-D*a)/b^2)/(b*x^2+a)+1/2/b/a*(D*a/b*ln(b*x^2+
a)+(A*b+C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.98.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.76

$$\int \frac{A+Bx+Cx^2+Dx^3}{\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{2\,Da^3-2\,Ba^2b-(Ca^2+Aab+(Cab+Ab^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-2\,(Ca^2b-Aab^2)x+2\,(Da^2bx^2+Da^3)\log(bx^2+a)}{4\,(a^2b^3x^2+a^3b^2)}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*D*a^3 - 2*B*a^2*b - (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(C*a^2*b - A*a*b^2)*x
 + 2*(D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a))/(a^2*b^3*x^2 + a^3*b^2), 1/2*(D
*a^3 - B*a^2*b + (C*a^2 + A*a*b + (C*a*b + A*b^2)*x^2)*sqrt(a*b)*arctan(sq
rt(a*b)*x/a) - (C*a^2*b - A*a*b^2)*x + (D*a^2*b*x^2 + D*a^3)*log(b*x^2 + a
))/(a^2*b^3*x^2 + a^3*b^2)]
```

3.98.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(78) = 156.

Time = 0.99 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.51

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}-\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right) + \left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)\log\left(x+\frac{-2Da^2+4a^2b^2\left(\frac{D}{2b^2}+\frac{\sqrt{-a^3b^5}(Ab+Ca)}{4a^3b^4}\right)}{Ab^2+Cab}\right) + \frac{-Bab+Da^2+x(Ab^2-Cab)}{2a^2b^2+2ab^3x^2}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**2,x)
```

output
```
(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))*log(x + (-2*D*a*
*2 + 4*a**2*b**2*(D/(2*b**2) - sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b**4))
)/(A*b**2 + C*a*b)) + (D/(2*b**2) + sqrt(-a**3*b**5)*(A*b + C*a)/(4*a**3*b
**4))*log(x + (-2*D*a**2 + 4*a**2*b**2*(D/(2*b**2) + sqrt(-a**3*b**5)*(A*b
 + C*a)/(4*a**3*b**4)))/(A*b**2 + C*a*b)) + (-B*a*b + D*a**2 + x*(A*b**2 -
 C*a*b))/(2*a**2*b**2 + 2*a*b**3*x**2)
```

3.98.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{Da^2-Bab-(Cab-Ab^2)x}{2\,(ab^3x^2+a^2b^2)} + \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(D*a^2 - B*a*b - (C*a*b - A*b^2)*x)/(a*b^3*x^2 + a^2*b^2) + 1/2*D*log(
b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b)
```

3.98.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{D\log(bx^2+a)}{2\,b^2} + \frac{(Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} - \frac{(Ca-Ab)x - \frac{Da^2-Bab}{b}}{2\,(bx^2+a)ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^2 + 1/2*(C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a*b) - 1/2*((C*a - A*b)*x - (D*a^2 - B*a*b)/b)/((b*x^2 + a)*a*b)
```

3.98.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^2}\,dx = \frac{\left(\ln(b\,x^2+a)+\frac{a}{b\,x^2+a}\right)\,D}{2\,b^2} - \frac{B}{2\,b\,(b\,x^2+a)} + \frac{A\,x}{2\,a\,(b\,x^2+a)} - \frac{C\,x}{2\,b\,(b\,x^2+a)} + \frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}} + \frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,\sqrt{a}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^2,x)
```

output
```
((log(a + b*x^2) + a/(a + b*x^2))*D)/(2*b^2) - B/(2*b*(a + b*x^2)) + (A*x)
/(2*a*(a + b*x^2)) - (C*x)/(2*b*(a + b*x^2)) + (A*atan((b^(1/2)*x)/a^(1/2)
))/(2*a^(3/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(1/2)*b^(3/2))
```

3.99 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2} dx$

3.99.1 Optimal result

Integrand size = 28, antiderivative size = 95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2} dx = \frac{Ab-aC+(bB-aD)x}{2ab(a+bx^2)} + \frac{(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} + \frac{A\log(x)}{a^2} - \frac{A\log(a+bx^2)}{2a^2}$$

output
```
1/2*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)+1/2*(B*b+D*a)*arctan(x*b^(1/2)/a^(
1/2))/a^(3/2)/b^(3/2)+A*ln(x)/a^2-1/2*A*ln(b*x^2+a)/a^2
```

3.99.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2} dx = \frac{\frac{a(Ab+bBx-a(C+Dx))}{b(a+bx^2)} + \frac{\sqrt{a}(bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}} + 2A\log(x) - A\log(a+bx^2)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

output
```
((a*(A*b + b*B*x - a*(C + D*x)))/(b*(a + b*x^2)) + (Sqrt[a]*(b*B + a*D)*Ar
cTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + 2*A*Log[x] - A*Log[a + b*x^2])/(2*a^2
)
```

3.99.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2336, 25, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}-\frac{\int -\frac{2Ab+(bB+aD)x}{bx\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2Ab+(bB+aD)x}{bx\left(bx^2+a\right)}dx}{2a}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{2Ab+(bB+aD)x}{x\left(bx^2+a\right)}dx}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{523}$$

$$\frac{\int \left(\frac{2Ab}{ax}+\frac{Da^2+bBa-2Ab^2x}{a\left(bx^2+a\right)}\right)dx}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{Ab\log\left(a+bx^2\right)}{a}+\frac{2Ab\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+bB)}{\sqrt{a}\sqrt{b}}}{2ab}+\frac{x(bB-aD)-aC+Ab}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^2),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(2*a*b*(a + b*x^2)) + (((b*B + a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (2*A*b*Log[x])/a - (A*b*Log[a + b*x
^2])/a)/(2*a*b)
```

3.99.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.99.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{A\ln\left(x\right)}{a^{2}}-\frac{\frac{-\frac{a\left(Bb-Da\right)x}{2b}-\frac{a\left(Ab-Ca\right)}{2b}}{b\,x^{2}+a}+\frac{bA\ln\left(b\,x^{2}+a\right)+\frac{\left(-abB-D\,a^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{2b}}{a^{2}}$	99

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^2-1/a^2*((-1/2*a*(B*b-D*a)/b*x-1/2*a*(A*b-C*a)/b)/(b*x^2+a)+1/2/
b*(b*A*ln(b*x^2+a)+(-B*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.12

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx$$
$$= \left[-\frac{2\,Ca^2b-2\,Aab^2+(Da^2+Bab+(Dab+Bb^2)x^2)\sqrt{-ab}\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(Da^2b-Bab^2)x+2}{4\,(a^2b^3x^2+a^3b^2)}\right.$$
$$-\frac{Ca^2b-Aab^2-(Da^2+Bab+(Dab+Bb^2)x^2)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Da^2b-Bab^2)x+(Ab^3x^2+Aab}{2\,(a^2b^3x^2+a^3b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(2*C*a^2*b - 2*A*a*b^2 + (D*a^2 + B*a*b + (D*a*b + B*b^2)*x^2)*sqrt(
-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(D*a^2*b - B*a*b^2
)*x + 2*(A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 4*(A*b^3*x^2 + A*a*b^2)*log
(x))/(a^2*b^3*x^2 + a^3*b^2), -1/2*(C*a^2*b - A*a*b^2 - (D*a^2 + B*a*b + (
D*a*b + B*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^2*b - B*a*b^2)*
x + (A*b^3*x^2 + A*a*b^2)*log(b*x^2 + a) - 2*(A*b^3*x^2 + A*a*b^2)*log(x))
/(a^2*b^3*x^2 + a^3*b^2)]
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{Ca-Ab+(Da-Bb)x}{2\left(ab^2x^2+a^2b\right)} - \frac{A\log\left(bx^2+a\right)}{2\,a^2} + \frac{A\log\left(x\right)}{a^2} + \frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(C*a - A*b + (D*a - B*b)*x)/(a*b^2*x^2 + a^2*b) - 1/2*A*log(b*x^2 + a
)/a^2 + A*log(x)/a^2 + 1/2*(D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*
b)
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx = -\frac{A\log\left(bx^2+a\right)}{2\,a^2} + \frac{A\log\left(|x|\right)}{a^2} + \frac{(Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}ab} - \frac{Ca^2-Aab+(Da^2-Bab)x}{2\left(bx^2+a\right)a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*A*log(b*x^2 + a)/a^2 + A*log(abs(x))/a^2 + 1/2*(D*a + B*b)*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*a*b) - 1/2*(C*a^2 - A*a*b + (D*a^2 - B*a*b)*x)/((b*
x^2 + a)*a^2*b)
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^2}\,dx=\int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,{\left(b\,x^2+a\right)}^2}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^2), x)
```

3.100 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx$

3.100.1 Optimal result

Integrand size = 28, antiderivative size = 110

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{A}{a^2x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{2ab\left(a+bx^2\right)}-\frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log\left(a+bx^2\right)}{2a^2}$$

output
```
-A/a^2/x+1/2*(B*b-D*a-b*(A*b/a-C)*x)/a/b/(b*x^2+a)+B*ln(x)/a^2-1/2*B*ln(b*
x^2+a)/a^2-1/2*(3*A*b-C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(1/2)
```

3.100.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx=-\frac{A}{a^2x}+\frac{abB-a^2D-Ab^2x+abCx}{2a^2b\left(a+bx^2\right)}+\frac{(-3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}+\frac{B\log(x)}{a^2}-\frac{B\log\left(a+bx^2\right)}{2a^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

output
```
-(A/(a^2*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(2*a^2*b*(a + b*x^2)) +
 ((-3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(5/2)*Sqrt[b]) + (B*Log
[x])/a^2 - (B*Log[a + b*x^2])/(2*a^2)
```

3.100.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2 \left(a+bx^2\right)^2} \, dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}-\frac{\int -\frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+2Bx+2A}{x^2\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\left(\left(\frac{Ab}{a}-C\right)x^2\right)+2Bx+2A}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{ax^2}+\frac{2B}{ax}+\frac{-3Ab-2Bxb+aC}{a\left(bx^2+a\right)}\right)dx}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(3Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{2A}{ax}-\frac{B\log\left(a+bx^2\right)}{a}+\frac{2B\log(x)}{a}}{2a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{2ab\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^2),x]
```

output
```
(b*B - a*D - b*((A*b)/a - C)*x)/(2*a*b*(a + b*x^2)) + ((-2*A)/(a*x) - ((3*
A*b - a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + (2*B*Log[x])/a
 - (B*Log[a + b*x^2])/a)/(2*a)
```

3.100.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.100.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A}{a^2 x}+\frac{B\ln(x)}{a^2}-\frac{\frac{\left(\frac{Ab}{2}-\frac{Ca}{2}\right)x-\frac{a(Bb-Da)}{2b}}{b\,x^2+a}+\frac{B\ln\left(b\,x^2+a\right)}{2}+\frac{(3Ab-Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2}$	96

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/a^2/x+B*ln(x)/a^2-1/a^2*(((1/2*A*b-1/2*C*a)*x-1/2*a*(B*b-D*a)/b)/(b*x^2
+a)+1/2*B*ln(b*x^2+a)+1/2*(3*A*b-C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 3.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx$$
$$= \left[-\frac{4\,Aa^2b-2\,(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{-ab}\log\left(\frac{bx^2+2\,\sqrt{-ab}x-a}{bx^2+a}\right)+2\,(\ldots}{4\,(a^3b^2x^3+a^4bx)}\right.$$
$$-\frac{2\,Aa^2b-(Ca^2b-3\,Aab^2)x^2-((Cab-3\,Ab^2)x^3+(Ca^2-3\,Aab)x)\sqrt{ab}\arctan\left(\frac{\sqrt{ab}x}{a}\right)+(Da^3-Ba^2\ldots}{2\,(a^3b^2x^3+a^4bx)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*A*a^2*b - 2*(C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 +
(C*a^2 - 3*A*a*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 +
a)) + 2*(D*a^3 - B*a^2*b)*x + 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(b*x^2 + a) -
 4*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^2*x^3 + a^4*b*x), -1/2*(2*A*a^
2*b - (C*a^2*b - 3*A*a*b^2)*x^2 - ((C*a*b - 3*A*b^2)*x^3 + (C*a^2 - 3*A*a*
b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (D*a^3 - B*a^2*b)*x + (B*a*b^2*x^3
 + B*a^2*b*x)*log(b*x^2 + a) - 2*(B*a*b^2*x^3 + B*a^2*b*x)*log(x))/(a^3*b^
2*x^3 + a^4*b*x)]
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{2\,Aab-(Cab-3\,Ab^2)x^2+(Da^2-Bab)x}{2\,(a^2b^2x^3+a^3bx)} - \frac{B\log\left(bx^2+a\right)}{2\,a^2} + \frac{B\log\left(x\right)}{a^2} + \frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*A*a*b - (C*a*b - 3*A*b^2)*x^2 + (D*a^2 - B*a*b)*x)/(a^2*b^2*x^3 +
a^3*b*x) - 1/2*B*log(b*x^2 + a)/a^2 + B*log(x)/a^2 + 1/2*(C*a - 3*A*b)*arc
tan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2)
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{B\log\left(bx^2+a\right)}{2\,a^2} + \frac{B\log\left(|x|\right)}{a^2} + \frac{(Ca-3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} + \frac{Cabx^2-3\,Ab^2x^2-Da^2x+Babx-2\,Aab}{2\,(bx^3+ax)a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*B*log(b*x^2 + a)/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a - 3*A*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^2) + 1/2*(C*a*b*x^2 - 3*A*b^2*x^2 - D*a^2*x + B
*a*b*x - 2*A*a*b)/((b*x^3 + a*x)*a^2*b)
```

3.100.9 Mupad [B] (verification not implemented)

Time = 6.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^2}\,dx = \frac{B}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{a}+\frac{3\,A\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{B\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{B\,\ln\left(x\right)}{a^2}-\frac{D}{2\,b\,\left(b\,x^2+a\right)}+\frac{C\,x}{2\,a\,\left(b\,x^2+a\right)}-\frac{3\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^2),x)
```

output
```
B/(2*a*(a + b*x^2)) - (A/a + (3*A*b*x^2)/(2*a^2))/(a*x + b*x^3) - (B*log(a
 + b*x^2))/(2*a^2) + (B*log(x))/a^2 - D/(2*b*(a + b*x^2)) + (C*x)/(2*a*(a
+ b*x^2)) - (3*A*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*a^(5/2)) + (C*atan(
(b^(1/2)*x)/a^(1/2)))/(2*a^(3/2)*b^(1/2))
```

3.101 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$

3.101.1 Optimal result

Integrand size = 28, antiderivative size = 135

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx=-\frac{A}{2a^2x^2}-\frac{B}{a^2x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{2a\left(a+bx^2\right)}-\frac{(3bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}\sqrt{b}}-\frac{(2Ab-aC)\log(x)}{a^3}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{2a^3}$$

output
```
-1/2*A/a^2/x^2-B/a^2/x+1/2*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)-(2*A*b-C*a)*
ln(x)/a^3+1/2*(2*A*b-C*a)*ln(b*x^2+a)/a^3-1/2*(3*B*b-D*a)*arctan(x*b^(1/2)
/a^(1/2))/a^(5/2)/b^(1/2)
```

3.101.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.83

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{-\frac{aA}{x^2}-\frac{2aB}{x}+\frac{a(-Ab-bBx+a(C+Dx))}{a+bx^2}+\frac{\sqrt{a}(-3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+2(-2Ab+aC)\log(x)+(2Ab-aC)\log(a+}{2a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

output
```
(-((a*A)/x^2) - (2*a*B)/x + (a*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)
 + (Sqrt[a]*(-3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 2*(-2*A*
b + a*C)*Log[x] + (2*A*b - a*C)*Log[a + b*x^2])/(2*a^3)
```

3.101.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-2\left(\frac{Ab}{a}-C\right)x^2+2Bx+2A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\left(\left(\frac{bB}{a}-D\right)x^3\right)-2\left(\frac{Ab}{a}-C\right)x^2+2Bx+2A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2A}{ax^3}+\frac{2(aC-2Ab)}{a^2x}+\frac{2b(2Ab-aC)x-a(3bB-aD)}{a^2\left(bx^2+a\right)}+\frac{2B}{ax^2}\right)dx}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3bB-aD)}{a^{3/2}\sqrt{b}}+\frac{(2Ab-aC)\log\left(a+bx^2\right)}{a^2}-\frac{2\log(x)(2Ab-aC)}{a^2}-\frac{A}{ax^2}-\frac{2B}{ax}}{2a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{2a\left(a+bx^2\right)}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^2),x]
```

output
```
-1/2*((A*b)/a - C + ((b*B)/a - D)*x)/(a*(a + b*x^2)) + (-(A/(a*x^2)) - (2*
B)/(a*x) - ((3*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) -
 (2*(2*A*b - a*C)*Log[x])/a^2 + ((2*A*b - a*C)*Log[a + b*x^2])/a^2)/(2*a)
```

3.101.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.101.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.94

method	result
default	$-\frac{A}{2a^{2}x^{2}}-\frac{B}{a^{2}x}+\frac{(-2Ab+Ca)\ln(x)}{a^{3}}+\frac{\frac{\left(-\frac{1}{2}abB+\frac{1}{2}Da^{2}\right)x-\frac{a(Ab-Ca)}{2}}{b\,x^{2}+a}+\frac{\left(4b^{2}A-2Cab\right)\ln\left(b\,x^{2}+a\right)}{4b}+\frac{\left(-3abB+Da^{2}\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^{3}}$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^2/x^2-B/a^2/x+(-2*A*b+C*a)/a^3*ln(x)+1/a^3*(((-1/2*a*b*B+1/2*D*a^
2)*x-1/2*a*(A*b-C*a))/(b*x^2+a)+1/4*(4*A*b^2-2*C*a*b)/b*ln(b*x^2+a)+1/2*(-
3*B*a*b+D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 441, normalized size of antiderivative = 3.27

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx$$

$$= \left[-\frac{4\,Ba^2bx+2\,Aa^2b-2\,(Da^2b-3\,Bab^2)x^3-2\,(Ca^2b-2\,Aab^2)x^2+((Dab-3\,Bb^2)x^4+(Da^2-3\,Bab\right.$$

$$-\frac{2\,Ba^2bx+Aa^2b-(Da^2b-3\,Bab^2)x^3-(Ca^2b-2\,Aab^2)x^2-((Dab-3\,Bb^2)x^4+(Da^2-3\,Bab)x^2)\sqrt{}}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*B*a^2*b*x + 2*A*a^2*b - 2*(D*a^2*b - 3*B*a*b^2)*x^3 - 2*(C*a^2*b
- 2*A*a*b^2)*x^2 + ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(-a
*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*((C*a*b^2 - 2*A*b^3)
*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*x^2 + a) - 4*((C*a*b^2 - 2*A*b^3)*
x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))/(a^3*b^2*x^4 + a^4*b*x^2), -1/2*(
2*B*a^2*b*x + A*a^2*b - (D*a^2*b - 3*B*a*b^2)*x^3 - (C*a^2*b - 2*A*a*b^2)*
x^2 - ((D*a*b - 3*B*b^2)*x^4 + (D*a^2 - 3*B*a*b)*x^2)*sqrt(a*b)*arctan(sqr
t(a*b)*x/a) + ((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(b*
x^2 + a) - 2*((C*a*b^2 - 2*A*b^3)*x^4 + (C*a^2*b - 2*A*a*b^2)*x^2)*log(x))
/(a^3*b^2*x^4 + a^4*b*x^2)]
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Da-3\,Bb)x^3-2\,Bax+(Ca-2\,Ab)x^2-Aa}{2\,(a^2bx^4+a^3x^2)} + \frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ca-2\,Ab)\log\left(x\right)}{a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((D*a - 3*B*b)*x^3 - 2*B*a*x + (C*a - 2*A*b)*x^2 - A*a)/(a^2*b*x^4 + a
^3*x^2) + 1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C
*a - 2*A*b)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(x)/a^3
```

3.101.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{(Da-3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2} - \frac{(Ca-2\,Ab)\log\left(bx^2+a\right)}{2\,a^3} + \frac{(Ca-2\,Ab)\log\left(|x|\right)}{a^3} - \frac{2\,Ba^2x-(Da^2-3\,Bab)x^3+Aa^2-(Ca^2-2\,Aab)x^2}{2\,(bx^2+a)a^3x^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(D*a - 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2) - 1/2*(C*a - 2*A*b
)*log(b*x^2 + a)/a^3 + (C*a - 2*A*b)*log(abs(x))/a^3 - 1/2*(2*B*a^2*x - (D
*a^2 - 3*B*a*b)*x^3 + A*a^2 - (C*a^2 - 2*A*a*b)*x^2)/((b*x^2 + a)*a^3*x^2)
```

3.101.9 Mupad [B] (verification not implemented)

Time = 6.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^2}\,dx = \frac{C}{2\,a\,\left(b\,x^2+a\right)}-\frac{\frac{A}{2\,a}+\frac{A\,b\,x^2}{a^2}}{b\,x^4+a\,x^2}-\frac{\frac{B}{a}+\frac{3\,B\,b\,x^2}{2\,a^2}}{b\,x^3+a\,x}-\frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^2}+\frac{C\,\ln\left(x\right)}{a^2}+\frac{A\,b\,\ln\left(b\,x^2+a\right)}{a^3}-\frac{2\,A\,b\,\ln\left(x\right)}{a^3}+\frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},2;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^2}-\frac{3\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{2\,a^{5/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^2),x)
```

output
```
C/(2*a*(a + b*x^2)) - (A/(2*a) + (A*b*x^2)/a^2)/(a*x^2 + b*x^4) - (B/a + (
3*B*b*x^2)/(2*a^2))/(a*x + b*x^3) - (C*log(a + b*x^2))/(2*a^2) + (C*log(x)
)/a^2 + (A*b*log(a + b*x^2))/a^3 - (2*A*b*log(x))/a^3 + (x*D*hypergeom([1/
2, 2], 3/2, -(b*x^2)/a))/a^2 - (3*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(2*
a^(5/2))
```

3.102 $\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

3.102.1 Optimal result

Integrand size = 28, antiderivative size = 185

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{3(Ab-5aC)x}{8ab^3} - \frac{(bB-3aD)x^2}{2ab^3} - \frac{x^4\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} + \frac{x^3(Ab-5aC+4(bB-2aD)x)}{8ab^2\,(a+bx^2)} + \frac{3(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{(bB-3aD)\log(a+bx^2)}{2b^4}$$

output
```
-3/8*(A*b-5*C*a)*x/a/b^3-1/2*(B*b-3*D*a)*x^2/a/b^3-1/4*x^4*(a*(B-a*D/b)-(A
*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*x^3*(A*b-5*C*a+4*(B*b-2*D*a)*x)/a/b^2/(b*x^
2+a)+1/2*(B*b-3*D*a)*ln(b*x^2+a)/b^4+3/8*(A*b-5*C*a)*arctan(x*b^(1/2)/a^(1
/2))/b^(7/2)/a^(1/2)
```

3.102.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.75

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{8bCx+4bDx^2+\frac{8abB-12a^2D-5Ab^2x+9abCx}{a+bx^2}+\frac{2a(a^2D+Ab^2x-ab(B+Cx))}{(a+bx^2)^2}+\frac{3\sqrt{b}(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}}+4(bB-3aD)}{8b^4}$$

input
```
Integrate[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(8*b*C*x + 4*b*D*x^2 + (8*a*b*B - 12*a^2*D - 5*A*b^2*x + 9*a*b*C*x)/(a + b
*x^2) + (2*a*(a^2*D + A*b^2*x - a*b*(B + C*x)))/(a + b*x^2)^2 + (3*Sqrt[b]
*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a] + 4*(b*B - 3*a*D)*Log[
a + b*x^2])/(8*b^4)
```

3.102.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2335, 25, 2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2335}$$

$$-\frac{\int -\frac{x^3\left(4aDx^2-(Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{x^3\left(4aDx^2-(Ab-5aC)x+4a\left(B-\frac{aD}{b}\right)\right)}{(bx^2+a)^2}dx}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \frac{ax^2(3(Ab-5aC)+8(bB-3aD)x)}{bx^2+a}dx}{2ab}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \frac{x^2(3(Ab-5aC)+8(bB-3aD)x)}{bx^2+a}dx}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 523$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{\int \left(3\left(A-\frac{5aC}{b}\right)+\frac{8(bB-3aD)x}{b}-\frac{3a(Ab-5aC)+8a(bB-3aD)x}{b\left(bx^2+a\right)}\right)dx}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{x^3(4x(bB-2aD)-5aC+Ab)}{2b(a+bx^2)}-\frac{-\frac{3\sqrt{a}(Ab-5aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+3x\left(A-\frac{5aC}{b}\right)-\frac{4a(bB-3aD)\log\left(a+bx^2\right)}{b^2}+\frac{4x^2(bB-3aD)}{b}}{2b}}{4ab}-\frac{x^4\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^4*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^4*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + ((x^3*(
A*b - 5*a*C + 4*(b*B - 2*a*D)*x))/(2*b*(a + b*x^2)) - (3*(A - (5*a*C)/b)*x
 + (4*(b*B - 3*a*D)*x^2)/b - (3*Sqrt[a]*(A*b - 5*a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(3/2) - (4*a*(b*B - 3*a*D)*Log[a + b*x^2])/b^2)/(2*b))/(4*a*b)
```

3.102.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.102.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.76

method	result
default	$\frac{\frac{1}{2} D x^2+C x}{b^3}+\frac{\frac{\left(-\frac{5}{8} b^2 A+\frac{9}{8} C a b\right) x^3+\left(a b B-\frac{3}{2} D a^2\right) x^2-\frac{a(3 A b-7 C a) x}{8}+\frac{a^2(3 B b-5 D a)}{4 b}}{\left(b x^2+a\right)^2}+\frac{(8 B b-24 D a) \ln \left(b x^2+a\right)}{16 b}+\frac{(3 A b-15 C a) \arctan \Big(\frac{b x}{\sqrt{a b}}}{8 \sqrt{a b}}}{b^3}$

input
```
int(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/2*D*x^2+C*x)+1/b^3*(((-5/8*b^2*A+9/8*C*a*b)*x^3+(a*b*B-3/2*D*a^2)
*x^2-1/8*a*(3*A*b-7*C*a)*x+1/4*a^2*(3*B*b-5*D*a)/b)/(b*x^2+a)^2+1/16*(8*B*
b-24*D*a)/b*ln(b*x^2+a)+1/8*(3*A*b-15*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1
/2)))
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 574, normalized size of antiderivative = 3.10

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{8\,Dab^3x^6+16\,Cab^3x^5+16\,Da^2b^2x^4-20\,Da^4+12\,Ba^3b+10\,(5\,Ca^2b^2-Aab^3)x^3-16\,(Da^3b-Ba^2b^2)}{}\right.$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*D*a*b^3*x^6 + 16*C*a*b^3*x^5 + 16*D*a^2*b^2*x^4 - 20*D*a^4 + 12*B
*a^3*b + 10*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 16*(D*a^3*b - B*a^2*b^2)*x^2 + 3
*((5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*x^
2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*C*a^3*b
 - A*a^2*b^2)*x - 8*(3*D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(
3*D*a^3*b - B*a^2*b^2)*x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a
^3*b^4), 1/8*(4*D*a*b^3*x^6 + 8*C*a*b^3*x^5 + 8*D*a^2*b^2*x^4 - 10*D*a^4 +
 6*B*a^3*b + 5*(5*C*a^2*b^2 - A*a*b^3)*x^3 - 8*(D*a^3*b - B*a^2*b^2)*x^2 -
 3*((5*C*a*b^2 - A*b^3)*x^4 + 5*C*a^3 - A*a^2*b + 2*(5*C*a^2*b - A*a*b^2)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(5*C*a^3*b - A*a^2*b^2)*x - 4*(3*
D*a^4 - B*a^3*b + (3*D*a^2*b^2 - B*a*b^3)*x^4 + 2*(3*D*a^3*b - B*a^2*b^2)*
x^2)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. 2(172) = 344.

Time = 98.08 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.93

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Cx}{b^3}+\frac{Dx^2}{2b^3}+\left(-\frac{-Bb+3Da}{2b^4} - \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)\log\left(x+\frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}-\frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right) + \left(-\frac{-Bb+3Da}{2b^4} + \frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)\log\left(x+\frac{8Bab-24Da^2-16ab^4\left(-\frac{-Bb+3Da}{2b^4}+\frac{3\sqrt{-ab^9}(-Ab+5Ca)}{16ab^8}\right)}{-3Ab^2+15Cab}\right) + \frac{6Ba^2b-10Da^3+x^3(-5Ab^3+9Cab^2)+x^2\cdot(8Bab^2-12Da^2b)+x(-3Aab^2+7Ca^2b)}{8a^2b^4+16ab^5x^2+8b^6x^4}$$

input
```
integrate(x**4*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
C*x/b**3 + D*x**2/(2*b**3) + (-(-B*b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(
-A*b + 5*C*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*
b + 3*D*a)/(2*b**4) - 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b
**2 + 15*C*a*b)) + (-(-B*b + 3*D*a)/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C
*a)/(16*a*b**8))*log(x + (8*B*a*b - 24*D*a**2 - 16*a*b**4*(-(-B*b + 3*D*a)
/(2*b**4) + 3*sqrt(-a*b**9)*(-A*b + 5*C*a)/(16*a*b**8)))/(-3*A*b**2 + 15*C
*a*b)) + (6*B*a**2*b - 10*D*a**3 + x**3*(-5*A*b**3 + 9*C*a*b**2) + x**2*(8
*B*a*b**2 - 12*D*a**2*b) + x*(-3*A*a*b**2 + 7*C*a**2*b))/(8*a**2*b**4 + 16
*a*b**5*x**2 + 8*b**6*x**4)
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.89

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(b^6x^4+2\,ab^5x^2+a^2b^4)} - \frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}+\frac{Dx^2+2\,Cx}{2\,b^3}-\frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4}$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(10*D*a^3 - 6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*
B*a*b^2)*x^2 - (7*C*a^2*b - 3*A*a*b^2)*x)/(b^6*x^4 + 2*a*b^5*x^2 + a^2*b^4
) - 3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) + 1/2*(D*x^2 +
 2*C*x)/b^3 - 1/2*(3*D*a - B*b)*log(b*x^2 + a)/b^4
```

3.102.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.85

$$
\begin{aligned}
\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx
&= -\frac{3\,(5\,Ca-Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} - \frac{(3\,Da-Bb)\log\left(bx^2+a\right)}{2\,b^4} + \frac{Db^3x^2+2\,Cb^3x}{2\,b^6} \\
&\quad - \frac{10\,Da^3-6\,Ba^2b-(9\,Cab^2-5\,Ab^3)x^3+4\,(3\,Da^2b-2\,Bab^2)x^2-(7\,Ca^2b-3\,Aab^2)x}{8\,(bx^2+a)^2b^4}
\end{aligned}
$$

input
```
integrate(x^4*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-3/8*(5*C*a - A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3) - 1/2*(3*D*a - B*
b)*log(b*x^2 + a)/b^4 + 1/2*(D*b^3*x^2 + 2*C*b^3*x)/b^6 - 1/8*(10*D*a^3 -
6*B*a^2*b - (9*C*a*b^2 - 5*A*b^3)*x^3 + 4*(3*D*a^2*b - 2*B*a*b^2)*x^2 - (7
*C*a^2*b - 3*A*a*b^2)*x)/((b*x^2 + a)^2*b^4)
```

3.102.9 Mupad [B] (verification not implemented)

Time = 6.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.25

$$\int \frac{x^4(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{7\,C\,a^2\,x}{8}+\frac{9\,C\,b\,a\,x^3}{8}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} - \frac{\frac{5\,A\,x^3}{8\,b}+\frac{3\,A\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} + \frac{\frac{3\,B\,a^2}{4\,b^3}+\frac{B\,a\,x^2}{b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{D\left(3\,a\,\ln\left(b\,x^2+a\right)-b\,x^2+\frac{3\,a^2}{b\,x^2+a}-\frac{a^3}{2\,(b\,x^2+a)^2}\right)}{2\,b^4} + \frac{B\,\ln\left(b\,x^2+a\right)}{2\,b^3} + \frac{C\,x}{b^3} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}} - \frac{15\,C\,\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,b^{7/2}}$$

input
```
int((x^4*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
((7*C*a^2*x)/8 + (9*C*a*b*x^3)/8)/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2) - ((5*
A*x^3)/(8*b) + (3*A*a*x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((3*B*a^2)
/(4*b^3) + (B*a*x^2)/b^2)/(a^2 + b^2*x^4 + 2*a*b*x^2) - (D*(3*a*log(a + b*
x^2) - b*x^2 + (3*a^2)/(a + b*x^2) - a^3/(2*(a + b*x^2)^2)))/(2*b^4) + (B*
log(a + b*x^2))/(2*b^3) + (C*x)/b^3 + (3*A*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(1/2)*b^(5/2)) - (15*C*a^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*b^(7/2))
```

3.103 $\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

3.103.1 Optimal result

Integrand size = 28, antiderivative size = 155

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{3(bB-5aD)x}{8ab^3} - \frac{x^3\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} - \frac{x^2(4aC-(3bB-7aD)x)}{8ab^2\,(a+bx^2)} + \frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{C\log\left(a+bx^2\right)}{2b^3}$$

output
```
-3/8*(B*b-5*D*a)*x/a/b^3-1/4*x^3*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2
-1/8*x^2*(4*C*a-(3*B*b-7*D*a)*x)/a/b^2/(b*x^2+a)+1/2*C*ln(b*x^2+a)/b^3+3/8
*(B*b-5*D*a)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.103.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3} + \frac{-4Ab+8aC-5bBx+9aDx}{8b^3\,(a+bx^2)} + \frac{a(Ab+bBx-a(C+Dx))}{4b^3\,(a+bx^2)^2} + \frac{3(bB-5aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{7/2}} + \frac{C\log\left(a+bx^2\right)}{2b^3}$$

input
```
Integrate[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
(D*x)/b^3 + (-4*A*b + 8*a*C - 5*b*B*x + 9*a*D*x)/(8*b^3*(a + b*x^2)) + (a*
(A*b + b*B*x - a*(C + D*x)))/(4*b^3*(a + b*x^2)^2) + (3*(b*B - 5*a*D)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqrt[a]*b^(7/2)) + (C*Log[a + b*x^2])/(2*b^3)
```

3.103.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {2335, 25, 2335, 25, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \left(A+B x+C x^2+D x^3\right)}{\left(a+b x^2\right)^3} \, dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{x^2 \left(4 a D x^2+4 a C x+3 a \left(B-\frac{a D}{b}\right)\right)}{\left(b x^2+a\right)^2}dx}{4 a b}-\frac{x^3 \left(a \left(B-\frac{a D}{b}\right)-x (A b-a C)\right)}{4 a b \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2 \left(4 a D x^2+4 a C x+3 a \left(B-\frac{a D}{b}\right)\right)}{\left(b x^2+a\right)^2}dx}{4 a b}-\frac{x^3 \left(a \left(B-\frac{a D}{b}\right)-x (A b-a C)\right)}{4 a b \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{2335}$$

$$\frac{-\frac{\int -\frac{a x (8 a C-3 (b B-5 a D) x)}{b x^2+a}dx}{2 a b}-\frac{x^2 (4 a C-x (3 b B-7 a D))}{2 b \left(a+b x^2\right)}}{4 a b}-\frac{x^3 \left(a \left(B-\frac{a D}{b}\right)-x (A b-a C)\right)}{4 a b \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a x (8 a C-3 (b B-5 a D) x)}{b x^2+a}dx}{2 a b}-\frac{x^2 (4 a C-x (3 b B-7 a D))}{2 b \left(a+b x^2\right)}}{4 a b}-\frac{x^3 \left(a \left(B-\frac{a D}{b}\right)-x (A b-a C)\right)}{4 a b \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x (8 a C-3 (b B-5 a D) x)}{b x^2+a}dx}{2 b}-\frac{x^2 (4 a C-x (3 b B-7 a D))}{2 b \left(a+b x^2\right)}}{4 a b}-\frac{x^3 \left(a \left(B-\frac{a D}{b}\right)-x (A b-a C)\right)}{4 a b \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{523}$$

$$\frac{\frac{\int \left(\frac{3a(bB-5aD)+8abCx}{b(bx^2+a)}-3\left(B-\frac{5aD}{b}\right)\right)dx}{2b}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{\frac{3\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(bB-5aD)}{b^{3/2}}-3x\left(B-\frac{5aD}{b}\right)+\frac{4aC\log\left(a+bx^2\right)}{b}}{2b}-\frac{x^2(4aC-x(3bB-7aD))}{2b(a+bx^2)}}{4ab}-\frac{x^3\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^3*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^3*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(
x^2*(4*a*C - (3*b*B - 7*a*D)*x))/(b*(a + b*x^2)) + (-3*(B - (5*a*D)/b)*x +
 (3*Sqrt[a]*(b*B - 5*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(3/2) + (4*a*C*Lo
g[a + b*x^2])/b)/(2*b))/(4*a*b)
```

3.103.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.103.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{Dx}{b^3}+\frac{\frac{\left(-\frac{5}{8}B\,b^2+\frac{9}{8}Dab\right)x^3+\left(-\frac{1}{2}b^2A+Cab\right)x^2-\frac{a(3Bb-7Da)x}{8}-\frac{abA}{4}+\frac{3C\,a^2}{4}}{\left(b\,x^2+a\right)^2}+\frac{C\ln\left(b\,x^2+a\right)}{2}+\frac{(3Bb-15Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{b^3}$	115

input
```
int(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
D/b^3*x+1/b^3*(((-5/8*B*b^2+9/8*D*a*b)*x^3+(-1/2*b^2*A+C*a*b)*x^2-1/8*a*(3
*B*b-7*D*a)*x-1/4*a*b*A+3/4*C*a^2)/(b*x^2+a)^2+1/2*C*ln(b*x^2+a)+1/8*(3*B*
b-15*D*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.103.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.10

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,Dab^3x^5+12\,Ca^3b-4\,Aa^2b^2+10\,(5\,Da^2b^2-Bab^3)x^3+8\,(2\,Ca^2b^2-Aab^3)x^2-3\,((5\,Dab^2-Bb^3)x\right.$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*D*a*b^3*x^5 + 12*C*a^3*b - 4*A*a^2*b^2 + 10*(5*D*a^2*b^2 - B*a*b
^3)*x^3 + 8*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^4 + 5*D
*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sq
rt(-a*b)*x - a)/(b*x^2 + a)) + 6*(5*D*a^3*b - B*a^2*b^2)*x + 8*(C*a*b^3*x^
4 + 2*C*a^2*b^2*x^2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2
+ a^3*b^4), 1/8*(8*D*a*b^3*x^5 + 6*C*a^3*b - 2*A*a^2*b^2 + 5*(5*D*a^2*b^2
- B*a*b^3)*x^3 + 4*(2*C*a^2*b^2 - A*a*b^3)*x^2 - 3*((5*D*a*b^2 - B*b^3)*x^
4 + 5*D*a^3 - B*a^2*b + 2*(5*D*a^2*b - B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt
(a*b)*x/a) + 3*(5*D*a^3*b - B*a^2*b^2)*x + 4*(C*a*b^3*x^4 + 2*C*a^2*b^2*x^
2 + C*a^3*b)*log(b*x^2 + a))/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4)]
```

3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(139) = 278$.

Time = 87.26 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.82

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{Dx}{b^3} + \left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} - \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)\log\left(x + \frac{8Ca - 16ab^3\left(\frac{C}{2b^3} + \frac{3\sqrt{-ab^7}(-Bb+5Da)}{16ab^7}\right)}{-3Bb+15Da}\right) + \frac{-2Aab + 6Ca^2 + x^3(-5Bb^2 + 9Dab) + x^2(-4Ab^2 + 8Cab) + x(-3Bab + 7Da^2)}{8a^2b^3 + 16ab^4x^2 + 8b^5x^4}$$

input
```
integrate(x**3*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
D*x/b**3 + (C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b**7))*log(x
 + (8*C*a - 16*a*b**3*(C/(2*b**3) - 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(16*a*b
**7)))/(-3*B*b + 15*D*a)) + (C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b + 5*D*a)/(
16*a*b**7))*log(x + (8*C*a - 16*a*b**3*(C/(2*b**3) + 3*sqrt(-a*b**7)*(-B*b
 + 5*D*a)/(16*a*b**7)))/(-3*B*b + 15*D*a)) + (-2*A*a*b + 6*C*a**2 + x**3*(
-5*B*b**2 + 9*D*a*b) + x**2*(-4*A*b**2 + 8*C*a*b) + x*(-3*B*a*b + 7*D*a**2
))/(8*a**2*b**3 + 16*a*b**4*x**2 + 8*b**5*x**4)
```

3.103.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(b^5x^4+2\,ab^4x^2+a^2b^3)} + \frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*(2*C*a*b - A*b^2)*x^2
 + (7*D*a^2 - 3*B*a*b)*x)/(b^5*x^4 + 2*a*b^4*x^2 + a^2*b^3) + D*x/b^3 + 1/
2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^3)
```

3.103.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{x^3(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{Dx}{b^3}+\frac{C\log\left(bx^2+a\right)}{2\,b^3}-\frac{3\,(5\,Da-Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^3} + \frac{(9\,Dab-5\,Bb^2)x^3+6\,Ca^2-2\,Aab+4\,(2\,Cab-Ab^2)x^2+(7\,Da^2-3\,Bab)x}{8\,(bx^2+a)^2b^3}$$

input
```
integrate(x^3*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
D*x/b^3 + 1/2*C*log(b*x^2 + a)/b^3 - 3/8*(5*D*a - B*b)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^3) + 1/8*((9*D*a*b - 5*B*b^2)*x^3 + 6*C*a^2 - 2*A*a*b + 4*
(2*C*a*b - A*b^2)*x^2 + (7*D*a^2 - 3*B*a*b)*x)/((b*x^2 + a)^2*b^3)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A + Bx + Cx^2 + Dx^3)}{(a + bx^2)^3}\,dx = \int \frac{x^3\,(A + B\,x + C\,x^2 + x^3\,D)}{(b\,x^2 + a)^3}\,dx$$

input
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int((x^3*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.104 $\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx$

3.104.1 Optimal result

Integrand size = 28, antiderivative size = 136

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = -\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} - \frac{x(Ab+3aC-2(bB-3aD)x)}{8ab^2\,(a+bx^2)} + \frac{(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}} + \frac{D\log\left(a+bx^2\right)}{2b^3}$$

output
```
-1/4*x^2*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2-1/8*x*(A*b+3*C*a-2*(B*b
-3*D*a)*x)/a/b^2/(b*x^2+a)+1/8*(A*b+3*C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/
2)/b^(5/2)+1/2*D*ln(b*x^2+a)/b^3
```

3.104.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.90

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3} dx = \frac{\frac{-2a^2D-2Ab^2x+2ab(B+Cx)}{(a+bx^2)^2} + \frac{8a^2D+Ab^2x-ab(4B+5Cx)}{a(a+bx^2)} + \frac{\sqrt{b}(Ab+3aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}} + 4D\log\left(a+bx^2\right)}{8b^3}$$

input
```
Integrate[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((-2*a^2*D - 2*A*b^2*x + 2*a*b*(B + C*x))/(a + b*x^2)^2 + (8*a^2*D + A*b^2
*x - a*b*(4*B + 5*C*x))/(a*(a + b*x^2)) + (Sqrt[b]*(A*b + 3*a*C)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/a^(3/2) + 4*D*Log[a + b*x^2])/(8*b^3)
```

3.104.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2335, 25, 2335, 25, 27, 452, 218, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(A+Bx+Cx^2+Dx^3\right)}{\left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \ 2335$$

$$-\frac{\int -\frac{x\left(4aDx^2+(Ab+3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{x\left(4aDx^2+(Ab+3aC)x+2a\left(B-\frac{aD}{b}\right)\right)}{\left(bx^2+a\right)^2}dx}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2335$$

$$\frac{-\frac{\int -\frac{a(Ab+3aC+8aDx)}{bx^2+a}dx}{2ab}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{a(Ab+3aC+8aDx)}{bx^2+a}dx}{2ab}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{Ab+3aC+8aDx}{bx^2+a}dx}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 452$$

$$\frac{\frac{(3aC+Ab)\int \frac{1}{bx^2+a}dx+8aD\int \frac{x}{bx^2+a}dx}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{8aD\int \frac{x}{bx^2+a}dx+\frac{(3aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 240$$

$$\frac{\frac{\frac{(3aC+Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}+\frac{4aD\log\left(a+bx^2\right)}{b}}{2b}-\frac{x(-2x(bB-3aD)+3aC+Ab)}{2b(a+bx^2)}}{4ab}-\frac{x^2\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x^2*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^2*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(
x*(A*b + 3*a*C - 2*(b*B - 3*a*D)*x))/(b*(a + b*x^2)) + (((A*b + 3*a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]) + (4*a*D*Log[a + b*x^2])/b)/(2
*b))/(4*a*b)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.104.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

method	result	size
default	$\frac{\frac{(Ab-5Ca)x^3}{8ab}-\frac{(Bb-2Da)x^2}{2b^2}-\frac{(Ab+3Ca)x}{8b^2}-\frac{a(Bb-3Da)}{4b^3}}{(b\,x^2+a)^2}+\frac{\frac{4Da\ln\left(b\,x^2+a\right)}{b}+\frac{(Ab+3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8a\,b^2}$	123

input
```
int(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(A*b-5*C*a)/a/b*x^3-1/2*(B*b-2*D*a)/b^2*x^2-1/8*(A*b+3*C*a)/b^2*x-1/4
*a*(B*b-3*D*a)/b^3)/(b*x^2+a)^2+1/8/a/b^2*(4*D*a/b*ln(b*x^2+a)+(A*b+3*C*a)
/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 447, normalized size of antiderivative = 3.29

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{12\,Da^4-4\,Ba^3b-2\,(5\,Ca^2b^2-Aab^3)x^3+8\,(2\,Da^3b-Ba^2b^2)x^2-((3\,Cab^2+Ab^3)x^4+3\,Ca^3+Aa^2b}{16\,(a^2b}\right.$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(12*D*a^4 - 4*B*a^3*b - 2*(5*C*a^2*b^2 - A*a*b^3)*x^3 + 8*(2*D*a^3*b
 - B*a^2*b^2)*x^2 - ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*
a^2*b + A*a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 +
 a)) - 2*(3*C*a^3*b + A*a^2*b^2)*x + 8*(D*a^2*b^2*x^4 + 2*D*a^3*b*x^2 + D*
a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), 1/8*(6*D*a^4
 - 2*B*a^3*b - (5*C*a^2*b^2 - A*a*b^3)*x^3 + 4*(2*D*a^3*b - B*a^2*b^2)*x^2
 + ((3*C*a*b^2 + A*b^3)*x^4 + 3*C*a^3 + A*a^2*b + 2*(3*C*a^2*b + A*a*b^2)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (3*C*a^3*b + A*a^2*b^2)*x + 4*(D*a^
2*b^2*x^4 + 2*D*a^3*b*x^2 + D*a^4)*log(b*x^2 + a))/(a^2*b^5*x^4 + 2*a^3*b^
4*x^2 + a^4*b^3)]
```

3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(119) = 238$.

Time = 74.79 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.24

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}-\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)\log\left(x+\frac{-8Da^2+16a^2b^3\left(\frac{D}{2b^3}+\frac{\sqrt{-a^3b^7}(Ab+3Ca)}{16a^3b^6}\right)}{Ab^2+3Cab}\right)$$
$$+\frac{-2Ba^2b+6Da^3+x^3(Ab^3-5Cab^2)+x^2(-4Bab^2+8Da^2b)+x(-Aab^2-3Ca^2b)}{8a^3b^3+16a^2b^4x^2+8ab^5x^4}$$

input
```
integrate(x**2*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6))*log(x + (-8*D
*a**2 + 16*a**2*b**3*(D/(2*b**3) - sqrt(-a**3*b**7)*(A*b + 3*C*a)/(16*a**3
*b**6)))/(A*b**2 + 3*C*a*b)) + (D/(2*b**3) + sqrt(-a**3*b**7)*(A*b + 3*C*a
)/(16*a**3*b**6))*log(x + (-8*D*a**2 + 16*a**2*b**3*(D/(2*b**3) + sqrt(-a*
*3*b**7)*(A*b + 3*C*a)/(16*a**3*b**6)))/(A*b**2 + 3*C*a*b)) + (-2*B*a**2*b
 + 6*D*a**3 + x**3*(A*b**3 - 5*C*a*b**2) + x**2*(-4*B*a*b**2 + 8*D*a**2*b)
 + x*(-A*a*b**2 - 3*C*a**2*b))/(8*a**3*b**3 + 16*a**2*b**4*x**2 + 8*a*b**5
*x**4)
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{6\,Da^3-2\,Ba^2b-(5\,Cab^2-Ab^3)x^3+4\,(2\,Da^2b-Bab^2)x^2-(3\,Ca^2b+Aab^2)x}{8\,(ab^5x^4+2\,a^2b^4x^2+a^3b^3)}$$
$$+\frac{D\log(bx^2+a)}{2\,b^3}+\frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(6*D*a^3 - 2*B*a^2*b - (5*C*a*b^2 - A*b^3)*x^3 + 4*(2*D*a^2*b - B*a*b^
2)*x^2 - (3*C*a^2*b + A*a*b^2)*x)/(a*b^5*x^4 + 2*a^2*b^4*x^2 + a^3*b^3) +
1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^2)
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \frac{D\log(bx^2+a)}{2\,b^3}+\frac{(3\,Ca+Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$
$$-\frac{(5\,Cab-Ab^2)x^3-4\,(2\,Da^2-Bab)x^2+(3\,Ca^2+Aab)x-\frac{2\,(3\,Da^3-Ba^2b)}{b}}{8\,(bx^2+a)^2ab^2}$$

input
```
integrate(x^2*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/2*D*log(b*x^2 + a)/b^3 + 1/8*(3*C*a + A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a
*b)*a*b^2) - 1/8*((5*C*a*b - A*b^2)*x^3 - 4*(2*D*a^2 - B*a*b)*x^2 + (3*C*a
^2 + A*a*b)*x - 2*(3*D*a^3 - B*a^2*b)/b)/((b*x^2 + a)^2*a*b^2)
```

3.104.9 Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.43

$$\int \frac{x^2(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{A\,x^3}{8\,a}-\frac{A\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{B\,x^2}{2\,b}+\frac{B\,a}{4\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}-\frac{\frac{5\,C\,x^3}{8\,b}+\frac{3\,C\,a\,x}{8\,b^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{D\left(\ln\left(b\,x^2+a\right)+\frac{2\,a}{b\,x^2+a}-\frac{a^2}{2\,\left(b\,x^2+a\right)^2}\right)}{2\,b^3}+\frac{A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}+\frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,\sqrt{a}\,b^{5/2}}$$

input
```
int((x^2*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
((A*x^3)/(8*a) - (A*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((B*x^2)/(2*b)
 + (B*a)/(4*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - ((5*C*x^3)/(8*b) + (3*C*a*
x)/(8*b^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + (D*(log(a + b*x^2) + (2*a)/(a +
b*x^2) - a^2/(2*(a + b*x^2)^2)))/(2*b^3) + (A*atan((b^(1/2)*x)/a^(1/2)))/(
8*a^(3/2)*b^(3/2)) + (3*C*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(1/2)*b^(5/2))
```

3.105 $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 119

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{x\left(a\left(B-\frac{aD}{b}\right)-(Ab-aC)x\right)}{4ab\,(a+bx^2)^2} - \frac{2(Ab+aC)-(bB-5aD)x}{8ab^2\,(a+bx^2)} + \frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}}$$

output
```
-1/4*x*(a*(B-a*D/b)-(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-2*A*b-2*C*a+(B*b-5*
D*a)*x)/a/b^2/(b*x^2+a)+1/8*(B*b+3*D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/
b^(5/2)
```

3.105.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.83

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{\frac{\sqrt{b}\left(b^2Bx^3-a^2(2C+3Dx)-ab\left(2A+x(B+4Cx+5Dx^2)\right)\right)}{a(a+bx^2)^2}+\frac{(bB+3aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}}}{8b^{5/2}}$$

input
```
Integrate[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
((Sqrt[b]*(b^2*B*x^3 - a^2*(2*C + 3*D*x) - a*b*(2*A + x*(B + 4*C*x + 5*D*x
^2))))/(a*(a + b*x^2)^2) + ((b*B + 3*a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(
3/2))/(8*b^(5/2))
```

3.105.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2335, 25, 2345, 25, 27, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{2335}$$

$$-\frac{\int -\frac{4aDx^2+2(Ab+aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^2}dx}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{4aDx^2+2(Ab+aC)x+\frac{a(bB-aD)}{b}}{(bx^2+a)^2}dx}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2345}$$

$$\frac{-\frac{\int -\frac{a(bB+3aD)}{b\left(bx^2+a\right)}dx}{2a}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a(bB+3aD)}{b\left(bx^2+a\right)}dx}{2a}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(3aD+bB)\int \frac{1}{bx^2+a}dx}{2b}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(3aD+bB)}{2\sqrt{a}b^{3/2}}-\frac{2(aC+Ab)-x(bB-5aD)}{2b(a+bx^2)}}{4ab}-\frac{x\left(a\left(B-\frac{aD}{b}\right)-x(Ab-aC)\right)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(x*(A + B*x + C*x^2 + D*x^3))/(a + b*x^2)^3,x]
```

output
```
-1/4*(x*(a*(B - (a*D)/b) - (A*b - a*C)*x))/(a*b*(a + b*x^2)^2) + (-1/2*(2*
(A*b + a*C) - (b*B - 5*a*D)*x)/(b*(a + b*x^2)) + ((b*B + 3*a*D)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(2*Sqrt[a]*b^(3/2)))/(4*a*b)
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.105. $\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$

3.105.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{\frac{(Bb-5Da)x^3}{8ab}-\frac{C\,x^2}{2b}-\frac{(Bb+3Da)x}{8b^2}-\frac{Ab+Ca}{4b^2}}{(b\,x^2+a)^2}+\frac{(Bb+3Da)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^2a\sqrt{ab}}$	97

input
```
int(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(B*b-5*D*a)/a/b*x^3-1/2*C*x^2/b-1/8*(B*b+3*D*a)/b^2*x-1/4*(A*b+C*a)/b
^2)/(b*x^2+a)^2+1/8*(B*b+3*D*a)/b^2/a/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.00

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Ca^2b^2x^2+4\,Ca^3b+4\,Aa^2b^2+2\,(5\,Da^2b^2-Bab^3)x^3+((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,D}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right.$$
$$-\frac{4\,Ca^2b^2x^2+2\,Ca^3b+2\,Aa^2b^2+(5\,Da^2b^2-Bab^3)x^3-((3\,Dab^2+Bb^3)x^4+3\,Da^3+Ba^2b+2\,(3\,Da^2}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*C*a^2*b^2*x^2 + 4*C*a^3*b + 4*A*a^2*b^2 + 2*(5*D*a^2*b^2 - B*a*b
^3)*x^3 + ((3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*
a*b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(
3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3), -1/8*(4
*C*a^2*b^2*x^2 + 2*C*a^3*b + 2*A*a^2*b^2 + (5*D*a^2*b^2 - B*a*b^3)*x^3 - (
(3*D*a*b^2 + B*b^3)*x^4 + 3*D*a^3 + B*a^2*b + 2*(3*D*a^2*b + B*a*b^2)*x^2)
*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + (3*D*a^3*b + B*a^2*b^2)*x)/(a^2*b^5*x^4
 + 2*a^3*b^4*x^2 + a^4*b^3)]
```

3.105.6 Sympy [A] (verification not implemented)

Time = 7.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.50

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{a^3b^5}}(Bb+3Da)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{16} + \frac{-2Aab-2Ca^2-4Cabx^2+x^3(Bb^2-5Dab)+x(-Bab-3Da^2)}{8a^3b^2+16a^2b^3x^2+8ab^4x^4}$$

input
```
integrate(x*(D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) +
x)/16 + sqrt(-1/(a**3*b**5))*(B*b + 3*D*a)*log(a**2*b**2*sqrt(-1/(a**3*b**
5)) + x)/16 + (-2*A*a*b - 2*C*a**2 - 4*C*a*b*x**2 + x**3*(B*b**2 - 5*D*a*b
) + x*(-B*a*b - 3*D*a**2))/(8*a**3*b**2 + 16*a**2*b**3*x**2 + 8*a*b**4*x**
4)
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = -\frac{4\,Cabx^2+(5\,Dab-Bb^2)x^3+2\,Ca^2+2\,Aab+(3\,Da^2+Bab)x}{8\,(ab^4x^4+2\,a^2b^3x^2+a^3b^2)} + \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*C*a*b*x^2 + (5*D*a*b - B*b^2)*x^3 + 2*C*a^2 + 2*A*a*b + (3*D*a^2 +
 B*a*b)*x)/(a*b^4*x^4 + 2*a^2*b^3*x^2 + a^3*b^2) + 1/8*(3*D*a + B*b)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \frac{(3\,Da+Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^2} - \frac{5\,Dabx^3-Bb^2x^3+4\,Cabx^2+3\,Da^2x+Babx+2\,Ca^2+2\,Aab}{8\,(bx^2+a)^2ab^2}$$

input
```
integrate(x*(D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*D*a + B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2) - 1/8*(5*D*a*b*x
^3 - B*b^2*x^3 + 4*C*a*b*x^2 + 3*D*a^2*x + B*a*b*x + 2*C*a^2 + 2*A*a*b)/((
b*x^2 + a)^2*a*b^2)
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx+Cx^2+Dx^3)}{(a+bx^2)^3}\,dx = \int \frac{x\,(A+B\,x+C\,x^2+x^3\,D)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3,x)
```

output
```
int((x*(A + B*x + C*x^2 + x^3*D))/(a + b*x^2)^3, x)
```

3.106 $\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx$

3.106.1 Optimal result

Integrand size = 25, antiderivative size = 116

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = \frac{-a\left(B-\frac{aD}{b}\right)+(Ab-aC)x}{4ab(a+bx^2)^2} - \frac{4a^2D-b(3Ab+aC)x}{8a^2b^2(a+bx^2)} + \frac{(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}$$

output
```
1/4*(-a*(B-a*D/b)+(A*b-C*a)*x)/a/b/(b*x^2+a)^2+1/8*(-4*a^2*D+b*(3*A*b+C*a)
*x)/a^2/b^2/(b*x^2+a)+1/8*(3*A*b+C*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^
(3/2)
```

3.106.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3} dx = \frac{\frac{\sqrt{a}(-2a^3D+3Ab^3x^3+ab^2x(5A+Cx^2)-a^2b(2B+x(C+4Dx)))}{(a+bx^2)^2}+\sqrt{b}(3Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^2}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
((Sqrt[a]*(-2*a^3*D + 3*A*b^3*x^3 + a*b^2*x*(5*A + C*x^2) - a^2*b*(2*B + x
*(C + 4*D*x))))/(a + b*x^2)^2 + Sqrt[b]*(3*A*b + a*C)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(8*a^(5/2)*b^2)
```

3.106.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2345, 25, 27, 454, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{ 2345}$$

$$-\frac{\int -\frac{b\left(3A+\frac{aC}{b}\right)+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{b(bx^2+a)^2}dx}{4a}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3Ab+aC+4aDx}{(bx^2+a)^2}dx}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 454}$$

$$\frac{\frac{(aC+3Ab)\int \frac{1}{bx^2+a}dx}{2a}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{(aC+3Ab)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}-\frac{4a^2D-bx(aC+3Ab)}{2ab(a+bx^2)}}{4ab}-\frac{a\left(B-\frac{aD}{b}\right)-x(Ab-aC)}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(a + b*x^2)^3,x]
```

output
```
-1/4*(a*(B - (a*D)/b) - (A*b - a*C)*x)/(a*b*(a + b*x^2)^2) + (-1/2*(4*a^2*
D - b*(3*A*b + a*C)*x)/(a*b*(a + b*x^2)) + ((3*A*b + a*C)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/(2*a^(3/2)*Sqrt[b]))/(4*a*b)
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 454
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*d
 - b*c*x)/(2*a*b*(p + 1)))*(a + b*x^2)^(p + 1), x] + Simp[c*((2*p + 3)/(2*a
*(p + 1)))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && L
tQ[p, -1] && NeQ[p, -3/2]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.106.4 Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{\frac{(3Ab+Ca)x^3}{8a^2}-\frac{Dx^2}{2b}+\frac{(5Ab-Ca)x}{8ab}-\frac{Bb+Da}{4b^2}}{(b\,x^2+a)^2}+\frac{(3Ab+Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2b\sqrt{ab}}$	98

input
```
int((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/8*(3*A*b+C*a)/a^2*x^3-1/2*D*x^2/b+1/8*(5*A*b-C*a)/a/b*x-1/4*(B*b+D*a)/b
^2)/(b*x^2+a)^2+1/8*(3*A*b+C*a)/a^2/b/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= \left[-\frac{8\,Da^3bx^2+4\,Da^4+4\,Ba^3b-2\,(Ca^2b^2+3\,Aab^3)x^3+((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+\right.}{16\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$
$$-\frac{4\,Da^3bx^2+2\,Da^4+2\,Ba^3b-(Ca^2b^2+3\,Aab^3)x^3-((Cab^2+3\,Ab^3)x^4+Ca^3+3\,Aa^2b+2\,(Ca^2b+3}{8\,(a^3b^4x^4+2\,a^4b^3x^2+a^5b^2)}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(8*D*a^3*b*x^2 + 4*D*a^4 + 4*B*a^3*b - 2*(C*a^2*b^2 + 3*A*a*b^3)*x^
3 + ((C*a*b^2 + 3*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)
*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 2*(C*a^3*
b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3*x^2 + a^5*b^2), -1/8*(4*D*a^3
*b*x^2 + 2*D*a^4 + 2*B*a^3*b - (C*a^2*b^2 + 3*A*a*b^3)*x^3 - ((C*a*b^2 + 3
*A*b^3)*x^4 + C*a^3 + 3*A*a^2*b + 2*(C*a^2*b + 3*A*a*b^2)*x^2)*sqrt(a*b)*a
rctan(sqrt(a*b)*x/a) + (C*a^3*b - 5*A*a^2*b^2)*x)/(a^3*b^4*x^4 + 2*a^4*b^3
*x^2 + a^5*b^2)]
```

3.106.6 Sympy [A] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^5b^3}}\cdot(3Ab+Ca)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{16}$$
$$+\frac{-2Ba^2b-2Da^3-4Da^2bx^2+x^3\cdot(3Ab^3+Cab^2)+x(5Aab^2-Ca^2b)}{8a^4b^2+16a^3b^3x^2+8a^2b^4x^4}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(-a**3*b*sqrt(-1/(a**5*b**3)) + x)/
16 + sqrt(-1/(a**5*b**3))*(3*A*b + C*a)*log(a**3*b*sqrt(-1/(a**5*b**3)) +
x)/16 + (-2*B*a**2*b - 2*D*a**3 - 4*D*a**2*b*x**2 + x**3*(3*A*b**3 + C*a*b
**2) + x*(5*A*a*b**2 - C*a**2*b))/(8*a**4*b**2 + 16*a**3*b**3*x**2 + 8*a**
2*b**4*x**4)
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = -\frac{4\,Da^2bx^2+2\,Da^3+2\,Ba^2b-(Cab^2+3\,Ab^3)x^3+(Ca^2b-5\,Aab^2)x}{8\,(a^2b^4x^4+2\,a^3b^3x^2+a^4b^2)} + \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(4*D*a^2*b*x^2 + 2*D*a^3 + 2*B*a^2*b - (C*a*b^2 + 3*A*b^3)*x^3 + (C*a
^2*b - 5*A*a*b^2)*x)/(a^2*b^4*x^4 + 2*a^3*b^3*x^2 + a^4*b^2) + 1/8*(C*a +
3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b)
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{(Ca+3\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b} + \frac{Cab^2x^3+3\,Ab^3x^3-4\,Da^2bx^2-Ca^2bx+5\,Aab^2x-2\,Da^3-2\,Ba^2b}{8\,(bx^2+a)^2a^2b^2}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(C*a + 3*A*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(C*a*b^2*x
^3 + 3*A*b^3*x^3 - 4*D*a^2*b*x^2 - C*a^2*b*x + 5*A*a*b^2*x - 2*D*a^3 - 2*B
*a^2*b)/((b*x^2 + a)^2*a^2*b^2)
```

3.106.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.41

$$\int \frac{A+Bx+Cx^2+Dx^3}{(a+bx^2)^3}\,dx = \frac{\frac{C\,x^3}{8\,a}-\frac{C\,x}{8\,b}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,A\,x}{8\,a}+\frac{3\,A\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{B}{4\,b\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}-\frac{\left(2\,b\,x^2+a\right)\,D}{4\,b^2\,{\left(b\,x^2+a\right)}^2} + \frac{3\,A\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}+\frac{C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{3/2}\,b^{3/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(a + b*x^2)^3,x)
```

output
```
((C*x^3)/(8*a) - (C*x)/(8*b))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*A*x)/(8*a)
 + (3*A*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - B/(4*b*(a^2 + b^2*x^
4 + 2*a*b*x^2)) - ((a + 2*b*x^2)*D)/(4*b^2*(a + b*x^2)^2) + (3*A*atan((b^(
1/2)*x)/a^(1/2)))/(8*a^(5/2)*b^(1/2)) + (C*atan((b^(1/2)*x)/a^(1/2)))/(8*a
^(3/2)*b^(3/2))
```

3.107 $\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx$

3.107.1 Optimal result

Integrand size = 28, antiderivative size = 130

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\frac{Ab-aC+(bB-aD)x}{4ab\left(a+bx^2\right)^2}+\frac{4Ab+(3bB+aD)x}{8a^2b\left(a+bx^2\right)}+\frac{(3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}}+\frac{A\log(x)}{a^3}-\frac{A\log\left(a+bx^2\right)}{2a^3}$$

output
```
1/4*(A*b-C*a+(B*b-D*a)*x)/a/b/(b*x^2+a)^2+1/8*(4*A*b+(3*B*b+D*a)*x)/a^2/b/
(b*x^2+a)+1/8*(3*B*b+D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)+A*ln(x
)/a^3-1/2*A*ln(b*x^2+a)/a^3
```

3.107.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx=\frac{\frac{a(4Ab+3bBx+aDx)}{b(a+bx^2)}+\frac{2a^2(Ab+bBx-a(C+Dx))}{b(a+bx^2)^2}+\frac{\sqrt{a}(3bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{3/2}}+8A\log(x)-4A\log\left(a+bx^2\right)}{8a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

output
```
((a*(4*A*b + 3*b*B*x + a*D*x))/(b*(a + b*x^2)) + (2*a^2*(A*b + b*B*x - a*(
C + D*x)))/(b*(a + b*x^2)^2) + (Sqrt[a]*(3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/b^(3/2) + 8*A*Log[x] - 4*A*Log[a + b*x^2])/(8*a^3)
```

3.107.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2336, 25, 27, 532, 25, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \ 2336$$

$$\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}-\frac{\int -\frac{4Ab+(3bB+aD)x}{bx\left(bx^2+a\right)^2}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{4Ab+(3bB+aD)x}{bx\left(bx^2+a\right)^2}dx}{4a}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{4Ab+(3bB+aD)x}{x\left(bx^2+a\right)^2}dx}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 532$$

$$\frac{\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}-\frac{\int -\frac{8Ab+(3bB+aD)x}{x\left(bx^2+a\right)}dx}{2a}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\int \frac{8Ab+(3bB+aD)x}{x\left(bx^2+a\right)}dx}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 523$$

$$\frac{\frac{\int \left(\frac{8Ab}{ax}+\frac{Da^2+3bBa-8Ab^2x}{a(bx^2+a)}\right)dx}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\,(a+bx^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{4Ab\log(a+bx^2)}{a}+\frac{8Ab\log(x)}{a}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(aD+3bB)}{\sqrt{a}\sqrt{b}}}{2a}+\frac{x(aD+3bB)+4Ab}{2a(a+bx^2)}}{4ab}+\frac{x(bB-aD)-aC+Ab}{4ab\,(a+bx^2)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x*(a + b*x^2)^3),x]
```

output
```
(A*b - a*C + (b*B - a*D)*x)/(4*a*b*(a + b*x^2)^2) + ((4*A*b + (3*b*B + a*D
)*x)/(2*a*(a + b*x^2)) + (((3*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqr
t[a]*Sqrt[b]) + (8*A*b*Log[x])/a - (4*A*b*Log[a + b*x^2])/a)/(2*a))/(4*a*b
)
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 532
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[x^m
*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*(Qx/x^m) + e*((2*p + 3)/x^m),
x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && ILtQ[m, 0] && LtQ[p,
 -1] && IntegerQ[2*p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.107.4 Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{A\ln(x)}{a^3}-\frac{\frac{\left(-\frac{3}{8}abB-\frac{1}{8}Da^2\right)x^3-\frac{aAb\,x^2}{2}-\frac{a^2(5Bb-Da)x}{8b}-\frac{a^2(3Ab-Ca)}{4b}}{\left(b\,x^2+a\right)^2}+\frac{4bA\ln\left(b\,x^2+a\right)+\frac{\left(-3abB-Da^2\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}}}{8b}}{a^3}$	130

input
```
int((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
A*ln(x)/a^3-1/a^3*(((-3/8*a*b*B-1/8*D*a^2)*x^3-1/2*a*A*b*x^2-1/8*a^2*(5*B*
b-D*a)/b*x-1/4*a^2*(3*A*b-C*a)/b)/(b*x^2+a)^2+1/8/b*(4*b*A*ln(b*x^2+a)+(-3
*B*a*b-D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))))
```

3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(114) = 228.

Time = 0.32 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.75

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx$$
$$=\left[\frac{8\,Aab^3x^2-4\,Ca^3b+12\,Aa^2b^2+2\,(Da^2b^2+3\,Bab^3)x^3-((Dab^2+3\,Bb^3)x^4+Da^3+3\,Ba^2b+2\,(Da^2b}{}\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*A*a*b^3*x^2 - 4*C*a^3*b + 12*A*a^2*b^2 + 2*(D*a^2*b^2 + 3*B*a*b^3
)*x^3 - ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*b + 2*(D*a^2*b + 3*B*a*
b^2)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(D*
a^3*b - 5*B*a^2*b^2)*x - 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x
^2 + a) + 16*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4
+ 2*a^4*b^3*x^2 + a^5*b^2), 1/8*(4*A*a*b^3*x^2 - 2*C*a^3*b + 6*A*a^2*b^2 +
 (D*a^2*b^2 + 3*B*a*b^3)*x^3 + ((D*a*b^2 + 3*B*b^3)*x^4 + D*a^3 + 3*B*a^2*
b + 2*(D*a^2*b + 3*B*a*b^2)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - (D*a^3*
b - 5*B*a^2*b^2)*x - 4*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(b*x^2 +
 a) + 8*(A*b^4*x^4 + 2*A*a*b^3*x^2 + A*a^2*b^2)*log(x))/(a^3*b^4*x^4 + 2*a
^4*b^3*x^2 + a^5*b^2)]
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \frac{A+Bx+Cx^2+Dx^3}{x\left(a+bx^2\right)^3}\,dx &= \frac{4\,Ab^2x^2+(Dab+3\,Bb^2)x^3-2\,Ca^2+6\,Aab-(Da^2-5\,Bab)x}{8\,(a^2b^3x^4+2\,a^3b^2x^2+a^4b)} \\ &\quad -\frac{A\log\left(bx^2+a\right)}{2\,a^3}+\frac{A\log\left(x\right)}{a^3}+\frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}\end{aligned}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*A*b^2*x^2 + (D*a*b + 3*B*b^2)*x^3 - 2*C*a^2 + 6*A*a*b - (D*a^2 - 5*
B*a*b)*x)/(a^2*b^3*x^4 + 2*a^3*b^2*x^2 + a^4*b) - 1/2*A*log(b*x^2 + a)/a^3
 + A*log(x)/a^3 + 1/8*(D*a + 3*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b
)
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx$$
$$= -\frac{A\log(bx^2+a)}{2\,a^3} + \frac{A\log(|x|)}{a^3} + \frac{(Da+3\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b}$$
$$+ \frac{4\,Aab^2x^2 - 2\,Ca^3 + 6\,Aa^2b + (Da^2b + 3\,Bab^2)x^3 - (Da^3 - 5\,Ba^2b)x}{8\,(bx^2+a)^2a^3b}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*A*log(b*x^2 + a)/a^3 + A*log(abs(x))/a^3 + 1/8*(D*a + 3*B*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 1/8*(4*A*a*b^2*x^2 - 2*C*a^3 + 6*A*a^2*b
 + (D*a^2*b + 3*B*a*b^2)*x^3 - (D*a^3 - 5*B*a^2*b)*x)/((b*x^2 + a)^2*a^3*b
)
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x(a+bx^2)^3}\,dx = \int \frac{A+B\,x+C\,x^2+x^3\,D}{x\,(b\,x^2+a)^3}\,dx$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3),x)
```

output
```
int((A + B*x + C*x^2 + x^3*D)/(x*(a + b*x^2)^3), x)
```

3.108 $\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx$

3.108.1 Optimal result

Integrand size = 28, antiderivative size = 144

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx=-\frac{A}{a^3x}+\frac{bB-aD-b\left(\frac{Ab}{a}-C\right)x}{4ab(a+bx^2)^2}+\frac{4B-\left(\frac{7Ab}{a}-3C\right)x}{8a^2(a+bx^2)}-\frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{B\log(x)}{a^3}-\frac{B\log(a+bx^2)}{2a^3}$$

output
```
-A/a^3/x+1/4*(B*b-D*a-b*(A*b/a-C)*x)/a/b/(b*x^2+a)^2+1/8*(4*B-(7*A*b/a-3*C
)*x)/a^2/(b*x^2+a)+B*ln(x)/a^3-1/2*B*ln(b*x^2+a)/a^3-3/8*(5*A*b-C*a)*arcta
n(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

3.108.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx=-\frac{A}{a^3x}+\frac{abB-a^2D-Ab^2x+abCx}{4a^2b(a+bx^2)^2}+\frac{4aB-7Abx+3aCx}{8a^3(a+bx^2)}+\frac{3(-5Ab+aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}}+\frac{B\log(x)}{a^3}-\frac{B\log(a+bx^2)}{2a^3}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

output
```
-(A/(a^3*x)) + (a*b*B - a^2*D - A*b^2*x + a*b*C*x)/(4*a^2*b*(a + b*x^2)^2)
 + (4*a*B - 7*A*b*x + 3*a*C*x)/(8*a^3*(a + b*x^2)) + (3*(-5*A*b + a*C)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]) + (B*Log[x])/a^3 - (B*Log[a
+ b*x^2])/(2*a^3)
```

3.108.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}-\frac{\int -\frac{-3\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^2\left(bx^2+a\right)^2}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^2\left(bx^2+a\right)^2}dx}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$\frac{\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a\left(a+bx^2\right)}-\frac{\int -\frac{-\left(\left(\frac{7Ab}{a}-3C\right)x^2\right)+8Bx+8A}{x^2\left(bx^2+a\right)}dx}{2a}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-\left(\left(\frac{7Ab}{a}-3C\right)x^2\right)+8Bx+8A}{x^2\left(bx^2+a\right)}dx}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a\left(a+bx^2\right)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\frac{\int \left(\frac{8A}{ax^2}+\frac{8B}{ax}+\frac{-15Ab-8Bxb+3aC}{a\left(bx^2+a\right)}\right)dx}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a\left(a+bx^2\right)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{-\frac{3(5Ab-aC)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}}-\frac{8A}{ax}-\frac{4B\log\left(a+bx^2\right)}{a}+\frac{8B\log(x)}{a}}{2a}+\frac{4B-x\left(\frac{7Ab}{a}-3C\right)}{2a(a+bx^2)}}{4a}+\frac{-bx\left(\frac{Ab}{a}-C\right)-aD+bB}{4ab\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^2*(a + b*x^2)^3),x]
```

output
```
(b*B - a*D - b*((A*b)/a - C)*x)/(4*a*b*(a + b*x^2)^2) + ((4*B - ((7*A*b)/a
 - 3*C)*x)/(2*a*(a + b*x^2)) + ((-8*A)/(a*x) - (3*(5*A*b - a*C)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) + (8*B*Log[x])/a - (4*B*Log[a + b*x^2
])/a)/(2*a))/(4*a)
```

3.108.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.108.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A}{a^{3}x}+\frac{B\ln(x)}{a^{3}}-\frac{\frac{\left(\frac{7}{8}b^{2}A-\frac{3}{8}Cab\right)x^{3}-\frac{Babx^{2}}{2}+\frac{a(9Ab-5Ca)x}{8}-\frac{a^{2}(3Bb-Da)}{4b}}{\left(b\,x^{2}+a\right)^{2}}+\frac{B\ln\left(b\,x^{2}+a\right)}{2}+\frac{(15Ab-3Ca)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}}}{a^{3}}$	125

input
```
int((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-A/a^3/x+B*ln(x)/a^3-1/a^3*(((7/8*b^2*A-3/8*C*a*b)*x^3-1/2*B*a*b*x^2+1/8*a
*(9*A*b-5*C*a)*x-1/4*a^2*(3*B*b-D*a)/b)/(b*x^2+a)^2+1/2*B*ln(b*x^2+a)+1/8*
(15*A*b-3*C*a)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(124) = 248.

Time = 0.31 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.64

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx$$
$$=\left[\frac{8\,Ba^2b^2x^3-16\,Aa^3b+6\,(Ca^2b^2-5\,Aab^3)x^4+10\,(Ca^3b-5\,Aa^2b^2)x^2+3\,((Cab^2-5\,Ab^3)x^5+2\,(Ca^2b\right.$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(8*B*a^2*b^2*x^3 - 16*A*a^3*b + 6*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 10*(
C*a^3*b - 5*A*a^2*b^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A
*a*b^2)*x^3 + (C*a^3 - 5*A*a^2*b)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*
x - a)/(b*x^2 + a)) - 4*(D*a^4 - 3*B*a^3*b)*x - 8*(B*a*b^3*x^5 + 2*B*a^2*b
^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 16*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B
*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), 1/8*(4*B*a^2*b^
2*x^3 - 8*A*a^3*b + 3*(C*a^2*b^2 - 5*A*a*b^3)*x^4 + 5*(C*a^3*b - 5*A*a^2*b
^2)*x^2 + 3*((C*a*b^2 - 5*A*b^3)*x^5 + 2*(C*a^2*b - 5*A*a*b^2)*x^3 + (C*a^
3 - 5*A*a^2*b)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 2*(D*a^4 - 3*B*a^3*b)*
x - 4*(B*a*b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(b*x^2 + a) + 8*(B*a*
b^3*x^5 + 2*B*a^2*b^2*x^3 + B*a^3*b*x)*log(x))/(a^4*b^3*x^5 + 2*a^5*b^2*x^
3 + a^6*b*x)]
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**2/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = \frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,(a^3b^3x^5+2\,a^4b^2x^3+a^5bx)} - \frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(x\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)*x^4 - 8*A*a^2*b + 5*(C*a^2*b -
5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/(a^3*b^3*x^5 + 2*a^4*b^2*x^3 + a
^5*b*x) - 1/2*B*log(b*x^2 + a)/a^3 + B*log(x)/a^3 + 3/8*(C*a - 5*A*b)*arct
an(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2\left(a+bx^2\right)^3}\,dx = -\frac{B\log\left(bx^2+a\right)}{2\,a^3}+\frac{B\log\left(|x|\right)}{a^3}+\frac{3\,(Ca-5\,Ab)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3} + \frac{4\,Bab^2x^3+3\,(Cab^2-5\,Ab^3)x^4-8\,Aa^2b+5\,(Ca^2b-5\,Aab^2)x^2-2\,(Da^3-3\,Ba^2b)x}{8\,\left(bx^2+a\right)^2a^3bx}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/2*B*log(b*x^2 + a)/a^3 + B*log(abs(x))/a^3 + 3/8*(C*a - 5*A*b)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*a^3) + 1/8*(4*B*a*b^2*x^3 + 3*(C*a*b^2 - 5*A*b^3)
*x^4 - 8*A*a^2*b + 5*(C*a^2*b - 5*A*a*b^2)*x^2 - 2*(D*a^3 - 3*B*a^2*b)*x)/
((b*x^2 + a)^2*a^3*b*x)
```

3.108.9 Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^2(a+bx^2)^3}\,dx = \frac{\frac{3B}{4a}+\frac{B\,b\,x^2}{2\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4}+\frac{\frac{5\,C\,x}{8\,a}+\frac{3\,C\,b\,x^3}{8\,a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{\frac{A}{a}+\frac{25\,A\,b\,x^2}{8\,a^2}+\frac{15\,A\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5} - \frac{D}{4\,b\,(b\,x^2+a)^2} - \frac{B\,\ln(b\,x^2+a)}{2\,a^3} + \frac{B\,\ln(x)}{a^3} - \frac{15\,A\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}} + \frac{3\,C\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{5/2}\,\sqrt{b}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^2*(a + b*x^2)^3),x)
```

output
```
((3*B)/(4*a) + (B*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) + ((5*C*x)/(
8*a) + (3*C*b*x^3)/(8*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/a + (25*A*b*x
^2)/(8*a^2) + (15*A*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x^5 + 2*a*b*x^3) - D/(4
*b*(a + b*x^2)^2) - (B*log(a + b*x^2))/(2*a^3) + (B*log(x))/a^3 - (15*A*b^
(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2)) + (3*C*atan((b^(1/2)*x)/a^(1/
2)))/(8*a^(5/2)*b^(1/2))
```

3.109 $\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx$

3.109.1 Optimal result

Integrand size = 28, antiderivative size = 174

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx = -\frac{A}{2a^3x^2}-\frac{B}{a^3x}-\frac{\frac{Ab}{a}-C+\left(\frac{bB}{a}-D\right)x}{4a(a+bx^2)^2} - \frac{4(2Ab-aC)+(7bB-3aD)x}{8a^3(a+bx^2)} - \frac{3(5bB-aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}} - \frac{(3Ab-aC)\log(x)}{a^4}+\frac{(3Ab-aC)\log(a+bx^2)}{2a^4}$$

output
```
-1/2*A/a^3/x^2-B/a^3/x+1/4*(-A*b/a+C-(b*B/a-D)*x)/a/(b*x^2+a)^2+1/8*(-8*A*
b+4*C*a-(7*B*b-3*D*a)*x)/a^3/(b*x^2+a)-(3*A*b-C*a)*ln(x)/a^4+1/2*(3*A*b-C*
a)*ln(b*x^2+a)/a^4-3/8*(5*B*b-D*a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/
2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^3} \, dx$$
$$= \frac{-\frac{4aA}{x^2}-\frac{8aB}{x}+\frac{a(-8Ab+4aC-7bBx+3aDx)}{a+bx^2}+\frac{2a^2(-Ab-bBx+a(C+Dx))}{(a+bx^2)^2}+\frac{3\sqrt{a}(-5bB+aD)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{b}}+8(-3Ab+aC)}{8a^4}$$

input
```
Integrate[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

output
```
((-4*a*A)/x^2 - (8*a*B)/x + (a*(-8*A*b + 4*a*C - 7*b*B*x + 3*a*D*x))/(a +
b*x^2) + (2*a^2*(-(A*b) - b*B*x + a*(C + D*x)))/(a + b*x^2)^2 + (3*Sqrt[a]
*(-5*b*B + a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b] + 8*(-3*A*b + a*C)*Lo
g[x] + 4*(3*A*b - a*C)*Log[a + b*x^2])/(8*a^4)
```

3.109.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-3\left(\frac{bB}{a}-D\right)x^3-4\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^3(bx^2+a)^2}dx}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-3\left(\frac{bB}{a}-D\right)x^3-4\left(\frac{Ab}{a}-C\right)x^2+4Bx+4A}{x^3(bx^2+a)^2}dx}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\frac{\int -\frac{-\left(\left(\frac{7bB}{a}-3D\right)x^3\right)-8\left(\frac{2Ab}{a}-C\right)x^2+8Bx+8A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{-\left(\left(\frac{7bB}{a}-3D\right)x^3\right)-8\left(\frac{2Ab}{a}-C\right)x^2+8Bx+8A}{x^3\left(bx^2+a\right)}dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 2333

$$\frac{\frac{\int \left(\frac{8A}{ax^3}+\frac{8(aC-3Ab)}{a^2x}+\frac{8b(3Ab-aC)x-3a(5bB-aD)}{a^2\left(bx^2+a\right)}+\frac{8B}{ax^2}\right)dx}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

$\Big\downarrow$ 2009

$$\frac{\frac{-\frac{3\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5bB-aD)}{a^{3/2}\sqrt{b}}+\frac{4(3Ab-aC)\log\left(a+bx^2\right)}{a^2}-\frac{8\log(x)(3Ab-aC)}{a^2}-\frac{4A}{ax^2}-\frac{8B}{ax}}{2a}-\frac{4\left(\frac{2Ab}{a}-C\right)+x\left(\frac{7bB}{a}-3D\right)}{2a(a+bx^2)}}{4a}-\frac{\frac{Ab}{a}+x\left(\frac{bB}{a}-D\right)-C}{4a\left(a+bx^2\right)^2}$$

input
```
Int[(A + B*x + C*x^2 + D*x^3)/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/4*((A*b)/a - C + ((b*B)/a - D)*x)/(a*(a + b*x^2)^2) + (-1/2*(4*((2*A*b)
/a - C) + ((7*b*B)/a - 3*D)*x)/(a*(a + b*x^2)) + ((-4*A)/(a*x^2) - (8*B)/(
a*x) - (3*(5*b*B - a*D)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]) - (
8*(3*A*b - a*C)*Log[x])/a^2 + (4*(3*A*b - a*C)*Log[a + b*x^2])/a^2)/(2*a))
/(4*a)
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.109.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.97

method	result
default	$-\frac{A}{2a^3x^2}-\frac{B}{a^3x}+\frac{(-3Ab+Ca)\ln(x)}{a^4}+\frac{\frac{\left(-\frac{7}{8}ab^2B+\frac{3}{8}Da^2b\right)x^3+\left(-ab^2A+\frac{1}{2}Ca^2b\right)x^2-\frac{a^2(9Bb-5Da)x}{8}-\frac{5a^2bA}{4}+\frac{3Ca^3}{4}}{\left(bx^2+a\right)^2}+\frac{\left(24b^2A-8C\right.}{}}{a^4}$

input
```
int((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/a^3/x^2-B/a^3/x+(-3*A*b+C*a)/a^4*ln(x)+1/a^4*(((-7/8*a*b^2*B+3/8*D*
a^2*b)*x^3+(-a*b^2*A+1/2*C*a^2*b)*x^2-1/8*a^2*(9*B*b-5*D*a)*x-5/4*a^2*b*A+
3/4*C*a^3)/(b*x^2+a)^2+1/16*(24*A*b^2-8*C*a*b)/b*ln(b*x^2+a)+1/8*(-15*B*a*
b+3*D*a^2)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(147) = 294$.

Time = 0.34 (sec) , antiderivative size = 696, normalized size of antiderivative = 4.00

$$\int \frac{A + Bx + Cx^2 + Dx^3}{x^3 (a + bx^2)^3}\,dx$$
$$= \left[-\frac{16\,Ba^3bx - 6\,(Da^2b^2 - 5\,Bab^3)x^5 + 8\,Aa^3b - 8\,(Ca^2b^2 - 3\,Aab^3)x^4 - 10\,(Da^3b - 5\,Ba^2b^2)x^3 - 12\,(C}{} \right.$$
$$-\frac{8\,Ba^3bx - 3\,(Da^2b^2 - 5\,Bab^3)x^5 + 4\,Aa^3b - 4\,(Ca^2b^2 - 3\,Aab^3)x^4 - 5\,(Da^3b - 5\,Ba^2b^2)x^3 - 6\,(Ca^3b}{}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*B*a^3*b*x - 6*(D*a^2*b^2 - 5*B*a*b^3)*x^5 + 8*A*a^3*b - 8*(C*a^
2*b^2 - 3*A*a*b^3)*x^4 - 10*(D*a^3*b - 5*B*a^2*b^2)*x^3 - 12*(C*a^3*b - 3*
A*a^2*b^2)*x^2 + 3*((D*a*b^2 - 5*B*b^3)*x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4
+ (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*
x^2 + a)) + 8*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (
C*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 16*((C*a*b^3 - 3*A*b^4)*x^6 +
 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(x))/(a^4
*b^3*x^6 + 2*a^5*b^2*x^4 + a^6*b*x^2), -1/8*(8*B*a^3*b*x - 3*(D*a^2*b^2 -
5*B*a*b^3)*x^5 + 4*A*a^3*b - 4*(C*a^2*b^2 - 3*A*a*b^3)*x^4 - 5*(D*a^3*b -
5*B*a^2*b^2)*x^3 - 6*(C*a^3*b - 3*A*a^2*b^2)*x^2 - 3*((D*a*b^2 - 5*B*b^3)*
x^6 + 2*(D*a^2*b - 5*B*a*b^2)*x^4 + (D*a^3 - 5*B*a^2*b)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + 4*((C*a*b^3 - 3*A*b^4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3
)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(b*x^2 + a) - 8*((C*a*b^3 - 3*A*b^
4)*x^6 + 2*(C*a^2*b^2 - 3*A*a*b^3)*x^4 + (C*a^3*b - 3*A*a^2*b^2)*x^2)*log(
x))/(a^4*b^3*x^6 + 2*a^5*b^2*x^4 + a^6*b*x^2)]
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((D*x**3+C*x**2+B*x+A)/x**3/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\,(Dab-5\,Bb^2)x^5+4\,(Cab-3\,Ab^2)x^4-8\,Ba^2x+5\,(Da^2-5\,Bab)x^3-4\,Aa^2+6\,(Ca^2-3\,Aab)x^2}{8\,(a^3b^2x^6+2\,a^4bx^4+a^5x^2)}$$
$$+\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(x\right)}{a^4}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*(3*(D*a*b - 5*B*b^2)*x^5 + 4*(C*a*b - 3*A*b^2)*x^4 - 8*B*a^2*x + 5*(D*
a^2 - 5*B*a*b)*x^3 - 4*A*a^2 + 6*(C*a^2 - 3*A*a*b)*x^2)/(a^3*b^2*x^6 + 2*a
^4*b*x^4 + a^5*x^2) + 3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^3) - 1/2*(C*a - 3*A*b)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(x)/a^4
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3\left(a+bx^2\right)^3}\,dx$$
$$=\frac{3\,(Da-5\,Bb)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3}-\frac{(Ca-3\,Ab)\log\left(bx^2+a\right)}{2\,a^4}+\frac{(Ca-3\,Ab)\log\left(|x|\right)}{a^4}$$
$$+\frac{3\,Dabx^5-15\,Bb^2x^5+4\,Cabx^4-12\,Ab^2x^4+5\,Da^2x^3-25\,Babx^3+6\,Ca^2x^2-18\,Aabx^2-8\,Ba^2x-4}{8\,(bx^3+ax)^2a^3}$$

input
```
integrate((D*x^3+C*x^2+B*x+A)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
3/8*(D*a - 5*B*b)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3) - 1/2*(C*a - 3*A*b
)*log(b*x^2 + a)/a^4 + (C*a - 3*A*b)*log(abs(x))/a^4 + 1/8*(3*D*a*b*x^5 -
15*B*b^2*x^5 + 4*C*a*b*x^4 - 12*A*b^2*x^4 + 5*D*a^2*x^3 - 25*B*a*b*x^3 + 6
*C*a^2*x^2 - 18*A*a*b*x^2 - 8*B*a^2*x - 4*A*a^2)/((b*x^3 + a*x)^2*a^3)
```

3.109.9 Mupad [B] (verification not implemented)

Time = 6.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.32

$$\int \frac{A+Bx+Cx^2+Dx^3}{x^3(a+bx^2)^3}\,dx = \frac{\frac{3C}{4a}+\frac{C\,b\,x^2}{2a^2}}{a^2+2\,a\,b\,x^2+b^2\,x^4} - \frac{\frac{A}{2a}+\frac{9\,A\,b\,x^2}{4\,a^2}+\frac{3\,A\,b^2\,x^4}{2\,a^3}}{a^2\,x^2+2\,a\,b\,x^4+b^2\,x^6} - \frac{\frac{B}{a}+\frac{25\,B\,b\,x^2}{8\,a^2}+\frac{15\,B\,b^2\,x^4}{8\,a^3}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5} - \frac{C\,\ln\left(b\,x^2+a\right)}{2\,a^3} + \frac{C\,\ln\left(x\right)}{a^3} + \frac{3\,A\,b\,\ln\left(b\,x^2+a\right)}{2\,a^4} - \frac{3\,A\,b\,\ln\left(x\right)}{a^4} + \frac{x\,D\,{}_2\mathrm{F}_1\left(\frac{1}{2},3;\ \frac{3}{2};\ -\frac{b\,x^2}{a}\right)}{a^3} - \frac{15\,B\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8\,a^{7/2}}$$

input
```
int((A + B*x + C*x^2 + x^3*D)/(x^3*(a + b*x^2)^3),x)
```

output
```
((3*C)/(4*a) + (C*b*x^2)/(2*a^2))/(a^2 + b^2*x^4 + 2*a*b*x^2) - (A/(2*a) +
 (9*A*b*x^2)/(4*a^2) + (3*A*b^2*x^4)/(2*a^3))/(a^2*x^2 + b^2*x^6 + 2*a*b*x
^4) - (B/a + (25*B*b*x^2)/(8*a^2) + (15*B*b^2*x^4)/(8*a^3))/(a^2*x + b^2*x
^5 + 2*a*b*x^3) - (C*log(a + b*x^2))/(2*a^3) + (C*log(x))/a^3 + (3*A*b*log
(a + b*x^2))/(2*a^4) - (3*A*b*log(x))/a^4 + (x*D*hypergeom([1/2, 3], 3/2,
-(b*x^2)/a))/a^3 - (15*B*b^(1/2)*atan((b^(1/2)*x)/a^(1/2)))/(8*a^(7/2))
```

3.110 $\int \frac{-x+4x^3}{(5+x^2)^2} \, dx$

3.110.1 Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \frac{-x+4x^3}{(5+x^2)^2} \, dx = \frac{21}{2(5+x^2)} + 2\log\left(5+x^2\right)$$

output
```
21/2/(x^2+5)+2*ln(x^2+5)
```

3.110.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2} \, dx = \frac{21}{2(5+x^2)} + 2\log\left(5+x^2\right)$$

input
```
Integrate[(-x + 4*x^3)/(5 + x^2)^2,x]
```

output
```
21/(2*(5 + x^2)) + 2*Log[5 + x^2]
```

3.110.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {2027, 353, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x^3-x}{(x^2+5)^2} \, dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x(4x^2-1)}{(x^2+5)^2}dx$$

$$\Big\downarrow \text{ 353}$$

$$\frac{1}{2}\int -\frac{1-4x^2}{(x^2+5)^2}dx^2$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{1}{2}\int \frac{1-4x^2}{(x^2+5)^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$-\frac{1}{2}\int \left(\frac{21}{(x^2+5)^2}-\frac{4}{x^2+5}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}\left(\frac{21}{x^2+5}+4\log\left(x^2+5\right)\right)$$

input
```
Int[(-x + 4*x^3)/(5 + x^2)^2,x]
```

output
```
(21/(5 + x^2) + 4*Log[5 + x^2])/2
```

3.110.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.110.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{21}{2(x^2+5)} + 2\ln(x^2+5)$	19
norman	$\frac{21}{2(x^2+5)} + 2\ln(x^2+5)$	19
risch	$\frac{21}{2(x^2+5)} + 2\ln(x^2+5)$	19
meijerg	$-\frac{21x^2}{50\left(1+\frac{x^2}{5}\right)} + 2\ln\left(1+\frac{x^2}{5}\right)$	26
parallelrisch	$\frac{4\ln(x^2+5)x^2+21+20\ln(x^2+5)}{2x^2+10}$	31

input
```
int((4*x^3-x)/(x^2+5)^2,x,method=_RETURNVERBOSE)
```

output
```
21/2/(x^2+5)+2*ln(x^2+5)
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{4\,(x^2+5)\log{(x^2+5)}+21}{2\,(x^2+5)}$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="fricas")
```

output
```
1/2*(4*(x^2 + 5)*log(x^2 + 5) + 21)/(x^2 + 5)
```

3.110.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = 2\log{\left(x^2+5\right)} + \frac{21}{2x^2+10}$$

input
```
integrate((4*x**3-x)/(x**2+5)**2,x)
```

output
```
2*log(x**2 + 5) + 21/(2*x**2 + 10)
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = \frac{21}{2\,(x^2+5)} + 2\,\log{\left(x^2+5\right)}$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="maxima")
```

output
```
21/2/(x^2 + 5) + 2*log(x^2 + 5)
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = -\frac{4\,x^2-1}{2\,(x^2+5)} + 2\,\log\left(x^2+5\right)$$

input
```
integrate((4*x^3-x)/(x^2+5)^2,x, algorithm="giac")
```

output
```
-1/2*(4*x^2 - 1)/(x^2 + 5) + 2*log(x^2 + 5)
```

3.110.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{-x+4x^3}{(5+x^2)^2}\,dx = 2\,\ln\left(x^2+5\right) + \frac{21}{2\,\left(x^2+5\right)}$$

input
```
int(-(x - 4*x^3)/(x^2 + 5)^2,x)
```

output
```
2*log(x^2 + 5) + 21/(2*(x^2 + 5))
```

3.111 $\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx$

3.111.1 Optimal result

Integrand size = 17, antiderivative size = 23

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx=\sqrt{-2+x^2}+\frac{1}{3}\left(-2+x^2\right)^{3/2}$$

output
```
1/3*(x^2-2)^(3/2)+(x^2-2)^(1/2)
```

3.111.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx=\frac{1}{3}\sqrt{-2+x^2}\left(1+x^2\right)$$

input
```
Integrate[(-x + x^3)/Sqrt[-2 + x^2],x]
```

output
```
(Sqrt[-2 + x^2]*(1 + x^2))/3
```

3.111.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2027, 353, 25, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3-x}{\sqrt{x^2-2}}dx$$
$$\Big\downarrow \text{2027}$$
$$\int \frac{x\left(x^2-1\right)}{\sqrt{x^2-2}}dx$$
$$\Big\downarrow \text{353}$$
$$\frac{1}{2}\int -\frac{1-x^2}{\sqrt{x^2-2}}dx^2$$
$$\Big\downarrow \text{25}$$
$$-\frac{1}{2}\int \frac{1-x^2}{\sqrt{x^2-2}}dx^2$$
$$\Big\downarrow \text{53}$$
$$-\frac{1}{2}\int \left(-\sqrt{x^2-2}-\frac{1}{\sqrt{x^2-2}}\right)dx^2$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{2}{3}\left(x^2-2\right)^{3/2}+2\sqrt{x^2-2}\right)$$

input
```
Int[(-x + x^3)/Sqrt[-2 + x^2],x]
```

output
```
(2*Sqrt[-2 + x^2] + (2*(-2 + x^2)^(3/2))/3)/2
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.111.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
risch	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
pseudoelliptic	$\frac{(x^2+1)\sqrt{x^2-2}}{3}$	15
trager	$\left(\frac{x^2}{3}+\frac{1}{3}\right)\sqrt{x^2-2}$	16
default	$\frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$	23
meijerg	$\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(2x^2+8\right)\sqrt{1-\frac{x^2}{2}}}{6}\right)}{\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}+\frac{\sqrt{2}\,\sqrt{-\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1-\frac{x^2}{2}}\right)}{2\sqrt{\pi}\,\sqrt{\operatorname{signum}\left(-1+\frac{x^2}{2}\right)}}$	108

input
```
int((x^3-x)/(x^2-2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^2+1)*(x^2-2)^(1/2)
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\left(x^2+1\right)\sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^2 + 1)*sqrt(x^2 - 2)
```

3.111.6 Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{x^2\sqrt{x^2-2}}{3}+\frac{\sqrt{x^2-2}}{3}$$

input
```
integrate((x**3-x)/(x**2-2)**(1/2),x)
```

output
```
x**2*sqrt(x**2 - 2)/3 + sqrt(x**2 - 2)/3
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\sqrt{x^2-2}x^2+\frac{1}{3}\sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(x^2 - 2)*x^2 + 1/3*sqrt(x^2 - 2)
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{1}{3}\left(x^2-2\right)^{\frac{3}{2}}+\sqrt{x^2-2}$$

input
```
integrate((x^3-x)/(x^2-2)^(1/2),x, algorithm="giac")
```

output
```
1/3*(x^2 - 2)^(3/2) + sqrt(x^2 - 2)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{-x+x^3}{\sqrt{-2+x^2}}\,dx = \frac{(x^2+1)\,\sqrt{x^2-2}}{3}$$

input
```
int(-(x - x^3)/(x^2 - 2)^(1/2),x)
```

output
```
((x^2 + 1)*(x^2 - 2)^(1/2))/3
```

3.112 $\int \frac{-x^2+2x^4}{1+2x^2} \, dx$

3.112.1 Optimal result

Integrand size = 21, antiderivative size = 25

$$\int \frac{-x^2+2x^4}{1+2x^2} \, dx = -x+\frac{x^3}{3}+\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-x+1/3*x^3+1/2*arctan(x*2^(1/2))*2^(1/2)
```

3.112.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{-x^2+2x^4}{1+2x^2} \, dx = -x+\frac{x^3}{3}+\frac{\arctan\left(\sqrt{2}x\right)}{\sqrt{2}}$$

input
```
Integrate[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

output
```
-x + x^3/3 + ArcTan[Sqrt[2]*x]/Sqrt[2]
```

3.112.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2027, 363, 262, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x^4-x^2}{2x^2+1}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^2(2x^2-1)}{2x^2+1}dx$$

$$\Big\downarrow \text{363}$$

$$\frac{x^3}{3}-2\int \frac{x^2}{2x^2+1}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{x^3}{3}-2\left(\frac{x}{2}-\frac{1}{2}\int \frac{1}{2x^2+1}dx\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{x^3}{3}-2\left(\frac{x}{2}-\frac{\arctan\left(\sqrt{2}x\right)}{2\sqrt{2}}\right)$$

input
```
Int[(-x^2 + 2*x^4)/(1 + 2*x^2),x]
```

output
```
x^3/3 - 2*(x/2 - ArcTan[Sqrt[2]*x]/(2*Sqrt[2]))
```

3.112.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.112.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.84

method	result	size
default	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
risch	$-x+\frac{x^{3}}{3}+\frac{\arctan\left(\sqrt{2}\, x\right)\sqrt{2}}{2}$	21
meijerg	$\frac{\sqrt{2}\left(-\frac{2x\sqrt{2}\left(-10x^{2}+15\right)}{15}+2\arctan\left(\sqrt{2}\, x\right)\right)}{8}-\frac{\sqrt{2}\left(2\sqrt{2}\, x-2\arctan\left(\sqrt{2}\, x\right)\right)}{8}$	49

input
```
int((2*x^4-x^2)/(2*x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/3*x^3+1/2*arctan(2^(1/2)*x)*2^(1/2)
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)-x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

3.112.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{x^3}{3}-x+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}x\right)}{2}$$

input
```
integrate((2*x**4-x**2)/(2*x**2+1),x)
```

output
```
x**3/3 - x + sqrt(2)*atan(sqrt(2)*x)/2
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3+\frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right)-x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{1}{3}\,x^3 + \frac{1}{2}\,\sqrt{2}\arctan\left(\sqrt{2}x\right) - x$$

input
```
integrate((2*x^4-x^2)/(2*x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*sqrt(2)*arctan(sqrt(2)*x) - x
```

3.112.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{-x^2+2x^4}{1+2x^2}\,dx = \frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,x\right)}{2} - x + \frac{x^3}{3}$$

input
```
int(-(x^2 - 2*x^4)/(2*x^2 + 1),x)
```

output
```
(2^(1/2)*atan(2^(1/2)*x))/2 - x + x^3/3
```

3.113 $\int \frac{x^3+x^4}{1+x^2} \, dx$

3.113.1 Optimal result

Integrand size = 15, antiderivative size = 30

$$\int \frac{x^3+x^4}{1+x^2} \, dx = -x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

output
```
-x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

3.113.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{x^3+x^4}{1+x^2} \, dx = -x+\frac{x^2}{2}+\frac{x^3}{3}+\arctan(x)-\frac{1}{2}\log\left(1+x^2\right)$$

input
```
Integrate[(x^3 + x^4)/(1 + x^2),x]
```

output
```
-x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

3.113.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4+x^3}{x^2+1}\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int \frac{x^3(x+1)}{x^2+1}dx$$

$$\Big\downarrow \text{ 523}$$

$$\int \left(x^2+\frac{1-x}{x^2+1}+x-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\arctan(x)+\frac{x^3}{3}+\frac{x^2}{2}-\frac{1}{2}\log\left(x^2+1\right)-x$$

input
```
Int[(x^3 + x^4)/(1 + x^2),x]
```

output
```
-x + x^2/2 + x^3/3 + ArcTan[x] - Log[1 + x^2]/2
```

3.113.3.1 Defintions of rubi rules used

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.113.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
default	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
risch	$-x+\frac{x^{2}}{2}+\frac{x^{3}}{3}+\arctan\left(x\right)-\frac{\ln\left(x^{2}+1\right)}{2}$	25
meijerg	$-\frac{x\left(-5x^{2}+15\right)}{15}+\arctan\left(x\right)+\frac{x^{2}}{2}-\frac{\ln\left(x^{2}+1\right)}{2}$	27
parallelrisch	$\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\ln\left(x-i\right)}{2}-\frac{i\ln\left(x-i\right)}{2}-\frac{\ln\left(x+i\right)}{2}+\frac{i\ln\left(x+i\right)}{2}$	45

input
```
int((x^4+x^3)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-x+1/2*x^2+1/3*x^3+arctan(x)-1/2*ln(x^2+1)
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3+x^4}{1+x^2}\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,x^{2}-x+\arctan\left(x\right)-\frac{1}{2}\,\log\left(x^{2}+1\right)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

3.113.6 Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.73

$$\int \frac{x^3+x^4}{1+x^2}\,dx=\frac{x^{3}}{3}+\frac{x^{2}}{2}-x-\frac{\log\left(x^{2}+1\right)}{2}+\operatorname{atan}\left(x\right)$$

input
```
integrate((x**4+x**3)/(x**2+1),x)
```

output
```
x**3/3 + x**2/2 - x - log(x**2 + 1)/2 + atan(x)
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3 + x^4}{1 + x^2} \, dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="maxima")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3 + x^4}{1 + x^2} \, dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 - x + \arctan(x) - \frac{1}{2}\log(x^2 + 1)$$

input
```
integrate((x^4+x^3)/(x^2+1),x, algorithm="giac")
```

output
```
1/3*x^3 + 1/2*x^2 - x + arctan(x) - 1/2*log(x^2 + 1)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \frac{x^3 + x^4}{1 + x^2} \, dx = \mathrm{atan}(x) - \frac{\ln(x^2 + 1)}{2} - x + \frac{x^2}{2} + \frac{x^3}{3}$$

input
```
int((x^3 + x^4)/(x^2 + 1),x)
```

output
```
atan(x) - log(x^2 + 1)/2 - x + x^2/2 + x^3/3
```

3.114 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

3.114.1 Optimal result

Integrand size = 30, antiderivative size = 210

$$\begin{aligned}\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx &= \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\,x}{b^6} \\ &\quad - \frac{a(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^5} \\ &\quad + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} \\ &\quad + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3}+\frac{(be-af)x^9}{9b^2}+\frac{fx^{11}}{11b} \\ &\quad - \frac{a^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}\end{aligned}$$

output
```
a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^6-1/3*a*(-a^3*f+a^2*b*e-a*b^2*d+b^3
*c)*x^3/b^5+1/5*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x^5/b^4+1/7*(a^2*f-a*b*e+b^
2*d)*x^7/b^3+1/9*(-a*f+b*e)*x^9/b^2+1/11*f*x^11/b-a^(5/2)*(-a^3*f+a^2*b*e-
a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{b^6} + \frac{a(-b^3c+ab^2d-a^2be+a^3f)\,x^3}{3b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^5}{5b^4} + \frac{(b^2d-abe+a^2f)\,x^7}{7b^3} + \frac{(be-af)x^9}{9b^2} + \frac{fx^{11}}{11b} + \frac{a^{5/2}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{13/2}}$$

input
```
Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
-((a^2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/b^6) + (a*(-(b^3*c) + a*b
^2*d - a^2*b*e + a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f
)*x^5)/(5*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)
/(9*b^2) + (f*x^11)/(11*b) + (a^(5/2)*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(13/2)
```

3.114.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{x^6\left(a^2f-abe+b^2d\right)}{b^3} + \frac{a^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^6} - \frac{ax^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^5} + \frac{x^4\left(a^3(-f)\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^7(a^2f-abe+b^2d)}{7b^3}+\frac{a^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{b^6}-\frac{ax^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^5}+\frac{x^5(a^3(-f)+a^2be-ab^2d+b^3c)}{5b^4}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{13/2}}+\frac{x^9(be-af)}{9b^2}+\frac{fx^{11}}{11b}$$

input
```
Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
(a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^6 - (a*(b^3*c - a*b^2*d + a^
2*b*e - a^3*f)*x^3)/(3*b^5) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^5)/(5
*b^4) + ((b^2*d - a*b*e + a^2*f)*x^7)/(7*b^3) + ((b*e - a*f)*x^9)/(9*b^2)
+ (f*x^11)/(11*b) - (a^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/b^(13/2)
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.114.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.11

method	result
default	$-\frac{-\frac{1}{11}fx^{11}b^5+\frac{1}{9}ab^4fx^9-\frac{1}{9}b^5ex^9-\frac{1}{7}a^2b^3fx^7+\frac{1}{7}ab^4ex^7-\frac{1}{7}b^5dx^7+\frac{1}{5}a^3b^2fx^5-\frac{1}{5}a^2b^3ex^5+\frac{1}{5}ab^4dx^5-\frac{1}{5}b^5cx^5-\frac{1}{3}a^4bfx^3+\frac{1}{3}a^3}{b^6}$
risch	$-\frac{\sqrt{-ab}\,a^2\ln(-\sqrt{-ab}\,x+a)c}{2b^4}-\frac{\sqrt{-ab}\,a^5\ln(\sqrt{-ab}\,x+a)f}{2b^7}-\frac{\sqrt{-ab}\,a^3\ln(\sqrt{-ab}\,x+a)d}{2b^5}+\frac{ex^9}{9b}+\frac{dx^7}{7b}+\frac{cx^5}{5b}+\sqrt{-ab}\,a^2\ln$

input
```
int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/11*f*x^11*b^5+1/9*a*b^4*f*x^9-1/9*b^5*e*x^9-1/7*a^2*b^3*f*x^7+1
/7*a*b^4*e*x^7-1/7*b^5*d*x^7+1/5*a^3*b^2*f*x^5-1/5*a^2*b^3*e*x^5+1/5*a*b^4
*d*x^5-1/5*b^5*c*x^5-1/3*a^4*b*f*x^3+1/3*a^3*b^2*e*x^3-1/3*a^2*b^3*d*x^3+1
/3*a*b^4*c*x^3+a^5*f*x-a^4*b*e*x+a^3*b^2*d*x-a^2*b^3*c*x)+a^3*(a^3*f-a^2*b
*e+a*b^2*d-b^3*c)/b^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.15

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$

$$= \left[\frac{630\,b^5fx^{11}+770\,(b^5e-ab^4f)x^9+990\,(b^5d-ab^4e+a^2b^3f)x^7+1386\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^5-}{}\right.$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6930*(630*b^5*f*x^11 + 770*(b^5*e - a*b^4*f)*x^9 + 990*(b^5*d - a*b^4*e
 + a^2*b^3*f)*x^7 + 1386*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 2
310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^
3*b^2*d + a^4*b*e - a^5*f)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)/(
b*x^2 + a)) + 6930*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6, 1/346
5*(315*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d - a*b^4*e + a^2
*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^5 - 1155*(a*
b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 - 3465*(a^2*b^3*c - a^3*b^2*d
 + a^4*b*e - a^5*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 3465*(a^2*b^3*c -
a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6]
```

3.114.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.83

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^9\left(-\frac{af}{9b^2}+\frac{e}{9b}\right)+x^7\left(\frac{a^2f}{7b^3}-\frac{ae}{7b^2}+\frac{d}{7b}\right)+x^5\left(-\frac{a^3f}{5b^4}+\frac{a^2e}{5b^3}-\frac{ad}{5b^2}+\frac{c}{5b}\right)$$
$$+x^3\left(\frac{a^4f}{3b^5}-\frac{a^3e}{3b^4}+\frac{a^2d}{3b^3}-\frac{ac}{3b^2}\right)+x\left(-\frac{a^5f}{b^6}+\frac{a^4e}{b^5}-\frac{a^3d}{b^4}+\frac{a^2c}{b^3}\right)$$
$$-\frac{\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{b^6\sqrt{-\frac{a^5}{b^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^5f-a^4be+a^3b^2d-a^2b^3c}+x\right)}{2}+\frac{fx^{11}}{11b}$$

input
```
integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

output
```
x**9*(-a*f/(9*b**2) + e/(9*b)) + x**7*(a**2*f/(7*b**3) - a*e/(7*b**2) + d/
(7*b)) + x**5*(-a**3*f/(5*b**4) + a**2*e/(5*b**3) - a*d/(5*b**2) + c/(5*b)
) + x**3*(a**4*f/(3*b**5) - a**3*e/(3*b**4) + a**2*d/(3*b**3) - a*c/(3*b**
2)) + x*(-a**5*f/b**6 + a**4*e/b**5 - a**3*d/b**4 + a**2*c/b**3) - sqrt(-a
**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**6*sqrt(-a**5/b*
*13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**5*f - a**4*b*e + a**3*b**
2*d - a**2*b**3*c) + x)/2 + sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)*log(b**6*sqrt(-a**5/b**13)*(a**3*f - a**2*b*e + a*b**2*d - b**
3*c)/(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c) + x)/2 + f*x**11/(11*
b)
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.01

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6}$$
$$+\frac{315\,b^5fx^{11}+385\,(b^5e-ab^4f)x^9+495\,(b^5d-ab^4e+a^2b^3f)x^7+693\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^5-}{3465\,b^6}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^6) + 1/3465*(315*b^5*f*x^11 + 385*(b^5*e - a*b^4*f)*x^9 + 495*(b^5*d -
 a*b^4*e + a^2*b^3*f)*x^7 + 693*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*
x^5 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^3 + 3465*(a^2*b^3
*c - a^3*b^2*d + a^4*b*e - a^5*f)*x)/b^6
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(a^3b^3c-a^4b^2d+a^5be-a^6f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^6} + \frac{315\,b^{10}fx^{11}+385\,b^{10}ex^9-385\,ab^9fx^9+495\,b^{10}dx^7-495\,ab^9ex^7+495\,a^2b^8fx^7+693\,b^{10}cx^5-693\,ab^9}{}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a^3*b^3*c - a^4*b^2*d + a^5*b*e - a^6*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*b^6) + 1/3465*(315*b^10*f*x^11 + 385*b^10*e*x^9 - 385*a*b^9*f*x^9 + 495*
b^10*d*x^7 - 495*a*b^9*e*x^7 + 495*a^2*b^8*f*x^7 + 693*b^10*c*x^5 - 693*a*
b^9*d*x^5 + 693*a^2*b^8*e*x^5 - 693*a^3*b^7*f*x^5 - 1155*a*b^9*c*x^3 + 115
5*a^2*b^8*d*x^3 - 1155*a^3*b^7*e*x^3 + 1155*a^4*b^6*f*x^3 + 3465*a^2*b^8*c
*x - 3465*a^3*b^7*d*x + 3465*a^4*b^6*e*x - 3465*a^5*b^5*f*x)/b^11
```

3.114.9 Mupad [B] (verification not implemented)

Time = 5.91 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.38

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^9\left(\frac{e}{9\,b}-\frac{a\,f}{9\,b^2}\right)+x^7\left(\frac{d}{7\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{7\,b}\right)+x^5\left(\frac{c}{5\,b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{5\,b}\right)$$
$$+\frac{f\,x^{11}}{11\,b}+\frac{a^{5/2}\,\mathrm{atan}\left(\frac{a^{5/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^6-e\,a^5\,b+d\,a^4\,b^2-c\,a^3\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{13/2}}$$
$$-\frac{a\,x^3\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{3\,b}+\frac{a^2\,x\,\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b^2}$$

input
```
int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

output
```
x^9*(e/(9*b) - (a*f)/(9*b^2)) + x^7*(d/(7*b) - (a*(e/b - (a*f)/b^2))/(7*b)
) + x^5*(c/(5*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(5*b)) + (f*x^11)/(
11*b) + (a^(5/2)*atan((a^(5/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*
e))/(a^6*f - a^3*b^3*c + a^4*b^2*d - a^5*b*e))*(b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/b^(13/2) - (a*x^3*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))
/(3*b) + (a^2*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b^2
```

3.115 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

3.115.1 Optimal result

Integrand size = 30, antiderivative size = 172

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\,x}{b^5} + \frac{(b^3c-ab^2d+a^2be-a^3f)\,x^3}{3b^4} + \frac{(b^2d-abe+a^2f)\,x^5}{5b^3} + \frac{(be-af)x^7}{7b^2} + \frac{fx^9}{9b} + \frac{a^{3/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

output

```
-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^5+1/3*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)
*x^3/b^4+1/5*(a^2*f-a*b*e+b^2*d)*x^5/b^3+1/7*(-a*f+b*e)*x^7/b^2+1/9*f*x^9/
b+a^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2
)
```

3.115.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= \frac{x(315a^4f-105a^3b(3e+fx^2)+21a^2b^2(15d+5ex^2+3fx^4)-3ab^3(105c+35dx^2+21ex^4+15fx^6)+b^4}{315b^5}$$
$$-\frac{a^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
(x*(315*a^4*f - 105*a^3*b*(3*e + f*x^2) + 21*a^2*b^2*(15*d + 5*e*x^2 + 3*f
*x^4) - 3*a*b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6) + b^4*x^2*(105*c
+ 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5) - (a^(3/2)*(-(b^3*c) + a*b^2
*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

3.115.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$\Big\downarrow \text{2333}$$
$$\int \left(\frac{x^4(a^2f-abe+b^2d)}{b^3}-\frac{a(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{x^2(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{a^5(-f)+a^4be}{b^5(a}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{x^5(a^2f-abe+b^2d)}{5b^3}-\frac{ax(a^3(-f)+a^2be-ab^2d+b^3c)}{b^5}+\frac{x^3(a^3(-f)+a^2be-ab^2d+b^3c)}{3b^4}+$$
$$\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{b^{11/2}}+\frac{x^7(be-af)}{7b^2}+\frac{fx^9}{9b}$$

input
```
Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
-((a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^5) + ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*x^3)/(3*b^4) + ((b^2*d - a*b*e + a^2*f)*x^5)/(5*b^3) + ((b*e
 - a*f)*x^7)/(7*b^2) + (f*x^9)/(9*b) + (a^(3/2)*(b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(11/2)
```

3.115.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.115.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.08

method	result
default	$\frac{\frac{1}{9} f\,x^{9} b^{4}-\frac{1}{7} a\,b^{3} f\,x^{7}+\frac{1}{7} b^{4} e\,x^{7}+\frac{1}{5} a^{2} b^{2} f\,x^{5}-\frac{1}{5} a\,b^{3} e\,x^{5}+\frac{1}{5} b^{4} d\,x^{5}-\frac{1}{3} a^{3} b f\,x^{3}+\frac{1}{3} a^{2} b^{2} e\,x^{3}-\frac{1}{3} a\,b^{3} d\,x^{3}+\frac{1}{3} b^{4} c\,x^{3}+a^{4} f x-a^{3} b e x+a^{2} b^{2} d x-}{b^{5}}$
risch	$\frac{f\,x^{9}}{9b}-\frac{a f\,x^{7}}{7b^{2}}+\frac{e\,x^{7}}{7b}+\frac{a^{2} f\,x^{5}}{5b^{3}}-\frac{a e\,x^{5}}{5b^{2}}+\frac{d\,x^{5}}{5b}-\frac{a^{3} f\,x^{3}}{3b^{4}}+\frac{a^{2} e\,x^{3}}{3b^{3}}-\frac{a d\,x^{3}}{3b^{2}}+\frac{c\,x^{3}}{3b}+\frac{a^{4} f x}{b^{5}}-\frac{a^{3} e x}{b^{4}}+\frac{a^{2} d x}{b^{3}}-\frac{a c x}{b^{2}}$

input
```
int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/9*f*x^9*b^4-1/7*a*b^3*f*x^7+1/7*b^4*e*x^7+1/5*a^2*b^2*f*x^5-1/5*a
*b^3*e*x^5+1/5*b^4*d*x^5-1/3*a^3*b*f*x^3+1/3*a^2*b^2*e*x^3-1/3*a*b^3*d*x^3
+1/3*b^4*c*x^3+a^4*f*x-a^3*b*e*x+a^2*b^2*d*x-a*b^3*c*x)-a^2*(a^3*f-a^2*b*e
+a*b^2*d-b^3*c)/b^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.14

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= \left[\frac{70\,b^4fx^9+90\,(b^4e-ab^3f)x^7+126\,(b^4d-ab^3e+a^2b^2f)x^5+210\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-315\,(}{630\,b^5}\right.$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/630*(70*b^4*f*x^9 + 90*(b^4*e - a*b^3*f)*x^7 + 126*(b^4*d - a*b^3*e + a
^2*b^2*f)*x^5 + 210*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 315*(a*b
^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(-a/b)*log((b*x^2 - 2*b*x*sqrt(-a/
b) - a)/(b*x^2 + a)) - 630*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5,
 1/315*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3*e + a^
2*b^2*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 + 315*(a*b^
3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315
*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5]
```

3.115.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(167) = 334.

Time = 0.45 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.96

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^7\left(-\frac{af}{7b^2}+\frac{e}{7b}\right)+x^5\left(\frac{a^2f}{5b^3}-\frac{ae}{5b^2}+\frac{d}{5b}\right)$$
$$+x^3\left(-\frac{a^3f}{3b^4}+\frac{a^2e}{3b^3}-\frac{ad}{3b^2}+\frac{c}{3b}\right)+x\left(\frac{a^4f}{b^5}-\frac{a^3e}{b^4}+\frac{a^2d}{b^3}-\frac{ac}{b^2}\right)$$
$$+\frac{\sqrt{-\frac{a^3}{b^{11}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}$$
$$-\frac{\sqrt{-\frac{a^3}{b^{11}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{b^5\sqrt{-\frac{a^3}{b^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^4f-a^3be+a^2b^2d-ab^3c}+x\right)}{2}+\frac{fx^9}{9b}$$

input
```
integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

output
```
x**7*(-a*f/(7*b**2) + e/(7*b)) + x**5*(a**2*f/(5*b**3) - a*e/(5*b**2) + d/
(5*b)) + x**3*(-a**3*f/(3*b**4) + a**2*e/(3*b**3) - a*d/(3*b**2) + c/(3*b)
) + x*(a**4*f/b**5 - a**3*e/b**4 + a**2*d/b**3 - a*c/b**2) + sqrt(-a**3/b*
*11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**5*sqrt(-a**3/b**11)*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**4*f - a**3*b*e + a**2*b**2*d -
a*b**3*c) + x)/2 - sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*
c)*log(b**5*sqrt(-a**3/b**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**
4*f - a**3*b*e + a**2*b**2*d - a*b**3*c) + x)/2 + f*x**9/(9*b)
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^4fx^9+45\,(b^4e-ab^3f)x^7+63\,(b^4d-ab^3e+a^2b^2f)x^5+105\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^3-315\,(}{315\,b^5}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - a*b^3*f)*x^7 + 63*(b^4*d - a*b^3
*e + a^2*b^2*f)*x^5 + 105*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^3 - 31
5*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/b^5
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^5} + \frac{35\,b^8fx^9+45\,b^8ex^7-45\,ab^7fx^7+63\,b^8dx^5-63\,ab^7ex^5+63\,a^2b^6fx^5+105\,b^8cx^3-105\,ab^7dx^3+105\,a}{315\,b^9}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*b^5) + 1/315*(35*b^8*f*x^9 + 45*b^8*e*x^7 - 45*a*b^7*f*x^7 + 63*b^8*d*x^5
 - 63*a*b^7*e*x^5 + 63*a^2*b^6*f*x^5 + 105*b^8*c*x^3 - 105*a*b^7*d*x^3 + 1
05*a^2*b^6*e*x^3 - 105*a^3*b^5*f*x^3 - 315*a*b^7*c*x + 315*a^2*b^6*d*x - 3
15*a^3*b^5*e*x + 315*a^4*b^4*f*x)/b^9
```

3.115.9 Mupad [B] (verification not implemented)

Time = 5.92 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^7\left(\frac{e}{7\,b}-\frac{a\,f}{7\,b^2}\right)+x^5\left(\frac{d}{5\,b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{5\,b}\right)$$
$$+x^3\left(\frac{c}{3\,b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{3\,b}\right)+\frac{f\,x^9}{9\,b}-\frac{a\,x\left(\frac{c}{b}-\frac{a\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)}{b}$$
$$-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^5-e\,a^4\,b+d\,a^3\,b^2-c\,a^2\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{11/2}}$$

input
```
int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

output
```
x^7*(e/(7*b) - (a*f)/(7*b^2)) + x^5*(d/(5*b) - (a*(e/b - (a*f)/b^2))/(5*b)
) + x^3*(c/(3*b) - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/(3*b)) + (f*x^9)/(9
*b) - (a*x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b))/b - (a^(3/2)*ata
n((a^(3/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^5*f - a^2*b^3
*c + a^3*b^2*d - a^4*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(11/2)
```

3.116 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$

3.116.1 Optimal result

Integrand size = 30, antiderivative size = 136

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{b^4} + \frac{(b^2d-abe+a^2f)\,x^3}{3b^3} + \frac{(be-af)x^5}{5b^2} + \frac{fx^7}{7b} - \frac{\sqrt{a}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

output
```
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/b^4+1/3*(a^2*f-a*b*e+b^2*d)*x^3/b^3+1/5*(
-a*f+b*e)*x^5/b^2+1/7*f*x^7/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1
/2)/a^(1/2))*a^(1/2)/b^(9/2)
```

3.116.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = \frac{x(-105a^3f+35a^2b(3e+fx^2)-7ab^2(15d+5ex^2+3fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4} + \frac{\sqrt{a}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
(x*(-105*a^3*f + 35*a^2*b*(3*e + f*x^2) - 7*a*b^2*(15*d + 5*e*x^2 + 3*f*x^
4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^4) + (Sqrt[a]*(
-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

3.116.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{x^2\left(a^2f-abe+b^2d\right)}{b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{b^4}+\frac{a^4f-a^3be+a^2b^2d-ab^3c}{b^4\left(a+bx^2\right)}+\frac{x^4(be-af)}{b^2}+\frac{fx^6}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3\left(a^2f-abe+b^2d\right)}{3b^3}-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^{9/2}}+\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{b^4}+\frac{x^5(be-af)}{5b^2}+\frac{fx^7}{7b}$$

input
```
Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x]
```

output
```
((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4 + ((b^2*d - a*b*e + a^2*f)*x^3
)/(3*b^3) + ((b*e - a*f)*x^5)/(5*b^2) + (f*x^7)/(7*b) - (Sqrt[a]*(b^3*c -
a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2)
```

3.116.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.116.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

method	result
default	$-\frac{-\frac{1}{7} f\, x^7 b^3+\frac{1}{5} a\, b^2 f\, x^5-\frac{1}{5} b^3 e\, x^5-\frac{1}{3} a^2 b f\, x^3+\frac{1}{3} a\, b^2 e\, x^3-\frac{1}{3} b^3 d\, x^3+f\, a^3 x-a^2 b e x+a\, b^2 d x-b^3 c x}{b^4}+\frac{a\left(f\, a^3-a^2 b e+a\, b^2 d-b^3 c\right) \operatorname{arctan}}{b^4 \sqrt{a b}}$
risch	$\frac{f\, x^7}{7 b}-\frac{a f\, x^5}{5 b^2}+\frac{e\, x^5}{5 b}+\frac{a^2 f\, x^3}{3 b^3}-\frac{a e\, x^3}{3 b^2}+\frac{d\, x^3}{3 b}-\frac{f\, a^3 x}{b^4}+\frac{a^2 e x}{b^3}-\frac{a d x}{b^2}+\frac{c x}{b}+\frac{\sqrt{-a b}\, \ln\left(-\sqrt{-a b}\, x+a\right) f\, a^3}{2 b^5}-\frac{\sqrt{-a b}}{}$

input
```
int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/7*f*x^7*b^3+1/5*a*b^2*f*x^5-1/5*b^3*e*x^5-1/3*a^2*b*f*x^3+1/3*a
*b^2*e*x^3-1/3*b^3*d*x^3+f*a^3*x-a^2*b*e*x+a*b^2*d*x-b^3*c*x)+a*(a^3*f-a^2
*b*e+a*b^2*d-b^3*c)/b^4/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.10

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$=\left[\frac{30\,b^3 f x^7+42\,(b^3 e-ab^2 f)x^5+70\,(b^3 d-ab^2 e+a^2 bf)x^3-105\,(b^3 c-ab^2 d+a^2 be-a^3 f)\sqrt{-\frac{a}{b}}\log\left(\frac{bx^2}{}\right.}{210\,b^4}\right.$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/210*(30*b^3*f*x^7 + 42*(b^3*e - a*b^2*f)*x^5 + 70*(b^3*d - a*b^2*e + a^
2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a/b)*log((b*x^2
 + 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*x)/b^4, 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d -
 a*b^2*e + a^2*b*f)*x^3 - 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
]
```

3.116.6 Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.36

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = x^5\left(-\frac{af}{5b^2}+\frac{e}{5b}\right)+x^3\left(\frac{a^2f}{3b^3}-\frac{ae}{3b^2}+\frac{d}{3b}\right) + x\left(-\frac{a^3f}{b^4}+\frac{a^2e}{b^3}-\frac{ad}{b^2}+\frac{c}{b}\right) - \frac{\sqrt{-\frac{a}{b^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2} + \frac{\sqrt{-\frac{a}{b^9}}(a^3f-a^2be+ab^2d-b^3c)\log\left(b^4\sqrt{-\frac{a}{b^9}}+x\right)}{2} + \frac{fx^7}{7b}$$

input
```
integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

output
```
x**5*(-a*f/(5*b**2) + e/(5*b)) + x**3*(a**2*f/(3*b**3) - a*e/(3*b**2) + d/
(3*b)) + x*(-a**3*f/b**4 + a**2*e/b**3 - a*d/b**2 + c/b) - sqrt(-a/b**9)*(
a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-b**4*sqrt(-a/b**9) + x)/2 + sq
rt(-a/b**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(b**4*sqrt(-a/b**9)
 + x)/2 + f*x**7/(7*b)
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^3fx^7+21\,(b^3e-ab^2f)x^5+35\,(b^3d-ab^2e+a^2bf)x^3+105\,(b^3c-ab^2d+a^2be-a^3f)x}{105\,b^4}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^4) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - a*b^2*f)*x^5 + 35*(b^3*d - a*b^2*
e + a^2*b*f)*x^3 + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/b^4
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.09

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^4} + \frac{15\,b^6fx^7+21\,b^6ex^5-21\,ab^5fx^5+35\,b^6dx^3-35\,ab^5ex^3+35\,a^2b^4fx^3+105\,b^6cx-105\,ab^5dx+105\,a^2b}{105\,b^7}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
-(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*
b^4) + 1/105*(15*b^6*f*x^7 + 21*b^6*e*x^5 - 21*a*b^5*f*x^5 + 35*b^6*d*x^3
- 35*a*b^5*e*x^3 + 35*a^2*b^4*f*x^3 + 105*b^6*c*x - 105*a*b^5*d*x + 105*a^
2*b^4*e*x - 105*a^3*b^3*f*x)/b^7
```

3.116.9 Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.42

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{a+bx^2}\,dx$$
$$= x^5\left(\frac{e}{5\,b}-\frac{a\,f}{5\,b^2}\right)+x^3\left(\frac{d}{3\,b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{3\,b}\right)+x\left(\frac{c}{b}-\frac{a\,\left(\frac{d}{b}-\frac{a\,\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)}{b}\right)$$
$$+\frac{f\,x^7}{7\,b}+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{f\,a^4-e\,a^3\,b+d\,a^2\,b^2-c\,a\,b^3}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{b^{9/2}}$$

input
```
int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2),x)
```

output
```
x^5*(e/(5*b) - (a*f)/(5*b^2)) + x^3*(d/(3*b) - (a*(e/b - (a*f)/b^2))/(3*b)
) + x*(c/b - (a*(d/b - (a*(e/b - (a*f)/b^2))/b))/b) + (f*x^7)/(7*b) + (a^(
1/2)*atan((a^(1/2)*b^(1/2)*x*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^4*f +
 a^2*b^2*d - a*b^3*c - a^3*b*e))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/b^(9
/2)
```

3.117 $\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$

3.117.1 Optimal result

Integrand size = 27, antiderivative size = 100

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{(b^2d-abe+a^2f)\,x}{b^3}+\frac{(be-af)x^3}{3b^2}+\frac{fx^5}{5b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

output
```
(a^2*f-a*b*e+b^2*d)*x/b^3+1/3*(-a*f+b*e)*x^3/b^2+1/5*f*x^5/b+(-a^3*f+a^2*b
*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

3.117.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{x(15a^2f-5ab(3e+fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

output
```
(x*(15*a^2*f - 5*a*b*(3*e + f*x^2) + b^2*(15*d + 5*e*x^2 + 3*f*x^4))/(15*
b^3) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
Sqrt[a]*b^(7/2))
```

3.117.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2} \, dx$$

$$\Big\downarrow \text{2341}$$

$$\int \left(\frac{a^2f-abe+b^2d}{b^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{b^3\left(a+bx^2\right)}+\frac{x^2(be-af)}{b^2}+\frac{fx^4}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(a^2f-abe+b^2d\right)}{b^3}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{\sqrt{a}b^{7/2}}+\frac{x^3(be-af)}{3b^2}+\frac{fx^5}{5b}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x]
```

output
```
((b^2*d - a*b*e + a^2*f)*x)/b^3 + ((b*e - a*f)*x^3)/(3*b^2) + (f*x^5)/(5*b
) + ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqr
t[a]*b^(7/2))
```

3.117.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.117.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

method	result
default	$\frac{\frac{1}{5} f\, x^{5} b^{2}-\frac{1}{3} a b f\, x^{3}+\frac{1}{3} b^{2} e\, x^{3}+a^{2} f x -a b e x +b^{2} d x}{b^{3}}+\frac{\left(-f\, a^{3}+a^{2} b e -a\, b^{2} d +b^{3} c \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{b^{3} \sqrt{a b}}$
risch	$\frac{f\, x^{5}}{5 b}-\frac{a f\, x^{3}}{3 b^{2}}+\frac{e\, x^{3}}{3 b}+\frac{a^{2} f x}{b^{3}}-\frac{a e x}{b^{2}}+\frac{d x}{b}-\frac{\ln \left(b x -\sqrt{-a b}\right) f\, a^{3}}{2 b^{3} \sqrt{-a b}}+\frac{\ln \left(b x -\sqrt{-a b}\right) a^{2} e}{2 b^{2} \sqrt{-a b}}-\frac{\ln \left(b x -\sqrt{-a b}\right) a d}{2 b \sqrt{-a b}}+\frac{\ln \left(b x -\sqrt{-a b}\right)}{2 \sqrt{-a b}}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/b^3*(1/5*f*x^5*b^2-1/3*a*b*f*x^3+1/3*b^2*e*x^3+a^2*f*x-a*b*e*x+b^2*d*x)+
(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/b^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.117.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.36

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$$
$$= \left[\frac{6\,ab^3fx^5+10\,(ab^3e-a^2b^2f)x^3+15\,(b^3c-ab^2d+a^2be-a^3f)\sqrt{-ab}\log\left(\frac{bx^2+2\sqrt{-ab}x-a}{bx^2+a}\right)+30\,(ab^3d-a}{30\,ab^4}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(6*a*b^3*f*x^5 + 10*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d
+ a^2*b*e - a^3*f)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)
) + 30*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4), 1/15*(3*a*b^3*f*x^5 + 5
*(a*b^3*e - a^2*b^2*f)*x^3 + 15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(a
*b)*arctan(sqrt(a*b)*x/a) + 15*(a*b^3*d - a^2*b^2*e + a^3*b*f)*x)/(a*b^4)]
```

3.117.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.60

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = x^3\left(-\frac{af}{3b^2}+\frac{e}{3b}\right)+x\left(\frac{a^2f}{b^3}-\frac{ae}{b^2}+\frac{d}{b}\right) + \frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{ab^7}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(ab^3\sqrt{-\frac{1}{ab^7}}+x\right)}{2} + \frac{fx^5}{5b}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a),x)
```

output
```
x**3*(-a*f/(3*b**2) + e/(3*b)) + x*(a**2*f/b**3 - a*e/b**2 + d/b) + sqrt(-
1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a*b**3*sqrt(-1/(a
*b**7)) + x)/2 - sqrt(-1/(a*b**7))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)
*log(a*b**3*sqrt(-1/(a*b**7)) + x)/2 + f*x**5/(5*b)
```

3.117.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3} + \frac{3\,b^2fx^5+5\,(b^2e-abf)x^3+15\,(b^2d-abe+a^2f)x}{15\,b^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^2*f*x^5 + 5*(b^2*e - a*b*f)*x^3 + 15*(b^2*d - a*b*e + a^2*f)*x
)/b^3
```

3.117.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx$$
$$=\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}$$
$$+\frac{3\,b^4fx^5+5\,b^4ex^3-5\,ab^3fx^3+15\,b^4dx-15\,ab^3ex+15\,a^2b^2fx}{15\,b^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^3)
+ 1/15*(3*b^4*f*x^5 + 5*b^4*e*x^3 - 5*a*b^3*f*x^3 + 15*b^4*d*x - 15*a*b^3*
e*x + 15*a^2*b^2*f*x)/b^5
```

3.117.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{a+bx^2}\,dx=x^3\left(\frac{e}{3\,b}-\frac{a\,f}{3\,b^2}\right)+x\left(\frac{d}{b}-\frac{a\left(\frac{e}{b}-\frac{a\,f}{b^2}\right)}{b}\right)+\frac{f\,x^5}{5\,b}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{\sqrt{a}\,b^{7/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2),x)
```

output
```
x^3*(e/(3*b) - (a*f)/(3*b^2)) + x*(d/b - (a*(e/b - (a*f)/b^2))/b) + (f*x^5
)/(5*b) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/
(a^(1/2)*b^(7/2))
```

3.118 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)} \, dx$

3.118.1 Optimal result

Integrand size = 30, antiderivative size = 84

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)} \, dx = -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

output
```
-c/a/x+(-a*f+b*e)*x/b^2+1/3*f*x^3/b-(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(
x*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)
```

3.118.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)} \, dx = -\frac{c}{ax}+\frac{(be-af)x}{b^2}+\frac{fx^3}{3b} + \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{3/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) + ((-(b^3*c) + a*b^2*d -
a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

3.118.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{a^3f-a^2be+ab^2d-b^3c}{ab^2\left(a+bx^2\right)}+\frac{be-af}{b^2}+\frac{c}{ax^2}+\frac{fx^2}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{3/2}b^{5/2}}+\frac{x(be-af)}{b^2}-\frac{c}{ax}+\frac{fx^3}{3b}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x]
```

output
```
-(c/(a*x)) + ((b*e - a*f)*x)/b^2 + (f*x^3)/(3*b) - ((b^3*c - a*b^2*d + a^2
*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2))
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.118.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.94

method	result
default	$-\frac{-\frac{1}{3} f\,x^{3} b+a f x-b e x}{b^{2}}+\frac{\left(f\,a^{3}-a^{2} b e+a\,b^{2} d-b^{3} c\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a\,b^{2} \sqrt{a b}}-\frac{c}{a x}$
risch	$\frac{f\,x^{3}}{3 b}-\frac{a f x}{b^{2}}+\frac{e x}{b}-\frac{c}{a x}-\frac{a^{2} \ln\left(-\sqrt{-a b}\,x+a\right) f}{2 b^{2} \sqrt{-a b}}+\frac{a \ln\left(-\sqrt{-a b}\,x+a\right) e}{2 b \sqrt{-a b}}-\frac{\ln\left(-\sqrt{-a b}\,x+a\right) d}{2 \sqrt{-a b}}+\frac{b \ln\left(-\sqrt{-a b}\,x+a\right) c}{2 \sqrt{-a b}\,a}+\frac{a^{2}}{}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/3*f*x^3*b+a*f*x-b*e*x)+1/a/b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/(a
*b)^(1/2)*arctan(b*x/(a*b)^(1/2))-c/a/x
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.51

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx$$
$$= \left[\frac{2\,a^2b^2fx^4-6\,ab^3c+3\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}x\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)+6\left(a^2b^2e-a^3bf\right)x^2}{6\,a^2b^3x},\ \frac{a^2b^2}{}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(2*a^2*b^2*f*x^4 - 6*a*b^3*c + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*
sqrt(-a*b)*x*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a^2*b^2*e
- a^3*b*f)*x^2)/(a^2*b^3*x), 1/3*(a^2*b^2*f*x^4 - 3*a*b^3*c - 3*(b^3*c - a
*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x*arctan(sqrt(a*b)*x/a) + 3*(a^2*b^2*e
 - a^3*b*f)*x^2)/(a^2*b^3*x)]
```

3.118.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(71) = 142$.

Time = 0.51 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.79

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = x\left(-\frac{af}{b^2}+\frac{e}{b}\right) - \frac{\sqrt{-\frac{1}{a^3b^5}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^3b^5}}\,(a^3f-a^2be+ab^2d-b^3c)\log\left(a^2b^2\sqrt{-\frac{1}{a^3b^5}}+x\right)}{2} + \frac{fx^3}{3b} - \frac{c}{ax}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a),x)
```

output
```
x*(-a*f/b**2 + e/b) - sqrt(-1/(a**3*b**5))*(a**3*f - a**2*b*e + a*b**2*d -
 b**3*c)*log(-a**2*b**2*sqrt(-1/(a**3*b**5)) + x)/2 + sqrt(-1/(a**3*b**5))
*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(a**2*b**2*sqrt(-1/(a**3*b**5)
) + x)/2 + f*x**3/(3*b) - c/(a*x)
```

3.118.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)}\,dx = \frac{bfx^3+3\,(be-af)x}{3\,b^2} - \frac{c}{ax} - \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/3*(b*f*x^3 + 3*(b*e - a*f)*x)/b^2 - c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e
 - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

3.118.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx = -\frac{c}{ax} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{b^2fx^3+3\,b^2ex-3\,abfx}{3\,b^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
-c/(a*x) - (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a*b^2) + 1/3*(b^2*f*x^3 + 3*b^2*e*x - 3*a*b*f*x)/b^3
```

3.118.9 Mupad [B] (verification not implemented)

Time = 5.76 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)}\,dx = x\left(\frac{e}{b}-\frac{a\,f}{b^2}\right) - \frac{c}{a\,x} + \frac{f\,x^3}{3\,b} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{3/2}\,b^{5/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)),x)
```

output
```
x*(e/b - (a*f)/b^2) - c/(a*x) + (f*x^3)/(3*b) - (atan((b^(1/2)*x)/a^(1/2))
*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(3/2)*b^(5/2))
```

3.119 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx$

3.119.1 Optimal result

Integrand size = 30, antiderivative size = 82

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b}+\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

output
```
-1/3*c/a/x^3+(-a*d+b*c)/a^2/x+f*x/b+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(
x*b^(1/2)/a^(1/2))/a^(5/2)/b^(3/2)
```

3.119.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = -\frac{c}{3ax^3}+\frac{bc-ad}{a^2x}+\frac{fx}{b} - \frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{5/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b - ((-(b^3*c) + a*b^2*d - a^
2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

3.119.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{ad-bc}{a^2x^2}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^2b\left(a+bx^2\right)}+\frac{c}{ax^4}+\frac{f}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{a^2x}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{5/2}b^{3/2}}-\frac{c}{3ax^3}+\frac{fx}{b}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x]
```

output
```
-1/3*c/(a*x^3) + (b*c - a*d)/(a^2*x) + (f*x)/b + ((b^3*c - a*b^2*d + a^2*b
*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2))
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.119.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

method	result
default	$\frac{fx}{b}-\frac{c}{3a\,x^{3}}-\frac{ad-bc}{a^{2}x}+\frac{\left(-f\,a^{3}+a^{2}be-a\,b^{2}d+b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{2}b\sqrt{ab}}$
risch	$\frac{fx}{b}+\frac{-\frac{(ad-bc)b\,x^{2}}{a^{2}}-\frac{cb}{3a}}{b\,x^{3}}-\frac{a\ln\left(-\sqrt{-ab}\,x-a\right)f}{2b\sqrt{-ab}}+\frac{\ln\left(-\sqrt{-ab}\,x-a\right)e}{2\sqrt{-ab}}-\frac{b\ln\left(-\sqrt{-ab}\,x-a\right)d}{2\sqrt{-ab}\,a}+\frac{b^{2}\ln\left(-\sqrt{-ab}\,x-a\right)c}{2\sqrt{-ab}\,a^{2}}+\frac{a\ln(-$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
f*x/b-1/3*c/a/x^3-(a*d-b*c)/a^2/x+1/a^2/b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/(
a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.119.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.63

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{6\,a^{3}bfx^{4}+3\,(b^{3}c-ab^{2}d+a^{2}be-a^{3}f)\sqrt{-ab}x^{3}\log\left(\frac{bx^{2}+2\,\sqrt{-ab}x-a}{bx^{2}+a}\right)-2\,a^{2}b^{2}c+6\,(ab^{3}c-a^{2}b^{2}d)x^{2}}{6\,a^{3}b^{2}x^{3}},\ \frac{3\,a^{3}\cdots}{\cdots}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/6*(6*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^3
*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*a^2*b^2*c + 6*(a*b^3*c
- a^2*b^2*d)*x^2)/(a^3*b^2*x^3), 1/3*(3*a^3*b*f*x^4 + 3*(b^3*c - a*b^2*d +
 a^2*b*e - a^3*f)*sqrt(a*b)*x^3*arctan(sqrt(a*b)*x/a) - a^2*b^2*c + 3*(a*b
^3*c - a^2*b^2*d)*x^2)/(a^3*b^2*x^3)]
```

3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(71) = 142$.

Time = 0.99 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.84

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{\sqrt{-\frac{1}{a^5b^3}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2} - \frac{\sqrt{-\frac{1}{a^5b^3}}(a^3f-a^2be+ab^2d-b^3c)\log\left(a^3b\sqrt{-\frac{1}{a^5b^3}}+x\right)}{2} + \frac{fx}{b} + \frac{-ac+x^2(-3ad+3bc)}{3a^2x^3}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a),x)
```

output
```
sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**3*b*s
qrt(-1/(a**5*b**3)) + x)/2 - sqrt(-1/(a**5*b**3))*(a**3*f - a**2*b*e + a*b
**2*d - b**3*c)*log(a**3*b*sqrt(-1/(a**5*b**3)) + x)/2 + f*x/b + (-a*c + x
**2*(-3*a*d + 3*b*c))/(3*a**2*x**3)
```

3.119.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{fx}{b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,(bc-ad)x^2-ac}{3\,a^2x^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="maxima")
```

output
```
f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^2*b) + 1/3*(3*(b*c - a*d)*x^2 - a*c)/(a^2*x^3)
```

3.119.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{fx}{b} + \frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{3\,bcx^2-3\,adx^2-ac}{3\,a^2x^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a),x, algorithm="giac")
```

output
```
f*x/b + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^2*b) + 1/3*(3*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^2*x^3)
```

3.119.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)}\,dx = \frac{f\,x}{b} - \frac{\frac{b\,c}{3\,a}+\frac{b\,x^2\,(a\,d-b\,c)}{a^2}}{b\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^{5/2}\,b^{3/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)),x)
```

output
```
(f*x)/b - ((b*c)/(3*a) + (b*x^2*(a*d - b*c))/a^2)/(b*x^3) + (atan((b^(1/2)
*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a^(5/2)*b^(3/2))
```

3.120 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx$

3.120.1 Optimal result

Integrand size = 30, antiderivative size = 104

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx=-\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}-\frac{b^2c-abd+a^2e}{a^3x}-\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

output
```
-1/5*c/a/x^5+1/3*(-a*d+b*c)/a^2/x^3+(-a^2*e+a*b*d-b^2*c)/a^3/x-(-a^3*f+a^2
*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)
```

3.120.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)}\,dx=-\frac{c}{5ax^5}+\frac{bc-ad}{3a^2x^3}+\frac{-b^2c+abd-a^2e}{a^3x}+\frac{(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) + (-(b^2*c) + a*b*d - a^2*e)/(a^3
*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(a^(7/2)*Sqrt[b])
```

3.120.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{ad-bc}{a^2x^4}+\frac{a^2e-abd+b^2c}{a^3x^2}+\frac{a^3f-a^2be+ab^2d-b^3c}{a^3\left(a+bx^2\right)}+\frac{c}{ax^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{3a^2x^3}-\frac{a^2e-abd+b^2c}{a^3x}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{c}{5ax^5}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x]
```

output
```
-1/5*c/(a*x^5) + (b*c - a*d)/(3*a^2*x^3) - (b^2*c - a*b*d + a^2*e)/(a^3*x)
 - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(7
/2)*Sqrt[b])
```

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.120.4 Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5 a\, x^{5}}-\frac{a d-b c}{3 a^{2} x^{3}}-\frac{a^{2} e-a b d+b^{2} c}{a^{3} x}+\frac{\left(f\, a^{3}-a^{2} b e+a\, b^{2} d-b^{3} c\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{a^{3} \sqrt{a b}}$
risch	$\frac{-\frac{\left(a^{2} e-a b d+b^{2} c\right) x^{4}}{a^{3}}-\frac{(a d-b c) x^{2}}{3 a^{2}}-\frac{c}{5 a}}{x^{5}}-\frac{\ln\left(-\sqrt{-a b}\, x+a\right) f}{2 \sqrt{-a b}}+\frac{\ln\left(-\sqrt{-a b}\, x+a\right) b e}{2 \sqrt{-a b}\, a}-\frac{\ln\left(-\sqrt{-a b}\, x+a\right) b^{2} d}{2 \sqrt{-a b}\, a^{2}}+\frac{\ln\left(-\sqrt{-a b}\, x+a\right) b^{3} c}{2 \sqrt{-a b}\, a^{3}} \cdots$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/5*c/a/x^5-1/3*(a*d-b*c)/a^2/x^3-(a^2*e-a*b*d+b^2*c)/a^3/x+(a^3*f-a^2*b*
e+a*b^2*d-b^3*c)/a^3/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.120.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.37

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)}\,dx$$
$$=\left[\frac{15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{-ab}x^5\log\left(\frac{bx^2-2\sqrt{-ab}x-a}{bx^2+a}\right)-6a^3bc-30\left(ab^3c-a^2b^2d+a^3be\right)x^4+10\left(a^2\cdots\right.}{30a^4bx^5}\right.$$
$$-\frac{15\left(b^3c-ab^2d+a^2be-a^3f\right)\sqrt{ab}x^5\arctan\left(\frac{\sqrt{ab}x}{a}\right)+3a^3bc+15\left(ab^3c-a^2b^2d+a^3be\right)x^4-5\left(a^2b^2c-\cdots\right.}{15a^4bx^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="fricas")
```

output
```
[1/30*(15*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(-a*b)*x^5*log((b*x^2 -
2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*a^3*b*c - 30*(a*b^3*c - a^2*b^2*d + a
^3*b*e)*x^4 + 10*(a^2*b^2*c - a^3*b*d)*x^2)/(a^4*b*x^5), -1/15*(15*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*sqrt(a*b)*x^5*arctan(sqrt(a*b)*x/a) + 3*a^3*b
*c + 15*(a*b^3*c - a^2*b^2*d + a^3*b*e)*x^4 - 5*(a^2*b^2*c - a^3*b*d)*x^2)
/(a^4*b*x^5)]
```

3.120.6 Sympy [A] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.61

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx = -\frac{\sqrt{-\frac{1}{a^7b}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2} + \frac{\sqrt{-\frac{1}{a^7b}}(a^3f-a^2be+ab^2d-b^3c)\log\left(a^4\sqrt{-\frac{1}{a^7b}}+x\right)}{2} + \frac{-3a^2c+x^4(-15a^2e+15abd-15b^2c)+x^2(-5a^2d+5abc)}{15a^3x^5}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a),x)
```

output
```
-sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**4*sqrt(
-1/(a**7*b)) + x)/2 + sqrt(-1/(a**7*b))*(a**3*f - a**2*b*e + a*b**2*d - b*
*3*c)*log(a**4*sqrt(-1/(a**7*b)) + x)/2 + (-3*a**2*c + x**4*(-15*a**2*e +
15*a*b*d - 15*b**2*c) + x**2*(-5*a**2*d + 5*a*b*c))/(15*a**3*x**5)
```

3.120.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,(b^2c-abd+a^2e)x^4+3\,a^2c-5\,(abc-a^2d)x^2}{15\,a^3x^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/15*(15*(b^2*c - a*b*d + a^2*e)*x^4 + 3*a^2*c - 5*(a*b*c - a^2*d)*x^2)
/(a^3*x^5)
```

3.120.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{15\,b^2cx^4-15\,abdx^4+15\,a^2ex^4-5\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^3x^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)
 - 1/15*(15*b^2*c*x^4 - 15*a*b*d*x^4 + 15*a^2*e*x^4 - 5*a*b*c*x^2 + 5*a^2*
d*x^2 + 3*a^2*c)/(a^3*x^5)
```

3.120.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)}\,dx = -\frac{\frac{c}{5\,a}+\frac{x^2\,(a\,d-b\,c)}{3\,a^2}+\frac{x^4\,(e\,a^2-d\,a\,b+c\,b^2)}{a^3}}{x^5} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^{7/2}\,\sqrt{b}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)),x)
```

output
```
- (c/(5*a) + (x^2*(a*d - b*c))/(3*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/a^3
)/x^5 - (atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(a
^(7/2)*b^(1/2))
```

3.121 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)} \, dx$

3.121.1 Optimal result

Integrand size = 30, antiderivative size = 137

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)} \, dx = -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}-\frac{b^2c-abd+a^2e}{3a^3x^3}+\frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} + \frac{\sqrt{b}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

output
```
-1/7*c/a/x^7+1/5*(-a*d+b*c)/a^2/x^5+1/3*(-a^2*e+a*b*d-b^2*c)/a^3/x^3+(-a^3
*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^
(1/2)/a^(1/2))*b^(1/2)/a^(9/2)
```

3.121.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)} \, dx = -\frac{c}{7ax^7}+\frac{bc-ad}{5a^2x^5}+\frac{-b^2c+abd-a^2e}{3a^3x^3} + \frac{b^3c-ab^2d+a^2be-a^3f}{a^4x} - \frac{\sqrt{b}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{9/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) + (-(b^2*c) + a*b*d - a^2*e)/(3*a
^3*x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) - (Sqrt[b]*(-(b^3*c)
 + a*b^2*d - a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

3.121.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{ad-bc}{a^2x^6}+\frac{a^2e-abd+b^2c}{a^3x^4}-\frac{b(a^3f-a^2be+ab^2d-b^3c)}{a^4(a+bx^2)}+\frac{a^3f-a^2be+ab^2d-b^3c}{a^4x^2}+\frac{c}{ax^8}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{5a^2x^5}-\frac{a^2e-abd+b^2c}{3a^3x^3}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(a^3(-f)+a^2be-ab^2d+b^3c)}{a^{9/2}}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{a^4x}-\frac{c}{7ax^7}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x]
```

output
```
-1/7*c/(a*x^7) + (b*c - a*d)/(5*a^2*x^5) - (b^2*c - a*b*d + a^2*e)/(3*a^3*
x^3) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(a^4*x) + (Sqrt[b]*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(9/2)
```

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.121.4 Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.94

method	result
default	$-\frac{c}{7a\,x^{7}}-\frac{ad-bc}{5a^{2}x^{5}}-\frac{a^{2}e-abd+b^{2}c}{3a^{3}x^{3}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{a^{4}x}-\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{a^{4}\sqrt{ab}}$
risch	$\frac{-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{6}}{a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{4}}{3a^{3}}-\frac{(ad-bc)x^{2}}{5a^{2}}-\frac{c}{7a}}{x^{7}}+\frac{\Bigg(\sum_{\_R=\mathrm{RootOf}\left(a^{9}\_Z^{2}+a^{6}b\,f^{2}-2a^{5}b^{2}ef+2a^{4}b^{3}df+a^{4}b^{3}e^{2}-2a^{3}b^{4}c\right.}}{}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a/x^7-1/5*(a*d-b*c)/a^2/x^5-1/3*(a^2*e-a*b*d+b^2*c)/a^3/x^3-(a^3*f-
a^2*b*e+a*b^2*d-b^3*c)/a^4/x-b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/(a*b)^(1/
2)*arctan(b*x/(a*b)^(1/2))
```

3.121.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)}\,dx$$
$$=\Bigg[-\frac{105\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{7}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^{2}-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^{2}+a}\right)-210\left(b^{3}c-ab^{2}d+a^{2}be-a^{3}f\right)x^{6}+70\,(a}{210\,a^{4}x^{7}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/210*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*sqrt(-b/a)*log((b*x^2
 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 210*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f)*x^6 + 70*(a*b^2*c - a^2*b*d + a^3*e)*x^4 + 30*a^3*c - 42*(a^2*b*c -
a^3*d)*x^2)/(a^4*x^7), 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^7*
sqrt(b/a)*arctan(x*sqrt(b/a)) + 105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^
6 - 35*(a*b^2*c - a^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x
^2)/(a^4*x^7)]
```

3.121.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(128) = 256$.

Time = 5.86 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.20

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8 (a + bx^2)}\, dx$$
$$= \frac{\sqrt{-\frac{b}{a^9}}(a^3f - a^2be + ab^2d - b^3c)\log\left(-\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c} + x\right)}{2}$$
$$- \frac{\sqrt{-\frac{b}{a^9}}(a^3f - a^2be + ab^2d - b^3c)\log\left(\frac{a^5\sqrt{-\frac{b}{a^9}}(a^3f-a^2be+ab^2d-b^3c)}{a^3bf-a^2b^2e+ab^3d-b^4c} + x\right)}{2}$$
$$+ \frac{-15a^3c + x^6(-105a^3f + 105a^2be - 105ab^2d + 105b^3c) + x^4(-35a^3e + 35a^2bd - 35ab^2c) + x^2(-21a^3d}{105a^4x^7}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a),x)
```

output
```
sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**5*sqrt(-b/a*
*9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b*f - a**2*b**2*e + a*b*
*3*d - b**4*c) + x)/2 - sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3
*c)*log(a**5*sqrt(-b/a**9)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b
*f - a**2*b**2*e + a*b**3*d - b**4*c) + x)/2 + (-15*a**3*c + x**6*(-105*a*
*3*f + 105*a**2*b*e - 105*a*b**2*d + 105*b**3*c) + x**4*(-35*a**3*e + 35*a
**2*b*d - 35*a*b**2*c) + x**2*(-21*a**3*d + 21*a**2*b*c))/(105*a**4*x**7)
```

3.121.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\,(a+bx^2)}\,dx = \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,(b^3c-ab^2d+a^2be-a^3f)x^6-35\,(ab^2c-a^2bd+a^3e)x^4-15\,a^3c+21\,(a^2bc-a^3d)x^2}{105\,a^4x^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^4) + 1/105*(105*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^6 - 35*(a*b^2*c - a
^2*b*d + a^3*e)*x^4 - 15*a^3*c + 21*(a^2*b*c - a^3*d)*x^2)/(a^4*x^7)
```

3.121.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\,(a+bx^2)}\,dx = \frac{(b^4c-ab^3d+a^2b^2e-a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^4} + \frac{105\,b^3cx^6-105\,ab^2dx^6+105\,a^2bex^6-105\,a^3fx^6-35\,ab^2cx^4+35\,a^2bdx^4-35\,a^3ex^4+21\,a^2bcx^2-21}{105\,a^4x^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a
^4) + 1/105*(105*b^3*c*x^6 - 105*a*b^2*d*x^6 + 105*a^2*b*e*x^6 - 105*a^3*f
*x^6 - 35*a*b^2*c*x^4 + 35*a^2*b*d*x^4 - 35*a^3*e*x^4 + 21*a^2*b*c*x^2 - 2
1*a^3*d*x^2 - 15*a^3*c)/(a^4*x^7)
```

3.121.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{9/2}} - \frac{\frac{c}{7\,a}-\frac{x^6\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^4}+\frac{x^2\,(a\,d-b\,c)}{5\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{3\,a^3}}{x^7}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)),x)
```

output
```
(b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^
(9/2) - (c/(7*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^4 + (x^2*(a
*d - b*c))/(5*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(3*a^3))/x^7
```

3.122 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$

3.122.1 Optimal result

Integrand size = 30, antiderivative size = 175

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}-\frac{b^2c-abd+a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{a^5x}-\frac{b^{3/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}$$

output
```
-1/9*c/a/x^9+1/7*(-a*d+b*c)/a^2/x^7+1/5*(-a^2*e+a*b*d-b^2*c)/a^3/x^5+1/3*(
-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^3-b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^5
/x-b^(3/2)*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/
2)
```

3.122.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = -\frac{c}{9ax^9}+\frac{bc-ad}{7a^2x^7}+\frac{-b^2c+abd-a^2e}{5a^3x^5}+\frac{b^3c-ab^2d+a^2be-a^3f}{3a^4x^3}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{a^5x}+\frac{b^{3/2}(-b^3c+ab^2d-a^2be+a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{11/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

output
```
-1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) + (-(b^2*c) + a*b*d - a^2*e)/(5*a
^3*x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) + (b*(-(b^3*c) +
 a*b^2*d - a^2*b*e + a^3*f))/(a^5*x) + (b^(3/2)*(-(b^3*c) + a*b^2*d - a^2*
b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

3.122.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{ad-bc}{a^2x^8}+\frac{a^2e-abd+b^2c}{a^3x^6}+\frac{b^2\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5\left(a+bx^2\right)}-\frac{b\left(a^3f-a^2be+ab^2d-b^3c\right)}{a^5x^2}+\frac{a^3f-a^2be+ab^2}{a^4x^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{7a^2x^7}-\frac{a^2e-abd+b^2c}{5a^3x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{11/2}}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^5x}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{3a^4x^3}-\frac{c}{9ax^9}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x]
```

output
```
-1/9*c/(a*x^9) + (b*c - a*d)/(7*a^2*x^7) - (b^2*c - a*b*d + a^2*e)/(5*a^3*
x^5) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(3*a^4*x^3) - (b*(b^3*c - a*b^2
*d + a^2*b*e - a^3*f))/(a^5*x) - (b^(3/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3
*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/a^(11/2)
```

3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.122.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.93

method	result
default	$-\frac{c}{9a\,x^{9}}-\frac{ad-bc}{7a^{2}x^{7}}-\frac{a^{2}e-abd+b^{2}c}{5a^{3}x^{5}}-\frac{f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c}{3a^{4}x^{3}}+\frac{b(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)}{a^{5}x}+\frac{b^{2}(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c)\,\text{arcta}}{a^{5}\sqrt{ab}}$
risch	$\frac{\frac{b\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{8}}{a^{5}}-\frac{\left(f\,a^{3}-a^{2}be+a\,b^{2}d-b^{3}c\right)x^{6}}{3a^{4}}-\frac{\left(a^{2}e-abd+b^{2}c\right)x^{4}}{5a^{3}}-\frac{(ad-bc)x^{2}}{7a^{2}}-\frac{c}{9a}}{x^{9}}+\frac{\sqrt{-ab}\,b\ln\left(-bx-\sqrt{-ab}\right)f}{2a^{3}}-\frac{\sqrt{-ab}\,b}{}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a/x^9-1/7*(a*d-b*c)/a^2/x^7-1/5*(a^2*e-a*b*d+b^2*c)/a^3/x^5-1/3*(a^
3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^3+b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x+b
^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.14

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)}\,dx$$

$$= \left[-\frac{315\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^9\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2+2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)+630\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^8-2}{630\,a^5x^9}\right.$$

$$-\frac{315\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^9\sqrt{\frac{b}{a}}\arctan\left(x\sqrt{\frac{b}{a}}\right)+315\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^8-105\,(ab^3c}{315\,a^5x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/630*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(-b/a)*log((b
*x^2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) + 630*(b^4*c - a*b^3*d + a^2*b^2
*e - a^3*b*f)*x^8 - 210*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 70*a
^4*c + 126*(a^2*b^2*c - a^3*b*d + a^4*e)*x^4 - 90*(a^3*b*c - a^4*d)*x^2)/(
a^5*x^9), -1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^9*sqrt(b/a
)*arctan(x*sqrt(b/a)) + 315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 -
105*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c
 - a^3*b*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)]
```

3.122.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(167) = 334$.

Time = 24.35 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.02

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$$
$$= -\frac{\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^6\sqrt{-\frac{b^3}{a^{11}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^2f-a^2b^3e+ab^4d-b^5c}+x\right)}{2}$$
$$+\frac{-35a^4c+x^8\cdot(315a^3bf-315a^2b^2e+315ab^3d-315b^4c)+x^6(-105a^4f+105a^3be-105a^2b^2d+105ab}{315a^5x^9}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a),x)
```

output
```
-sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**6*sqrt(
-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**2*f - a**2*b
**3*e + a*b**4*d - b**5*c) + x)/2 + sqrt(-b**3/a**11)*(a**3*f - a**2*b*e +
 a*b**2*d - b**3*c)*log(a**6*sqrt(-b**3/a**11)*(a**3*f - a**2*b*e + a*b**2
*d - b**3*c)/(a**3*b**2*f - a**2*b**3*e + a*b**4*d - b**5*c) + x)/2 + (-35
*a**4*c + x**8*(315*a**3*b*f - 315*a**2*b**2*e + 315*a*b**3*d - 315*b**4*c
) + x**6*(-105*a**4*f + 105*a**3*b*e - 105*a**2*b**2*d + 105*a*b**3*c) + x
**4*(-63*a**4*e + 63*a**3*b*d - 63*a**2*b**2*c) + x**2*(-45*a**4*d + 45*a*
*3*b*c))/(315*a**5*x**9)
```

3.122.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = -\frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{315\,(b^4c-ab^3d+a^2b^2e-a^3bf)x^8-105\,(ab^3c-a^2b^2d+a^3be-a^4f)x^6+35\,a^4c+63\,(a^2b^2c-a^3bd+}{315\,a^5x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="maxima")
```

output
```
-(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^5) - 1/315*(315*(b^4*c - a*b^3*d + a^2*b^2*e - a^3*b*f)*x^8 - 105*(a*b
^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x^6 + 35*a^4*c + 63*(a^2*b^2*c - a^3*b
*d + a^4*e)*x^4 - 45*(a^3*b*c - a^4*d)*x^2)/(a^5*x^9)
```

3.122.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx = -\frac{(b^5c-ab^4d+a^2b^3e-a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^5} - \frac{315\,b^4cx^8-315\,ab^3dx^8+315\,a^2b^2ex^8-315\,a^3bfx^8-105\,ab^3cx^6+105\,a^2b^2dx^6-105\,a^3bex^6+105\,a^4f}{315\,a^5x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a),x, algorithm="giac")
```

output
```
-(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b
)*a^5) - 1/315*(315*b^4*c*x^8 - 315*a*b^3*d*x^8 + 315*a^2*b^2*e*x^8 - 315*
a^3*b*f*x^8 - 105*a*b^3*c*x^6 + 105*a^2*b^2*d*x^6 - 105*a^3*b*e*x^6 + 105*
a^4*f*x^6 + 63*a^2*b^2*c*x^4 - 63*a^3*b*d*x^4 + 63*a^4*e*x^4 - 45*a^3*b*c*
x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^5*x^9)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)}\,dx$$
$$= -\frac{\frac{c}{9a} - \frac{x^6(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{3\,a^4} + \frac{x^2\,(a\,d-b\,c)}{7\,a^2} + \frac{x^4\,(e\,a^2-d\,a\,b+c\,b^2)}{5\,a^3} + \frac{b\,x^8\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^5}}{x^9}$$
$$- \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{a^{11/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)),x)
```

output
```
- (c/(9*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^4) + (x^2*(a*d
 - b*c))/(7*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(5*a^3) + (b*x^8*(b^3*c -
 a^3*f - a*b^2*d + a^2*b*e))/a^5)/x^9 - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))
*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^(11/2)
```

3.123 $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$

3.123.1 Optimal result

Integrand size = 30, antiderivative size = 211

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = &-\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7}\\ &+\frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}-\frac{b(b^3c-ab^2d+a^2be-a^3f)}{3a^5x^3}\\ &+\frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x}\\ &+\frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}\end{aligned}$$

output
```
-1/11*c/a/x^11+1/9*(-a*d+b*c)/a^2/x^9+1/7*(-a^2*e+a*b*d-b^2*c)/a^3/x^7+1/5
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^4/x^5-1/3*b*(-a^3*f+a^2*b*e-a*b^2*d+b^3*
c)/a^5/x^3+b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)/a^6/x+b^(5/2)*(-a^3*f+a^2*b*
e-a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(13/2)
```

3.123.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = -\frac{c}{11ax^{11}}+\frac{bc-ad}{9a^2x^9}-\frac{b^2c-abd+a^2e}{7a^3x^7} + \frac{b^3c-ab^2d+a^2be-a^3f}{5a^4x^5}+\frac{b(-b^3c+ab^2d-a^2be+a^3f)}{3a^5x^3} + \frac{b^2(b^3c-ab^2d+a^2be-a^3f)}{a^6x} + \frac{b^{5/2}(b^3c-ab^2d+a^2be-a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{a^{13/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

output
```
-1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^
3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) + (b*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e
- a^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(S
qrt[b]*x)/Sqrt[a]])/a^(13/2)
```

3.123.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$$

$$\Big\downarrow \text{2333}$$

$$\int \left(\frac{ad-bc}{a^2x^{10}}+\frac{a^2e-abd+b^2c}{a^3x^8}-\frac{b^3(a^3f-a^2be+ab^2d-b^3c)}{a^6(a+bx^2)}+\frac{b^2(a^3f-a^2be+ab^2d-b^3c)}{a^6x^2}-\frac{b(a^3f-a^2be+}{a^5x^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{bc-ad}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^{13/2}}+\frac{b^2\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{a^6x}-\frac{b\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{3a^5x^3}+\frac{a^3(-f)+a^2be-ab^2d+b^3c}{5a^4x^5}-\frac{c}{11ax^{11}}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x]
```

output
```
-1/11*c/(a*x^11) + (b*c - a*d)/(9*a^2*x^9) - (b^2*c - a*b*d + a^2*e)/(7*a^
3*x^7) + (b^3*c - a*b^2*d + a^2*b*e - a^3*f)/(5*a^4*x^5) - (b*(b^3*c - a*b
^2*d + a^2*b*e - a^3*f))/(3*a^5*x^3) + (b^2*(b^3*c - a*b^2*d + a^2*b*e - a
^3*f))/(a^6*x) + (b^(5/2)*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(13/2)
```

3.123.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

3.123.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c}{11a\,x^{11}}-\frac{ad-bc}{9a^2x^9}-\frac{a^2e-abd+b^2c}{7a^3x^7}-\frac{f\,a^3-a^2be+a\,b^2d-b^3c}{5a^4x^5}-\frac{b^2\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{a^6x}+\frac{b\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)}{3a^5x^3}-$
risch	$\frac{-\frac{b^2\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^{10}}{a^6}+\frac{b\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^8}{3a^5}-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x^6}{5a^4}-\frac{\left(a^2e-abd+b^2c\right)x^4}{7a^3}-\frac{(ad-bc)x^2}{9a^2}-\frac{c}{11a}}{x^{11}}+\sqrt{-}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/11*c/a/x^11-1/9*(a*d-b*c)/a^2/x^9-1/7*(a^2*e-a*b*d+b^2*c)/a^3/x^7-1/5*(
a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^4/x^5-b^2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^6
/x+1/3*b*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)/a^5/x^3-b^3*(a^3*f-a^2*b*e+a*b^2*d-
b^3*c)/a^6/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2))
```

3.123.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.17

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx$$
$$= \left[-\frac{3465\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x^{11}\sqrt{-\frac{b}{a}}\log\left(\frac{bx^2-2\,ax\sqrt{-\frac{b}{a}}-a}{bx^2+a}\right)-6930\left(b^5c-ab^4d+a^2b^3e-a^3b^2f\right)x\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="fricas")
```

output
```
[-1/6930*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(-b/a)*l
og((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)) - 6930*(b^5*c - a*b^4*d + a
^2*b^3*e - a^3*b^2*f)*x^10 + 2310*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b
*f)*x^8 - 1386*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x^6 + 630*a^5*c +
 990*(a^3*b^2*c - a^4*b*d + a^5*e)*x^4 - 770*(a^4*b*c - a^5*d)*x^2)/(a^6*x
^11), 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^11*sqrt(b/a
)*arctan(x*sqrt(b/a)) + 3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^1
0 - 1155*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c
- a^3*b^2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d
+ a^5*e)*x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)]
```

3.123.6 Sympy [A] (verification not implemented)

Time = 38.92 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx$$
$$= \frac{\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(-\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}$$
$$-\frac{\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)\log\left(\frac{a^7\sqrt{-\frac{b^5}{a^{13}}}(a^3f-a^2be+ab^2d-b^3c)}{a^3b^3f-a^2b^4e+ab^5d-b^6c}+x\right)}{2}$$
$$+\frac{-315a^5c+x^{10}(-3465a^3b^2f+3465a^2b^3e-3465ab^4d+3465b^5c)+x^8\cdot(1155a^4bf-1155a^3b^2e+1155a$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**12/(b*x**2+a),x)
```

output
```
sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)*log(-a**7*sqrt(-
b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(a**3*b**3*f - a**2*b*
*4*e + a*b**5*d - b**6*c) + x)/2 - sqrt(-b**5/a**13)*(a**3*f - a**2*b*e +
a*b**2*d - b**3*c)*log(a**7*sqrt(-b**5/a**13)*(a**3*f - a**2*b*e + a*b**2*
d - b**3*c)/(a**3*b**3*f - a**2*b**4*e + a*b**5*d - b**6*c) + x)/2 + (-315
*a**5*c + x**10*(-3465*a**3*b**2*f + 3465*a**2*b**3*e - 3465*a*b**4*d + 34
65*b**5*c) + x**8*(1155*a**4*b*f - 1155*a**3*b**2*e + 1155*a**2*b**3*d - 1
155*a*b**4*c) + x**6*(-693*a**5*f + 693*a**4*b*e - 693*a**3*b**2*d + 693*a
**2*b**3*c) + x**4*(-495*a**5*e + 495*a**4*b*d - 495*a**3*b**2*c) + x**2*(
-385*a**5*d + 385*a**4*b*c))/(3465*a**6*x**11)
```

3.123.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}\left(a+bx^2\right)}\,dx = \frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6}$$
$$+\frac{3465\,(b^5c-ab^4d+a^2b^3e-a^3b^2f)x^{10}-1155\,(ab^4c-a^2b^3d+a^3b^2e-a^4bf)x^8+693\,(a^2b^3c-a^3b^2d+a}{3465\,a^6x^{11}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="maxima")
```

output
```
(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^6) + 1/3465*(3465*(b^5*c - a*b^4*d + a^2*b^3*e - a^3*b^2*f)*x^10 - 1155
*(a*b^4*c - a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^8 + 693*(a^2*b^3*c - a^3*b^
2*d + a^4*b*e - a^5*f)*x^6 - 315*a^5*c - 495*(a^3*b^2*c - a^4*b*d + a^5*e)
*x^4 + 385*(a^4*b*c - a^5*d)*x^2)/(a^6*x^11)
```

3.123.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.16

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = \frac{(b^6c-ab^5d+a^2b^4e-a^3b^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}a^6} + \frac{3465\,b^5cx^{10}-3465\,ab^4dx^{10}+3465\,a^2b^3ex^{10}-3465\,a^3b^2fx^{10}-1155\,ab^4cx^8+1155\,a^2b^3dx^8-1155\,a^3b^2}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^12/(b*x^2+a),x, algorithm="giac")
```

output
```
(b^6*c - a*b^5*d + a^2*b^4*e - a^3*b^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a^6) + 1/3465*(3465*b^5*c*x^10 - 3465*a*b^4*d*x^10 + 3465*a^2*b^3*e*x^10
- 3465*a^3*b^2*f*x^10 - 1155*a*b^4*c*x^8 + 1155*a^2*b^3*d*x^8 - 1155*a^3*b
^2*e*x^8 + 1155*a^4*b*f*x^8 + 693*a^2*b^3*c*x^6 - 693*a^3*b^2*d*x^6 + 693*
a^4*b*e*x^6 - 693*a^5*f*x^6 - 495*a^3*b^2*c*x^4 + 495*a^4*b*d*x^4 - 495*a^
5*e*x^4 + 385*a^4*b*c*x^2 - 385*a^5*d*x^2 - 315*a^5*c)/(a^6*x^11)
```

3.123.9 Mupad [B] (verification not implemented)

Time = 5.86 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx = \frac{b^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{a^{13/2}} - \frac{\frac{c}{11\,a}-\frac{x^6\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{5\,a^4}+\frac{x^2\,\left(a\,d-b\,c\right)}{9\,a^2}+\frac{x^4\,\left(e\,a^2-d\,a\,b+c\,b^2\right)}{7\,a^3}+\frac{b\,x^8\,\left(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3\right)}{3\,a^5}-\frac{b^2\,x^{10}\,\left(-f\,a^3+e\,a^2\right.}{a^6}}{x^{11}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^12*(a + b*x^2)),x)
```

output
```
(b^(5/2)*atan((b^(1/2)*x)/a^(1/2))*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/a^
(13/2) - (c/(11*a) - (x^6*(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(5*a^4) + (
x^2*(a*d - b*c))/(9*a^2) + (x^4*(b^2*c + a^2*e - a*b*d))/(7*a^3) + (b*x^8*
(b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(3*a^5) - (b^2*x^10*(b^3*c - a^3*f -
a*b^2*d + a^2*b*e))/a^6)/x^11
```

3.123. $\int \frac{c+dx^2+ex^4+fx^6}{x^{12}(a+bx^2)}\,dx$

3.124 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

3.124.1 Optimal result

Integrand size = 30, antiderivative size = 240

$$\begin{aligned}\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx =& -\frac{a(5b^3c-7ab^2d+9a^2be-11a^3f)\,x}{2b^6} \\ &+ \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^3}{6b^5} \\ &- \frac{(5b^3c-7ab^2d+9a^2be-11a^3f)\,x^5}{10ab^4} \\ &+ \frac{(be-2af)x^7}{7b^3} + \frac{fx^9}{9b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{2a\,(a+bx^2)} \\ &+ \frac{a^{3/2}(5b^3c-7ab^2d+9a^2be-11a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}\end{aligned}$$

output
```
-1/2*a*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c)*x/b^6+1/6*(-11*a^3*f+9*a^2*
b*e-7*a*b^2*d+5*b^3*c)*x^3/b^5-1/10*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^3*c
)*x^5/a/b^4+1/7*(-2*a*f+b*e)*x^7/b^3+1/9*f*x^9/b^2+1/2*(c-a*(a^2*f-a*b*e+b
^2*d)/b^3)*x^7/a/(b*x^2+a)+1/2*a^(3/2)*(-11*a^3*f+9*a^2*b*e-7*a*b^2*d+5*b^
3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(13/2)
```

3.124.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{a(-2b^3c+3ab^2d-4a^2be+5a^3f)\,x}{b^6} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x^3}{3b^5} + \frac{(b^2d-2abe+3a^2f)\,x^5}{5b^4} + \frac{(be-2af)x^7}{7b^3} + \frac{fx^9}{9b^2} - \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\,x}{2b^6\,(a+bx^2)} - \frac{a^{3/2}(-5b^3c+7ab^2d-9a^2be+11a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{13/2}}$$

input
```
Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
(a*(-2*b^3*c + 3*a*b^2*d - 4*a^2*b*e + 5*a^3*f)*x)/b^6 + ((b^3*c - 2*a*b^2
*d + 3*a^2*b*e - 4*a^3*f)*x^3)/(3*b^5) + ((b^2*d - 2*a*b*e + 3*a^2*f)*x^5)
/(5*b^4) + ((b*e - 2*a*f)*x^7)/(7*b^3) + (f*x^9)/(9*b^2) - ((a^2*b^3*c - a
^3*b^2*d + a^4*b*e - a^5*f)*x)/(2*b^6*(a + b*x^2)) - (a^(3/2)*(-5*b^3*c +
7*a*b^2*d - 9*a^2*b*e + 11*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*b^(13/2)
)
```

3.124.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2335, 9, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a\,(a+bx^2)} - \frac{\int \frac{x^5\left(-2afx^5-2a\left(e-\frac{af}{b}\right)x^3+\left(-\frac{7fa^3}{b^2}+\frac{7ea^2}{b}-7da+5bc\right)x\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^7\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^6\left(-2afx^4-2a\left(e-\frac{af}{b}\right)x^2+5bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 1584$$

$$\frac{x^7\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-$$

$$\frac{\int \left(-\frac{2afx^8}{b}-\frac{2a(be-2af)x^6}{b^2}+\frac{\left(-11fa^3+9bea^2-7b^2da+5b^3c\right)x^4}{b^3}-\frac{a\left(-11fa^3+9bea^2-7b^2da+5b^3c\right)x^2}{b^4}+\frac{a^2\left(-11fa^3+9bea^2-7b^2da+5b^3\right.}{b^5}}{2ab}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^7\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-$$

$$\frac{\frac{x^5\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{5b^3}+\frac{a^2x\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{b^5}-\frac{ax^3\left(-11a^3f+9a^2be-7ab^2d+5b^3c\right)}{3b^4}-\frac{a^{5/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-11a^3f-\right.}{b^{11/2}}}{2ab}$$

input
```
Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(2*a*(a + b*x^2)) - ((a^2*(5*b
^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*x))/b^5 - (a*(5*b^3*c - 7*a*b^2*d
+ 9*a^2*b*e - 11*a^3*f)*x^3)/(3*b^4) + ((5*b^3*c - 7*a*b^2*d + 9*a^2*b*e -
 11*a^3*f)*x^5)/(5*b^3) - (2*a*(b*e - 2*a*f)*x^7)/(7*b^2) - (2*a*f*x^9)/(9
*b) - (a^(5/2)*(5*b^3*c - 7*a*b^2*d + 9*a^2*b*e - 11*a^3*f)*ArcTan[(Sqrt[b
]*x)/Sqrt[a]])/b^(11/2))/(2*a*b)
```

3.124.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

3.124. $\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}\,dx$

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.124.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{1}{9}f\,x^9b^4-\frac{2}{7}a\,b^3f\,x^7+\frac{1}{7}b^4e\,x^7+\frac{3}{5}a^2b^2f\,x^5-\frac{2}{5}a\,b^3e\,x^5+\frac{1}{5}b^4d\,x^5-\frac{4}{3}a^3bf\,x^3+a^2b^2e\,x^3-\frac{2}{3}a\,b^3d\,x^3+\frac{1}{3}b^4c\,x^3+5a^4fx-4a^3bex+3a^2b^2dx}{b^6}$
risch	$\frac{f\,x^9}{9b^2}-\frac{2af\,x^7}{7b^3}+\frac{e\,x^7}{7b^2}+\frac{3a^2f\,x^5}{5b^4}-\frac{2ae\,x^5}{5b^3}+\frac{d\,x^5}{5b^2}-\frac{4a^3f\,x^3}{3b^5}+\frac{a^2e\,x^3}{b^4}-\frac{2ad\,x^3}{3b^3}+\frac{c\,x^3}{3b^2}+\frac{5a^4fx}{b^6}-\frac{4a^3ex}{b^5}+\frac{3a^2dx}{b^4}$

input
```
int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^6*(1/9*f*x^9*b^4-2/7*a*b^3*f*x^7+1/7*b^4*e*x^7+3/5*a^2*b^2*f*x^5-2/5*a
*b^3*e*x^5+1/5*b^4*d*x^5-4/3*a^3*b*f*x^3+a^2*b^2*e*x^3-2/3*a*b^3*d*x^3+1/3
*b^4*c*x^3+5*a^4*f*x-4*a^3*b*e*x+3*a^2*b^2*d*x-2*a*b^3*c*x)-a^2/b^6*((-1/2
*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(11*a^3*f-9*a^2*
b*e+7*a*b^2*d-5*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.124.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 572, normalized size of antiderivative = 2.38

$$\int \frac{x^6(c + dx^2 + ex^4 + fx^6)}{(a + bx^2)^2} \, dx$$

$$= \left[\frac{140\, b^5 f x^{11} + 20\,(9\, b^5 e - 11\, ab^4 f)x^9 + 36\,(7\, b^5 d - 9\, ab^4 e + 11\, a^2 b^3 f)x^7 + 84\,(5\, b^5 c - 7\, ab^4 d + 9\, a^2 b^3 e - 1}{} \right.$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/1260*(140*b^5*f*x^11 + 20*(9*b^5*e - 11*a*b^4*f)*x^9 + 36*(7*b^5*d - 9*
a*b^4*e + 11*a^2*b^3*f)*x^7 + 84*(5*b^5*c - 7*a*b^4*d + 9*a^2*b^3*e - 11*a
^3*b^2*f)*x^5 - 420*(5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x
^3 - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f + (5*a*b^4*c -
7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 - 2*b*x
*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e
 - 11*a^5*f)*x)/(b^7*x^2 + a*b^6), 1/630*(70*b^5*f*x^11 + 10*(9*b^5*e - 11
*a*b^4*f)*x^9 + 18*(7*b^5*d - 9*a*b^4*e + 11*a^2*b^3*f)*x^7 + 42*(5*b^5*c
- 7*a*b^4*d + 9*a^2*b^3*e - 11*a^3*b^2*f)*x^5 - 210*(5*a*b^4*c - 7*a^2*b^3
*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^3 + 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^
4*b*e - 11*a^5*f + (5*a*b^4*c - 7*a^2*b^3*d + 9*a^3*b^2*e - 11*a^4*b*f)*x^
2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) - 315*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*
a^4*b*e - 11*a^5*f)*x)/(b^7*x^2 + a*b^6)]
```

3.124.6 Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.85

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= x^7\left(-\frac{2af}{7b^3}+\frac{e}{7b^2}\right)+x^5\cdot\left(\frac{3a^2f}{5b^4}-\frac{2ae}{5b^3}+\frac{d}{5b^2}\right)+x^3\left(-\frac{4a^3f}{3b^5}+\frac{a^2e}{b^4}-\frac{2ad}{3b^3}+\frac{c}{3b^2}\right)$$
$$+x\left(\frac{5a^4f}{b^6}-\frac{4a^3e}{b^5}+\frac{3a^2d}{b^4}-\frac{2ac}{b^3}\right)+\frac{x(a^5f-a^4be+a^3b^2d-a^2b^3c)}{2ab^6+2b^7x^2}$$
$$+\frac{\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)\log\left(-\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)}{11a^4f-9a^3be+7a^2b^2d-5ab^3c}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)\log\left(\frac{b^6\sqrt{-\frac{a^3}{b^{13}}}\cdot(11a^3f-9a^2be+7ab^2d-5b^3c)}{11a^4f-9a^3be+7a^2b^2d-5ab^3c}+x\right)}{4}$$
$$+\frac{fx^9}{9b^2}$$

input
```
integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x**7*(-2*a*f/(7*b**3) + e/(7*b**2)) + x**5*(3*a**2*f/(5*b**4) - 2*a*e/(5*b
**3) + d/(5*b**2)) + x**3*(-4*a**3*f/(3*b**5) + a**2*e/b**4 - 2*a*d/(3*b**
3) + c/(3*b**2)) + x*(5*a**4*f/b**6 - 4*a**3*e/b**5 + 3*a**2*d/b**4 - 2*a*
c/b**3) + x*(a**5*f - a**4*b*e + a**3*b**2*d - a**2*b**3*c)/(2*a*b**6 + 2*
b**7*x**2) + sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b*
*3*c)*log(-b**6*sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5
*b**3*c)/(11*a**4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 - sq
rt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)*log(b**6*
sqrt(-a**3/b**13)*(11*a**3*f - 9*a**2*b*e + 7*a*b**2*d - 5*b**3*c)/(11*a**
4*f - 9*a**3*b*e + 7*a**2*b**2*d - 5*a*b**3*c) + x)/4 + f*x**9/(9*b**2)
```

3.124.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= -\frac{(a^2b^3c - a^3b^2d + a^4be - a^5f)x}{2\,(b^7x^2 + ab^6)} + \frac{(5\,a^2b^3c - 7\,a^3b^2d + 9\,a^4be - 11\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6}$$
$$+ \frac{35\,b^4fx^9 + 45\,(b^4e - 2\,ab^3f)x^7 + 63\,(b^4d - 2\,ab^3e + 3\,a^2b^2f)x^5 + 105\,(b^4c - 2\,ab^3d + 3\,a^2b^2e - 4\,a^3bf)x}{315\,b^6}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*x/(b^7*x^2 + a*b^6) + 1/2*(
5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b))/(s
qrt(a*b)*b^6) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e - 2*a*b^3*f)*x^7 + 63*(b^4
*d - 2*a*b^3*e + 3*a^2*b^2*f)*x^5 + 105*(b^4*c - 2*a*b^3*d + 3*a^2*b^2*e -
 4*a^3*b*f)*x^3 - 315*(2*a*b^3*c - 3*a^2*b^2*d + 4*a^3*b*e - 5*a^4*f)*x)/b
^6
```

3.124.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.02

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= \frac{(5\,a^2b^3c - 7\,a^3b^2d + 9\,a^4be - 11\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^6} - \frac{a^2b^3cx - a^3b^2dx + a^4bex - a^5fx}{2\,(bx^2+a)b^6}$$
$$+ \frac{35\,b^{16}fx^9 + 45\,b^{16}ex^7 - 90\,ab^{15}fx^7 + 63\,b^{16}dx^5 - 126\,ab^{15}ex^5 + 189\,a^2b^{14}fx^5 + 105\,b^{16}cx^3 - 210\,ab^{15}dx}{315\,b^{18}}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*a^2*b^3*c - 7*a^3*b^2*d + 9*a^4*b*e - 11*a^5*f)*arctan(b*x/sqrt(a*b
))/(sqrt(a*b)*b^6) - 1/2*(a^2*b^3*c*x - a^3*b^2*d*x + a^4*b*e*x - a^5*f*x)
/((b*x^2 + a)*b^6) + 1/315*(35*b^16*f*x^9 + 45*b^16*e*x^7 - 90*a*b^15*f*x^
7 + 63*b^16*d*x^5 - 126*a*b^15*e*x^5 + 189*a^2*b^14*f*x^5 + 105*b^16*c*x^3
 - 210*a*b^15*d*x^3 + 315*a^2*b^14*e*x^3 - 420*a^3*b^13*f*x^3 - 630*a*b^15
*c*x + 945*a^2*b^14*d*x - 1260*a^3*b^13*e*x + 1575*a^4*b^12*f*x)/b^18
```

3.124.9 Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.72

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= x^7\left(\frac{e}{7\,b^2}-\frac{2\,a\,f}{7\,b^3}\right)-x\left(\frac{2\,a\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)}{b}-\frac{a^2\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b^2}\right)-x^5\left(\frac{a^2\,f}{5\,b^4}-\frac{d}{5\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{5\,b}\right)$$
$$+x^3\left(\frac{c}{3\,b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{3\,b}\right)$$
$$+\frac{f\,x^9}{9\,b^2}+\frac{x\left(\frac{f\,a^5}{2}-\frac{e\,a^4\,b}{2}+\frac{d\,a^3\,b^2}{2}-\frac{c\,a^2\,b^3}{2}\right)}{b^7\,x^2+a\,b^6}$$
$$-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{11\,f\,a^5-9\,e\,a^4\,b+7\,d\,a^3\,b^2-5\,c\,a^2\,b^3}\right)\,\left(-11\,f\,a^3+9\,e\,a^2\,b-7\,d\,a\,b^2+5\,c\,b^3\right)}{2\,b^{13/2}}$$

input
```
int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

output
```
x^7*(e/(7*b^2) - (2*a*f)/(7*b^3)) - x*((2*a*(c/b^2 - (a^2*(e/b^2 - (2*a*f)
/b^3))/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b
))/b - (a^2*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b^2) -
x^5*((a^2*f)/(5*b^4) - d/(5*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(5*b)) + x^
3*(c/(3*b^2) - (a^2*(e/b^2 - (2*a*f)/b^3))/(3*b^2) + (2*a*((a^2*f)/b^4 - d
/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/(3*b)) + (f*x^9)/(9*b^2) + (x*((a^5
*f)/2 - (a^2*b^3*c)/2 + (a^3*b^2*d)/2 - (a^4*b*e)/2))/(a*b^6 + b^7*x^2) -
(a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(5*b^3*c - 11*a^3*f - 7*a*b^2*d + 9*a^2*b
*e))/(11*a^5*f - 5*a^2*b^3*c + 7*a^3*b^2*d - 9*a^4*b*e))*(5*b^3*c - 11*a^3
*f - 7*a*b^2*d + 9*a^2*b*e))/(2*b^(13/2))
```

3.124. $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

3.125 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

3.125.1 Optimal result

Integrand size = 30, antiderivative size = 202

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x}{2b^5} - \frac{(3b^3c-5ab^2d+7a^2be-9a^3f)\,x^3}{6ab^4} + \frac{(be-2af)x^5}{5b^3} + \frac{fx^7}{7b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{2a\,(a+bx^2)} - \frac{\sqrt{a}(3b^3c-5ab^2d+7a^2be-9a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

output
```
1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2*d+3*b^3*c)*x/b^5-1/6*(-9*a^3*f+7*a^2*b*e-5
*a*b^2*d+3*b^3*c)*x^3/a/b^4+1/5*(-2*a*f+b*e)*x^5/b^3+1/7*f*x^7/b^2+1/2*(c-
a*(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)-1/2*(-9*a^3*f+7*a^2*b*e-5*a*b^2
*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(11/2)
```

3.125.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\,x}{b^5}+\frac{(b^2d-2abe+3a^2f)\,x^3}{3b^4} + \frac{(be-2af)x^5}{5b^3}+\frac{fx^7}{7b^2}+\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\,x}{2b^5\,(a+bx^2)} + \frac{\sqrt{a}(-3b^3c+5ab^2d-7a^2be+9a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
((b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*x)/b^5 + ((b^2*d - 2*a*b*e + 3*
a^2*f)*x^3)/(3*b^4) + ((b*e - 2*a*f)*x^5)/(5*b^3) + (f*x^7)/(7*b^2) + ((a*
b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x)/(2*b^5*(a + b*x^2)) + (Sqrt[a]*(-3
*b^3*c + 5*a*b^2*d - 7*a^2*b*e + 9*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*
b^(11/2))
```

3.125.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2335, 9, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^3\left(-2afx^5-2a\left(e-\frac{af}{b}\right)x^3+\left(-\frac{5fa^3}{b^2}+\frac{5ea^2}{b}-5da+3bc\right)x\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \ 9$$

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \frac{x^4\left(-2afx^4-2a\left(e-\frac{af}{b}\right)x^2+3bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)}{bx^2+a}dx}{2ab}$$

$$\Big\downarrow \; 1584$$

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\int \left(-\frac{2afx^6}{b}-\frac{2a(be-2af)x^4}{b^2}+\frac{(-9fa^3+7bea^2-5b^2da+3b^3c)x^2}{b^3}-\frac{a(-9fa^3+7bea^2-5b^2da+3b^3c)}{b^4}+\frac{-9fa^5+7bea^4-5b^2da^3+3b^3ca^2}{b^4(bx^2+a)}\right)dx}{2ab}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{2a\left(a+bx^2\right)}-\frac{\frac{x^3(-9a^3f+7a^2be-5ab^2d+3b^3c)}{3b^3}-\frac{ax(-9a^3f+7a^2be-5ab^2d+3b^3c)}{b^4}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-9a^3f+7a^2be-5ab^2d+3b^3c)}{b^{9/2}}-\frac{2ax^5(be-2af)}{5b^2}-}{2ab}$$

input
```
Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(2*a*(a + b*x^2)) - (-((a*(3*b
^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*x)/b^4) + ((3*b^3*c - 5*a*b^2*d +
7*a^2*b*e - 9*a^3*f)*x^3)/(3*b^3) - (2*a*(b*e - 2*a*f)*x^5)/(5*b^2) - (2*a
*f*x^7)/(7*b) + (a^(3/2)*(3*b^3*c - 5*a*b^2*d + 7*a^2*b*e - 9*a^3*f)*ArcTa
n[(Sqrt[b]*x)/Sqrt[a]])/b^(9/2))/(2*a*b)
```

3.125.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.125.4 Maple [A] (verified)

Time = 3.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.90

method	result
default	$-\frac{-\frac{1}{7} f x^7 b^3+\frac{2}{5} a b^2 f x^5-\frac{1}{5} b^3 e x^5-a^2 b f x^3+\frac{2}{3} a b^2 e x^3-\frac{1}{3} b^3 d x^3+4 f a^3 x-3 a^2 b e x+2 a b^2 d x-b^3 c x}{b^5}+\frac{a\left(\frac{\left(-\frac{1}{2} f a^3+\frac{1}{2} a^2 b e-\frac{1}{2} a b^2 d+\frac{1}{2}\right.}{b x^2+a}\right.}{}$
risch	$\frac{f x^7}{7b^2}-\frac{2af x^5}{5b^3}+\frac{e x^5}{5b^2}+\frac{a^2 f x^3}{b^4}-\frac{2ae x^3}{3b^3}+\frac{d x^3}{3b^2}-\frac{4f a^3 x}{b^5}+\frac{3a^2 ex}{b^4}-\frac{2adx}{b^3}+\frac{cx}{b^2}+\frac{\left(-\frac{1}{2}a^4 f+\frac{1}{2}a^3 be-\frac{1}{2}a^2 b^2 d+\frac{1}{2}a b^3\right.}{b^5(b x^2+a)}$

input
```
int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^5*(-1/7*f*x^7*b^3+2/5*a*b^2*f*x^5-1/5*b^3*e*x^5-a^2*b*f*x^3+2/3*a*b^2
*e*x^3-1/3*b^3*d*x^3+4*f*a^3*x-3*a^2*b*e*x+2*a*b^2*d*x-b^3*c*x)+a/b^5*((-1
/2*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*(9*a^3*f-7*a^2
*b*e+5*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.125.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.37

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$

$$= \left[\frac{60\, b^4 f x^9 + 12\,(7\, b^4 e - 9\, ab^3 f) x^7 + 28\,(5\, b^4 d - 7\, ab^3 e + 9\, a^2 b^2 f) x^5 + 140\,(3\, b^4 c - 5\, ab^3 d + 7\, a^2 b^2 e - 9\, a^3}{}\right.$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/420*(60*b^4*f*x^9 + 12*(7*b^4*e - 9*a*b^3*f)*x^7 + 28*(5*b^4*d - 7*a*b^
3*e + 9*a^2*b^2*f)*x^5 + 140*(3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*
f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f + (3*b^4*c - 5
*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqr
t(-a/b) - a)/(b*x^2 + a)) + 210*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a
^4*f)*x)/(b^6*x^2 + a*b^5), 1/210*(30*b^4*f*x^9 + 6*(7*b^4*e - 9*a*b^3*f)*
x^7 + 14*(5*b^4*d - 7*a*b^3*e + 9*a^2*b^2*f)*x^5 + 70*(3*b^4*c - 5*a*b^3*d
 + 7*a^2*b^2*e - 9*a^3*b*f)*x^3 - 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e
 - 9*a^4*f + (3*b^4*c - 5*a*b^3*d + 7*a^2*b^2*e - 9*a^3*b*f)*x^2)*sqrt(a/b
)*arctan(b*x*sqrt(a/b)/a) + 105*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a
^4*f)*x)/(b^6*x^2 + a*b^5)]
```

3.125.6 Sympy [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.27

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= x^5\left(-\frac{2af}{5b^3}+\frac{e}{5b^2}\right)+x^3\left(\frac{a^2f}{b^4}-\frac{2ae}{3b^3}+\frac{d}{3b^2}\right)$$
$$+x\left(-\frac{4a^3f}{b^5}+\frac{3a^2e}{b^4}-\frac{2ad}{b^3}+\frac{c}{b^2}\right)+\frac{x(-a^4f+a^3be-a^2b^2d+ab^3c)}{2ab^5+2b^6x^2}$$
$$-\frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^3f-7a^2be+5ab^2d-3b^3c)\log\left(-b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{a}{b^{11}}}\cdot(9a^3f-7a^2be+5ab^2d-3b^3c)\log\left(b^5\sqrt{-\frac{a}{b^{11}}}+x\right)}{4}+\frac{fx^7}{7b^2}$$

input
```
integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x**5*(-2*a*f/(5*b**3) + e/(5*b**2)) + x**3*(a**2*f/b**4 - 2*a*e/(3*b**3) +
 d/(3*b**2)) + x*(-4*a**3*f/b**5 + 3*a**2*e/b**4 - 2*a*d/b**3 + c/b**2) +
x*(-a**4*f + a**3*b*e - a**2*b**2*d + a*b**3*c)/(2*a*b**5 + 2*b**6*x**2) -
 sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d - 3*b**3*c)*log(-b**5*
sqrt(-a/b**11) + x)/4 + sqrt(-a/b**11)*(9*a**3*f - 7*a**2*b*e + 5*a*b**2*d
 - 3*b**3*c)*log(b**5*sqrt(-a/b**11) + x)/4 + f*x**7/(7*b**2)
```

3.125.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.91

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= \frac{(ab^3c-a^2b^2d+a^3be-a^4f)x}{2\,(b^6x^2+ab^5)} - \frac{(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5}$$
$$+ \frac{15\,b^3fx^7+21\,(b^3e-2\,ab^2f)x^5+35\,(b^3d-2\,ab^2e+3\,a^2bf)x^3+105\,(b^3c-2\,ab^2d+3\,a^2be-4\,a^3f)x}{105\,b^5}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*x/(b^6*x^2 + a*b^5) - 1/2*(3*a
*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*b^5) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 2*a*b^2*f)*x^5 + 35*(b^3*d - 2
*a*b^2*e + 3*a^2*b*f)*x^3 + 105*(b^3*c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*
x)/b^5
```

3.125.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.97

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= -\frac{(3\,ab^3c-5\,a^2b^2d+7\,a^3be-9\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^5} + \frac{ab^3cx-a^2b^2dx+a^3bex-a^4fx}{2\,(bx^2+a)b^5}$$
$$+ \frac{15\,b^{12}fx^7+21\,b^{12}ex^5-42\,ab^{11}fx^5+35\,b^{12}dx^3-70\,ab^{11}ex^3+105\,a^2b^{10}fx^3+105\,b^{12}cx-210\,ab^{11}dx+}{105\,b^{14}}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(3*a*b^3*c - 5*a^2*b^2*d + 7*a^3*b*e - 9*a^4*f)*arctan(b*x/sqrt(a*b))
/(sqrt(a*b)*b^5) + 1/2*(a*b^3*c*x - a^2*b^2*d*x + a^3*b*e*x - a^4*f*x)/((b
*x^2 + a)*b^5) + 1/105*(15*b^12*f*x^7 + 21*b^12*e*x^5 - 42*a*b^11*f*x^5 +
35*b^12*d*x^3 - 70*a*b^11*e*x^3 + 105*a^2*b^10*f*x^3 + 105*b^12*c*x - 210*
a*b^11*d*x + 315*a^2*b^10*e*x - 420*a^3*b^9*f*x)/b^14
```

3.125.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.43

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$= x^5\left(\frac{e}{5\,b^2}-\frac{2\,a\,f}{5\,b^3}\right)+x\left(\frac{c}{b^2}-\frac{a^2\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b^2}+\frac{2\,a\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right)}{b}\right)$$
$$-x^3\left(\frac{a^2\,f}{3\,b^4}-\frac{d}{3\,b^2}+\frac{2\,a\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{3\,b}\right)-\frac{x\left(\frac{f\,a^4}{2}-\frac{e\,a^3\,b}{2}+\frac{d\,a^2\,b^2}{2}-\frac{c\,a\,b^3}{2}\right)}{b^6\,x^2+a\,b^5}+\frac{f\,x^7}{7\,b^2}$$
$$+\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-9\,f\,a^3+7\,e\,a^2\,b-5\,d\,a\,b^2+3\,c\,b^3\right)}{9\,f\,a^4-7\,e\,a^3\,b+5\,d\,a^2\,b^2-3\,c\,a\,b^3}\right)\,\left(-9\,f\,a^3+7\,e\,a^2\,b-5\,d\,a\,b^2+3\,c\,b^3\right)}{2\,b^{11/2}}$$

input
```
int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

output
```
x^5*(e/(5*b^2) - (2*a*f)/(5*b^3)) + x*(c/b^2 - (a^2*(e/b^2 - (2*a*f)/b^3))
/b^2 + (2*a*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 - (2*a*f)/b^3))/b))/b) - x^
3*((a^2*f)/(3*b^4) - d/(3*b^2) + (2*a*(e/b^2 - (2*a*f)/b^3))/(3*b)) - (x*(
(a^4*f)/2 + (a^2*b^2*d)/2 - (a*b^3*c)/2 - (a^3*b*e)/2))/(a*b^5 + b^6*x^2)
+ (f*x^7)/(7*b^2) + (a^(1/2)*atan((a^(1/2)*b^(1/2)*x*(3*b^3*c - 9*a^3*f -
5*a*b^2*d + 7*a^2*b*e))/(9*a^4*f + 5*a^2*b^2*d - 3*a*b^3*c - 7*a^3*b*e))*(
3*b^3*c - 9*a^3*f - 5*a*b^2*d + 7*a^2*b*e))/(2*b^(11/2))
```

3.126 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$

3.126.1 Optimal result

Integrand size = 30, antiderivative size = 163

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = -\frac{(b^3c-3ab^2d+5a^2be-7a^3f)\,x}{2ab^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{2a\,(a+bx^2)} + \frac{(b^3c-3ab^2d+5a^2be-7a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

output
```
-1/2*(-7*a^3*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*x/a/b^4+1/3*(-2*a*f+b*e)*x^3/b^3
+1/5*f*x^5/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(b*x^2+a)+1/2*(-7*a
^3*f+5*a^2*b*e-3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(9/2)/a^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(b^2d-2abe+3a^2f)\,x}{b^4} + \frac{(be-2af)x^3}{3b^3} + \frac{fx^5}{5b^2} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{2b^4\,(a+bx^2)} - \frac{(-b^3c+3ab^2d-5a^2be+7a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2\sqrt{a}b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
((b^2*d - 2*a*b*e + 3*a^2*f)*x)/b^4 + ((b*e - 2*a*f)*x^3)/(3*b^3) + (f*x^5
)/(5*b^2) - ((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*b^4*(a + b*x^2)) -
((-(b^3*c) + 3*a*b^2*d - 5*a^2*b*e + 7*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(2*Sqrt[a]*b^(9/2))
```

3.126.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.01, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2335, 9, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+d x^2+e x^4+f x^6\right)}{\left(a+b x^2\right)^2} \, dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{2 a \left(a+b x^2\right)}-\frac{\int \frac{x \left(-2 a f x^5-2 a \left(e-\frac{a f}{b}\right) x^3+\left(-\frac{3 f a^3}{b^2}+\frac{3 e a^2}{b}-3 d a+b c\right) x\right)}{b x^2+a}dx}{2 a b}$$

$$\Big\downarrow \ 9$$

$$\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{2 a \left(a+b x^2\right)}-\frac{\int \frac{x^2 \left(-2 a f x^4-2 a \left(e-\frac{a f}{b}\right) x^2+b c-3 a d+\frac{3 a^2 e}{b}-\frac{3 a^3 f}{b^2}\right)}{b x^2+a}dx}{2 a b}$$

$$\Big\downarrow \ 1584$$

$$\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{2 a \left(a+b x^2\right)}-\frac{\int \left(-\frac{2 a f x^4}{b}-\frac{2 a (b e-2 a f) x^2}{b^2}+c-\frac{a \left(7 f a^2-5 b e a+3 b^2 d\right)}{b^3}+\frac{7 f a^4-5 b e a^3+3 b^2 d a^2-b^3 c a}{b^3 \left(b x^2+a\right)}\right)dx}{2 a b}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{x\left(c-\frac{a(7a^2f-5abe+3b^2d)}{b^3}\right)-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-7a^3f+5a^2be-3ab^2d+b^3c)}{b^{7/2}}-\frac{2ax^3(be-2af)}{3b^2}-\frac{2afx^5}{5b}}{2ab}}{}$$

input
```
Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)/(2*a*(a + b*x^2)) - ((c - (a*(
3*b^2*d - 5*a*b*e + 7*a^2*f))/b^3)*x - (2*a*(b*e - 2*a*f)*x^3)/(3*b^2) - (
2*a*f*x^5)/(5*b) - (Sqrt[a]*(b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*ArcT
an[(Sqrt[b]*x)/Sqrt[a]])/b^(7/2))/(2*a*b)
```

3.126.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.126.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.85

method	result
default	$\frac{\frac{1}{5} f x^5 b^2-\frac{2}{3} a b f x^3+\frac{1}{3} b^2 e x^3+3 a^2 f x-2 a b e x+b^2 d x}{b^4}-\frac{\frac{\left(-\frac{1}{2} f a^3+\frac{1}{2} a^2 b e-\frac{1}{2} a b^2 d+\frac{1}{2} b^3 c\right) x}{b x^2+a}+\frac{\left(7 f a^3-5 a^2 b e+3 a b^2 d-b^3 c\right) \arctan\left(\frac{b x}{\sqrt{a b}}\right)}{2 \sqrt{a b}}}{b^4}$
risch	$\frac{f x^5}{5 b^2}-\frac{2 a f x^3}{3 b^3}+\frac{e x^3}{3 b^2}+\frac{3 a^2 f x}{b^4}-\frac{2 a e x}{b^3}+\frac{d x}{b^2}+\frac{\left(\frac{1}{2} f a^3-\frac{1}{2} a^2 b e+\frac{1}{2} a b^2 d-\frac{1}{2} b^3 c\right) x}{b^4\left(b x^2+a\right)}-\frac{7 \ln \left(b x-\sqrt{-a b}\right) f a^3}{4 b^4 \sqrt{-a b}}+\frac{5 \ln \left(b x-\sqrt{-a\right.}}{4 b^3 \sqrt{-a}}$

input
```
int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^4*(1/5*f*x^5*b^2-2/3*a*b*f*x^3+1/3*b^2*e*x^3+3*a^2*f*x-2*a*b*e*x+b^2*d
*x)-1/b^4*((-1/2*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*x/(b*x^2+a)+1/2*
(7*a^3*f-5*a^2*b*e+3*a*b^2*d-b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.126.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.56

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx$$
$$=\left[\frac{12\,ab^4fx^7+4\,(5\,ab^4e-7\,a^2b^3f)x^5+20\,(3\,ab^4d-5\,a^2b^3e+7\,a^3b^2f)x^3+15\,(ab^3c-3\,a^2b^2d+5\,a^3be-7}{60\,(a}\right.$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/60*(12*a*b^4*f*x^7 + 4*(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 20*(3*a*b^4*d -
5*a^2*b^3*e + 7*a^3*b^2*f)*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7
*a^4*f + (b^4*c - 3*a*b^3*d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(-a*b)*log
((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(a*b^4*c - 3*a^2*b^3*d + 5
*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5), 1/30*(6*a*b^4*f*x^7 + 2*
(5*a*b^4*e - 7*a^2*b^3*f)*x^5 + 10*(3*a*b^4*d - 5*a^2*b^3*e + 7*a^3*b^2*f)
*x^3 + 15*(a*b^3*c - 3*a^2*b^2*d + 5*a^3*b*e - 7*a^4*f + (b^4*c - 3*a*b^3*
d + 5*a^2*b^2*e - 7*a^3*b*f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 15*(a*
b^4*c - 3*a^2*b^3*d + 5*a^3*b^2*e - 7*a^4*b*f)*x)/(a*b^6*x^2 + a^2*b^5)]
```

3.126.6 Sympy [A] (verification not implemented)

Time = 0.98 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.36

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = x^3\left(-\frac{2af}{3b^3}+\frac{e}{3b^2}\right)+x\left(\frac{3a^2f}{b^4}-\frac{2ae}{b^3}+\frac{d}{b^2}\right)+\frac{x(a^3f-a^2be+ab^2d-b^3c)}{2ab^4+2b^5x^2} + \frac{\sqrt{-\frac{1}{ab^9}}\cdot(7a^3f-5a^2be+3ab^2d-b^3c)\log\left(-ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4} - \frac{\sqrt{-\frac{1}{ab^9}}\cdot(7a^3f-5a^2be+3ab^2d-b^3c)\log\left(ab^4\sqrt{-\frac{1}{ab^9}}+x\right)}{4}+\frac{fx^5}{5b^2}$$

input
```
integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x**3*(-2*a*f/(3*b**3) + e/(3*b**2)) + x*(3*a**2*f/b**4 - 2*a*e/b**3 + d/b*
*2) + x*(a**3*f - a**2*b*e + a*b**2*d - b**3*c)/(2*a*b**4 + 2*b**5*x**2) +
 sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3*a*b**2*d - b**3*c)*log(-a*b*
*4*sqrt(-1/(a*b**9)) + x)/4 - sqrt(-1/(a*b**9))*(7*a**3*f - 5*a**2*b*e + 3
*a*b**2*d - b**3*c)*log(a*b**4*sqrt(-1/(a*b**9)) + x)/4 + f*x**5/(5*b**2)
```

3.126.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = -\frac{(b^3c-ab^2d+a^2be-a^3f)x}{2\,(b^5x^2+ab^4)} + \frac{(b^3c-3\,ab^2d+5\,a^2be-7\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}b^4} + \frac{3\,b^2fx^5+5\,(b^2e-2\,abf)x^3+15\,(b^2d-2\,abe+3\,a^2f)x}{15\,b^4}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(b^5*x^2 + a*b^4) + 1/2*(b^3*c
- 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^4) +
 1/15*(3*b^2*f*x^5 + 5*(b^2*e - 2*a*b*f)*x^3 + 15*(b^2*d - 2*a*b*e + 3*a^2
*f)*x)/b^4
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.91

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = \frac{(b^3c-3\,ab^2d+5\,a^2be-7\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}b^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)b^4} + \frac{3\,b^8fx^5+5\,b^8ex^3-10\,ab^7fx^3+15\,b^8dx-30\,ab^7ex+45\,a^2b^6fx}{15\,b^{10}}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c - 3*a*b^2*d + 5*a^2*b*e - 7*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^4) - 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*b
^4) + 1/15*(3*b^8*f*x^5 + 5*b^8*e*x^3 - 10*a*b^7*f*x^3 + 15*b^8*d*x - 30*a
*b^7*e*x + 45*a^2*b^6*f*x)/b^10
```

3.126.9 Mupad [B] (verification not implemented)

Time = 5.87 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.94

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^2}\,dx = x^3\left(\frac{e}{3\,b^2}-\frac{2\,a\,f}{3\,b^3}\right) - x\left(\frac{a^2\,f}{b^4}-\frac{d}{b^2}+\frac{2\,a\,\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)}{b}\right) - \frac{x\,\left(-\frac{f\,a^3}{2}+\frac{e\,a^2\,b}{2}-\frac{d\,a\,b^2}{2}+\frac{c\,b^3}{2}\right)}{b^5\,x^2+a\,b^4} + \frac{f\,x^5}{5\,b^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-7\,f\,a^3+5\,e\,a^2\,b-3\,d\,a\,b^2+c\,b^3\right)}{2\,\sqrt{a}\,b^{9/2}}$$

input
```
int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^2,x)
```

output
```
x^3*(e/(3*b^2) - (2*a*f)/(3*b^3)) - x*((a^2*f)/b^4 - d/b^2 + (2*a*(e/b^2 -
 (2*a*f)/b^3))/b) - (x*((b^3*c)/2 - (a^3*f)/2 - (a*b^2*d)/2 + (a^2*b*e)/2)
)/(a*b^4 + b^5*x^2) + (f*x^5)/(5*b^2) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c
- 7*a^3*f - 3*a*b^2*d + 5*a^2*b*e))/(2*a^(1/2)*b^(9/2))
```

3.127 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2} dx$

3.127.1 Optimal result

Integrand size = 27, antiderivative size = 118

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2} dx = \frac{(be-2af)x}{b^3} + \frac{fx^3}{3b^2} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{2a(a+bx^2)} + \frac{(b^3c+ab^2d-3a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

output
```
(-2*a*f+b*e)*x/b^3+1/3*f*x^3/b^2+1/2*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*
x^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(
3/2)/b^(7/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2} dx = \frac{(be-2af)x}{b^3} + \frac{fx^3}{3b^2} - \frac{(-b^3c+ab^2d-a^2be+a^3f)x}{2ab^3(a+bx^2)} + \frac{(b^3c+ab^2d-3a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{3/2}b^{7/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

output
```
((b*e - 2*a*f)*x)/b^3 + (f*x^3)/(3*b^2) - ((-(b^3*c) + a*b^2*d - a^2*b*e +
 a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(3/2)*b^(7/2))
```

3.127.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {2345, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}-\frac{\int -\frac{\frac{2afx^4}{b}+\frac{2a(be-af)x^2}{b^2}+\frac{fa^3-bea^2+b^2da+b^3c}{b^3}}{bx^2+a}dx}{2a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{2afx^4}{b}+\frac{2a(be-af)x^2}{b^2}+c+\frac{a(fa^2-bea+b^2d)}{b^3}}{bx^2+a}dx}{2a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}$$

$$\Big\downarrow \text{ 1467}$$

$$\frac{\int \left(\frac{2afx^2}{b^2}+\frac{2a(be-2af)}{b^3}+\frac{5fa^3-3bea^2+b^2da+b^3c}{b^3(bx^2+a)}\right)dx}{2a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{2a(a+bx^2)}+\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(5a^3f-3a^2be+ab^2d+b^3c)}{\sqrt{a}b^{7/2}}+\frac{2ax(be-2af)}{b^3}+\frac{2afx^3}{3b^2}}{2a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(2*a*(a + b*x^2)) + ((2*a*(b*e -
 2*a*f)*x)/b^3 + (2*a*f*x^3)/(3*b^2) + ((b^3*c + a*b^2*d - 3*a^2*b*e + 5*a
^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(7/2)))/(2*a)
```

3.127.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.127.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

method	result
default	$-\frac{-\frac{1}{3} f \,x^{3} b +2 a f x -b e x}{b^{3}}+\frac{-\frac{\left(f \,a^{3}-a^{2} b e +a \,b^{2} d -b^{3} c \right) x}{2 a \left(b \,x^{2}+a \right)}+\frac{\left(5 f \,a^{3}-3 a^{2} b e +a \,b^{2} d +b^{3} c \right) \arctan \left(\frac{b x}{\sqrt{a b}}\right)}{2 a \sqrt{a b}}}{b^{3}}$
risch	$\frac{f \,x^{3}}{3 b^{2}}-\frac{2 a f x}{b^{3}}+\frac{e x}{b^{2}}-\frac{\left(f \,a^{3}-a^{2} b e +a \,b^{2} d -b^{3} c \right) x}{2 a \,b^{3} \left(b \,x^{2}+a \right)}-\frac{5 a^{2} \ln \left(b x +\sqrt{-a b}\right) f}{4 b^{3} \sqrt{-a b}}+\frac{3 a \ln \left(b x +\sqrt{-a b}\right) e}{4 b^{2} \sqrt{-a b}}-\frac{\ln \left(b x +\sqrt{-a b}\right) d}{4 b \sqrt{-a b}}-\frac{c \ln(bx+}{4\sqrt{-}}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*(-1/3*f*x^3*b+2*a*f*x-b*e*x)+1/b^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3
*c)/a*x/(b*x^2+a)+1/2*(5*a^3*f-3*a^2*b*e+a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arct
an(b*x/(a*b)^(1/2)))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.08

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$$
$$= \left[\frac{4\,a^2b^3fx^5+4\,(3\,a^2b^3e-5\,a^3b^2f)x^3-3\,(ab^3c+a^2b^2d-3\,a^3be+5\,a^4f+(b^4c+ab^3d-3\,a^2b^2e+5\,a^3bf)}{12\,(a^2b^5x^2+a^3b^4)}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/12*(4*a^2*b^3*f*x^5 + 4*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 - 3*(a*b^3*c +
a^2*b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b
*f)*x^2)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) + 6*(a*b
^4*c - a^2*b^3*d + 3*a^3*b^2*e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4), 1/
6*(2*a^2*b^3*f*x^5 + 2*(3*a^2*b^3*e - 5*a^3*b^2*f)*x^3 + 3*(a*b^3*c + a^2*
b^2*d - 3*a^3*b*e + 5*a^4*f + (b^4*c + a*b^3*d - 3*a^2*b^2*e + 5*a^3*b*f)*
x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) + 3*(a*b^4*c - a^2*b^3*d + 3*a^3*b^2*
e - 5*a^4*b*f)*x)/(a^2*b^5*x^2 + a^3*b^4)]
```

3.127.6 Sympy [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.70

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx$$
$$= x\left(-\frac{2af}{b^3}+\frac{e}{b^2}\right)+\frac{x(-a^3f+a^2be-ab^2d+b^3c)}{2a^2b^3+2ab^4x^2}$$
$$-\frac{\sqrt{-\frac{1}{a^3b^7}}\cdot(5a^3f-3a^2be+ab^2d+b^3c)\log\left(-a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}$$
$$+\frac{\sqrt{-\frac{1}{a^3b^7}}\cdot(5a^3f-3a^2be+ab^2d+b^3c)\log\left(a^2b^3\sqrt{-\frac{1}{a^3b^7}}+x\right)}{4}+\frac{fx^3}{3b^2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**2,x)
```

output
```
x*(-2*a*f/b**3 + e/b**2) + x*(-a**3*f + a**2*b*e - a*b**2*d + b**3*c)/(2*a
**2*b**3 + 2*a*b**4*x**2) - sqrt(-1/(a**3*b**7))*(5*a**3*f - 3*a**2*b*e +
a*b**2*d + b**3*c)*log(-a**2*b**3*sqrt(-1/(a**3*b**7)) + x)/4 + sqrt(-1/(a
**3*b**7))*(5*a**3*f - 3*a**2*b*e + a*b**2*d + b**3*c)*log(a**2*b**3*sqrt(
-1/(a**3*b**7)) + x)/4 + f*x**3/(3*b**2)
```

3.127.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)x}{2\,(ab^4x^2+a^2b^3)}+\frac{bfx^3+3\,(be-2\,af)x}{3\,b^3}+\frac{(b^3c+ab^2d-3\,a^2be+5\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x/(a*b^4*x^2 + a^2*b^3) + 1/3*(b*f
*x^3 + 3*(b*e - 2*a*f)*x)/b^3 + 1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f
)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^3)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = \frac{(b^3c+ab^2d-3\,a^2be+5\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}ab^3}+\frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)ab^3}+\frac{b^4fx^3+3\,b^4ex-6\,ab^3fx}{3\,b^6}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(b^3*c + a*b^2*d - 3*a^2*b*e + 5*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a*b^3) + 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*a
*b^3) + 1/3*(b^4*f*x^3 + 3*b^4*e*x - 6*a*b^3*f*x)/b^6
```

3.127.9 Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^2}\,dx = x\left(\frac{e}{b^2}-\frac{2\,a\,f}{b^3}\right)+\frac{f\,x^3}{3\,b^2}+\frac{x\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+c\,b^3)}{2\,a\,(b^4\,x^2+a\,b^3)} + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(5\,f\,a^3-3\,e\,a^2\,b+d\,a\,b^2+c\,b^3)}{2\,a^{3/2}\,b^{7/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^2,x)
```

output
```
x*(e/b^2 - (2*a*f)/b^3) + (f*x^3)/(3*b^2) + (x*(b^3*c - a^3*f - a*b^2*d +
a^2*b*e))/(2*a*(a*b^3 + b^4*x^2)) + (atan((b^(1/2)*x)/a^(1/2))*(b^3*c + 5*
a^3*f + a*b^2*d - 3*a^2*b*e))/(2*a^(3/2)*b^(7/2))
```

3.128 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2} dx$

3.128.1 Optimal result

Integrand size = 30, antiderivative size = 112

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2} dx = -\frac{c}{a^2x}+\frac{fx}{b^2}-\frac{\left(\frac{bc}{a}-d+\frac{ae}{b}-\frac{a^2f}{b^2}\right)x}{2a(a+bx^2)} - \frac{(3b^3c-ab^2d-a^2be+3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

output
```
-c/a^2/x+f*x/b^2-1/2*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)-1/2*(3*a^3*f-
a^2*b*e-a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(5/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2} dx = -\frac{c}{a^2x}+\frac{fx}{b^2}+\frac{(-b^3c+ab^2d-a^2be+a^3f)x}{2a^2b^2(a+bx^2)} - \frac{(3b^3c-ab^2d-a^2be+3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{5/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

output
```
-(c/(a^2*x)) + (f*x)/b^2 + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a
^2*b^2*(a + b*x^2)) - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(2*a^(5/2)*b^(5/2))
```

3.128.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\int -\frac{\frac{2afx^4}{b}-\left(\frac{fa^2}{b^2}-\frac{ea}{b}-d+\frac{bc}{a}\right)x^2+2c}{x^2(bx^2+a)}dx}{2a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\frac{2afx^4}{b}-\left(\frac{fa^2}{b^2}-\frac{ea}{b}-d+\frac{bc}{a}\right)x^2+2c}{x^2(bx^2+a)}dx}{2a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 1584}$$

$$\frac{\int \left(\frac{2c}{ax^2}+\frac{2af}{b^2}+\frac{-3fa^3+bea^2+b^2da-3b^3c}{ab^2(bx^2+a)}\right)dx}{2a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(3a^3f-a^2be-ab^2d+3b^3c\right)}{a^{3/2}b^{5/2}}+\frac{2afx}{b^2}-\frac{2c}{ax}}{2a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{2a\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x]
```

output
```
-1/2*(((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(a*(a + b*x^2)) + ((-2*c)/(
a*x) + (2*a*f*x)/b^2 - ((3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*ArcTan[(Sq
rt[b]*x)/Sqrt[a]])/(a^(3/2)*b^(5/2)))/(2*a)
```

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.128.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.96

method	result
default	$\frac{fx}{b^2}-\frac{c}{a^2x}-\frac{\frac{\left(-\frac{1}{2}f\,a^3+\frac{1}{2}a^2be-\frac{1}{2}a\,b^2d+\frac{1}{2}b^3c\right)x}{b\,x^2+a}+\frac{\left(3f\,a^3-a^2be-a\,b^2d+3b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^2b^2}$
risch	$\frac{fx}{b^2}+\frac{\frac{\left(f\,a^3-a^2be+a\,b^2d-3b^3c\right)x^2}{2a^2}-\frac{b^2c}{a}}{b^2x(b\,x^2+a)}-\frac{3a\ln(-\sqrt{-ab}\,x-a)f}{4b^2\sqrt{-ab}}+\frac{\ln(-\sqrt{-ab}\,x-a)e}{4b\sqrt{-ab}}+\frac{\ln(-\sqrt{-ab}\,x-a)d}{4\sqrt{-ab}\,a}-\frac{3b\ln(-\sqrt{-ab}\,x-$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
f*x/b^2-c/a^2/x-1/a^2/b^2*((-1/2*f*a^3+1/2*a^2*b*e-1/2*a*b^2*d+1/2*b^3*c)*
x/(b*x^2+a)+1/2*(3*a^3*f-a^2*b*e-a*b^2*d+3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(
a*b)^(1/2)))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 354, normalized size of antiderivative = 3.16

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx$$
$$=\left[\frac{4\,a^3b^2fx^4-4\,a^2b^3c-2\,(3\,ab^4c-a^2b^3d+a^3b^2e-3\,a^4bf)x^2-((3\,b^4c-ab^3d-a^2b^2e+3\,a^3bf)x^3+(3\,a\right.}{4\,(a^3b^4x^3+a^4b^3x)}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a^3*b^2*f*x^4 - 4*a^2*b^3*c - 2*(3*a*b^4*c - a^2*b^3*d + a^3*b^2*e
 - 3*a^4*b*f)*x^2 - ((3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*
a*b^3*c - a^2*b^2*d - a^3*b*e + 3*a^4*f)*x)*sqrt(-a*b)*log((b*x^2 + 2*sqrt
(-a*b)*x - a)/(b*x^2 + a)))/(a^3*b^4*x^3 + a^4*b^3*x), 1/2*(2*a^3*b^2*f*x^
4 - 2*a^2*b^3*c - (3*a*b^4*c - a^2*b^3*d + a^3*b^2*e - 3*a^4*b*f)*x^2 - ((
3*b^4*c - a*b^3*d - a^2*b^2*e + 3*a^3*b*f)*x^3 + (3*a*b^3*c - a^2*b^2*d -
a^3*b*e + 3*a^4*f)*x)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^3*b^4*x^3 + a^4*
b^3*x)]
```

3.128.6 Sympy [A] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.76

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = \frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^3f-a^2be-ab^2d+3b^3c)\log\left(-a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4}$$
$$-\frac{\sqrt{-\frac{1}{a^5b^5}}\cdot(3a^3f-a^2be-ab^2d+3b^3c)\log\left(a^3b^2\sqrt{-\frac{1}{a^5b^5}}+x\right)}{4}$$
$$+\frac{-2ab^2c+x^2(a^3f-a^2be+ab^2d-3b^3c)}{2a^3b^2x+2a^2b^3x^3}+\frac{fx}{b^2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**2,x)
```

output
```
sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*b*e - a*b**2*d + 3*b**3*c)*log(-a**3
*b**2*sqrt(-1/(a**5*b**5)) + x)/4 - sqrt(-1/(a**5*b**5))*(3*a**3*f - a**2*
b*e - a*b**2*d + 3*b**3*c)*log(a**3*b**2*sqrt(-1/(a**5*b**5)) + x)/4 + (-2
*a*b**2*c + x**2*(a**3*f - a**2*b*e + a*b**2*d - 3*b**3*c))/(2*a**3*b**2*x
 + 2*a**2*b**3*x**3) + f*x/b**2
```

3.128.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = -\frac{2\,ab^2c+(3\,b^3c-ab^2d+a^2be-a^3f)x^2}{2\left(a^2b^3x^3+a^3b^2x\right)}+\frac{fx}{b^2} - \frac{(3\,b^3c-ab^2d-a^2be+3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2b^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(2*a*b^2*c + (3*b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x^2)/(a^2*b^3*x^3
+ a^3*b^2*x) + f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arcta
n(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b^2)
```

3.128.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^2}\,dx = \frac{fx}{b^2} - \frac{(3\,b^3c-ab^2d-a^2be+3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}a^2b^2} - \frac{3\,b^3cx^2-ab^2dx^2+a^2bex^2-a^3fx^2+2\,ab^2c}{2\left(bx^3+ax\right)a^2b^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
f*x/b^2 - 1/2*(3*b^3*c - a*b^2*d - a^2*b*e + 3*a^3*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*a^2*b^2) - 1/2*(3*b^3*c*x^2 - a*b^2*d*x^2 + a^2*b*e*x^2 - a^3
*f*x^2 + 2*a*b^2*c)/((b*x^3 + a*x)*a^2*b^2)
```

3.128.9 Mupad [B] (verification not implemented)

Time = 6.08 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^2}\,dx = \frac{f\,x}{b^2} - \frac{\frac{x^2\,(-f\,a^3+e\,a^2\,b-d\,a\,b^2+3\,c\,b^3)}{2\,a^2} + \frac{b^2\,c}{a}}{b^3\,x^3+a\,b^2\,x} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,f\,a^3-e\,a^2\,b-d\,a\,b^2+3\,c\,b^3)}{2\,a^{5/2}\,b^{5/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^2),x)
```

output
```
(f*x)/b^2 - ((x^2*(3*b^3*c - a^3*f - a*b^2*d + a^2*b*e))/(2*a^2) + (b^2*c)
/a)/(b^3*x^3 + a*b^2*x) - (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c + 3*a^3*f -
a*b^2*d - a^2*b*e))/(2*a^(5/2)*b^(5/2))
```

3.129 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx$

3.129.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{2a\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

output
```
-1/3*c/a^2/x^3+(-a*d+2*b*c)/a^3/x+1/2*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2
+a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2
)/b^(3/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx=-\frac{c}{3a^2x^3}+\frac{2bc-ad}{a^3x}-\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^3b\left(a+bx^2\right)}+\frac{\left(5b^3c-3ab^2d+a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{7/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

output
```
-1/3*c/(a^2*x^3) + (2*b*c - a*d)/(a^3*x) - ((-(b^3*c) + a*b^2*d - a^2*b*e
+ a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(7/2)*b^(3/2))
```

3.129.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \text{2336}$$

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}-\frac{\int -\frac{\left(\frac{cb^2}{a^2}-\frac{db}{a}+e+\frac{af}{b}\right)x^4-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^4\left(bx^2+a\right)}dx}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\left(\frac{cb^2}{a^2}-\frac{db}{a}+e+\frac{af}{b}\right)x^4-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^4\left(bx^2+a\right)}dx}{2a}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\int \left(\frac{2c}{ax^4}+\frac{fa^3+bea^2-3b^2da+5b^3c}{a^2b\left(bx^2+a\right)}+\frac{2(ad-2bc)}{a^2x^2}\right)dx}{2a}+\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{2a\left(a+bx^2\right)}+\frac{\frac{2(2bc-ad)}{a^2x}+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+a^2be-3ab^2d+5b^3c\right)}{a^{5/2}b^{3/2}}-\frac{2c}{3ax^3}}{2a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x]
```

output
```
(((b^2*c)/a^2 - (b*d)/a + e - (a*f)/b)*x)/(2*a*(a + b*x^2)) + ((-2*c)/(3*a
*x^3) + (2*(2*b*c - a*d))/(a^2*x) + ((5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(5/2)*b^(3/2)))/(2*a)
```

3.129.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.129.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c}{3a^2x^3}-\frac{ad-2bc}{a^3x}+\frac{-\frac{\left(f\,a^3-a^2be+a\,b^2d-b^3c\right)x}{2b\left(b\,x^2+a\right)}+\frac{\left(f\,a^3+a^2be-3a\,b^2d+5b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2b\sqrt{ab}}}{a^3}$
risch	$\frac{-\frac{\left(f\,a^3-a^2be+3a\,b^2d-5b^3c\right)x^4}{2a^3b}-\frac{(3ad-5bc)x^2}{3a^2}-\frac{c}{3a}}{x^3(b\,x^2+a)}-\frac{\ln(-\sqrt{-ab}\,x+a)f}{4\sqrt{-ab}\,b}-\frac{\ln(-\sqrt{-ab}\,x+a)e}{4\sqrt{-ab}\,a}+\frac{3b\ln(-\sqrt{-ab}\,x+a)d}{4\sqrt{-ab}\,a^2}-\frac{5b^2\ln(-}{4\sqrt{}}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^2/x^3-(a*d-2*b*c)/a^3/x+1/a^3*(-1/2*(a^3*f-a^2*b*e+a*b^2*d-b^3*c)
/b*x/(b*x^2+a)+1/2*(a^3*f+a^2*b*e-3*a*b^2*d+5*b^3*c)/b/(a*b)^(1/2)*arctan(
b*x/(a*b)^(1/2)))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.12

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4 (a + bx^2)^2} \, dx$$
$$= \left[-\frac{4\,a^3b^2c - 6\,(5\,ab^4c - 3\,a^2b^3d + a^3b^2e - a^4bf)x^4 - 4\,(5\,a^2b^3c - 3\,a^3b^2d)x^2 + 3\,((5\,b^4c - 3\,ab^3d + a^2b^2e -}{12\,(a^4b^3x^5 + a^5b^2x^3)} \right.$$
$$-\frac{2\,a^3b^2c - 3\,(5\,ab^4c - 3\,a^2b^3d + a^3b^2e - a^4bf)x^4 - 2\,(5\,a^2b^3c - 3\,a^3b^2d)x^2 - 3\,((5\,b^4c - 3\,ab^3d + a^2b^2e +}{6\,(a^4b^3x^5 + a^5b^2x^3)}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/12*(4*a^3*b^2*c - 6*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*f)*x^
4 - 4*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 + 3*((5*b^4*c - 3*a*b^3*d + a^2*b^2*
e + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*sqrt(-
a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^4*b^3*x^5 + a^5*b^2
*x^3), -1/6*(2*a^3*b^2*c - 3*(5*a*b^4*c - 3*a^2*b^3*d + a^3*b^2*e - a^4*b*
f)*x^4 - 2*(5*a^2*b^3*c - 3*a^3*b^2*d)*x^2 - 3*((5*b^4*c - 3*a*b^3*d + a^2
*b^2*e + a^3*b*f)*x^5 + (5*a*b^3*c - 3*a^2*b^2*d + a^3*b*e + a^4*f)*x^3)*s
qrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^4*b^3*x^5 + a^5*b^2*x^3)]
```

3.129.6 Sympy [A] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.75

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^4 (a + bx^2)^2} \, dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^3}}(a^3f + a^2be - 3ab^2d + 5b^3c)\log\left(-a^4b\sqrt{-\frac{1}{a^7b^3}} + x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{a^7b^3}}(a^3f + a^2be - 3ab^2d + 5b^3c)\log\left(a^4b\sqrt{-\frac{1}{a^7b^3}} + x\right)}{4}$$
$$+ \frac{-2a^2bc + x^4(-3a^3f + 3a^2be - 9ab^2d + 15b^3c) + x^2(-6a^2bd + 10ab^2c)}{6a^4bx^3 + 6a^3b^2x^5}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e - 3*a*b**2*d + 5*b**3*c)*log(-a**
4*b*sqrt(-1/(a**7*b**3)) + x)/4 + sqrt(-1/(a**7*b**3))*(a**3*f + a**2*b*e
- 3*a*b**2*d + 5*b**3*c)*log(a**4*b*sqrt(-1/(a**7*b**3)) + x)/4 + (-2*a**2
*b*c + x**4*(-3*a**3*f + 3*a**2*b*e - 9*a*b**2*d + 15*b**3*c) + x**2*(-6*a
**2*b*d + 10*a*b**2*c))/(6*a**4*b*x**3 + 6*a**3*b**2*x**5)
```

3.129.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx = \frac{3\,(5\,b^3c-3\,ab^2d+a^2be-a^3f)x^4-2\,a^2bc+2\,(5\,ab^2c-3\,a^2bd)x^2}{6\,(a^3b^2x^5+a^4bx^3)} + \frac{(5\,b^3c-3\,ab^2d+a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*(5*b^3*c - 3*a*b^2*d + a^2*b*e - a^3*f)*x^4 - 2*a^2*b*c + 2*(5*a*b^
2*c - 3*a^2*b*d)*x^2)/(a^3*b^2*x^5 + a^4*b*x^3) + 1/2*(5*b^3*c - 3*a*b^2*d
 + a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^2}\,dx = \frac{(5\,b^3c-3\,ab^2d+a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^3b} + \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)a^3b} + \frac{6\,bcx^2-3\,adx^2-ac}{3\,a^3x^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(5*b^3*c - 3*a*b^2*d + a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*
b)*a^3*b) + 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*a
^3*b) + 1/3*(6*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^3*x^3)
```

3.129.9 Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^2}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^{7/2}\,b^{3/2}} - \frac{\frac{c}{3\,a}+\frac{x^2\,\left(3\,a\,d-5\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-f\,a^3+e\,a^2\,b-3\,d\,a\,b^2+5\,c\,b^3\right)}{2\,a^3\,b}}{b\,x^5+a\,x^3}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^2),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(5*b^3*c + a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^(
7/2)*b^(3/2)) - (c/(3*a) + (x^2*(3*a*d - 5*b*c))/(3*a^2) - (x^4*(5*b^3*c -
 a^3*f - 3*a*b^2*d + a^2*b*e))/(2*a^3*b))/(a*x^3 + b*x^5)
```

3.130 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^2}\,dx$

3.130.1 Optimal result

Integrand size = 30, antiderivative size = 152

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}-\frac{3b^2c-2abd+a^2e}{a^4x} - \frac{\left(b^3c-ab^2d+a^2be-a^3f\right)x}{2a^4\left(a+bx^2\right)} - \frac{\left(7b^3c-5ab^2d+3a^2be-a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

output
```
-1/5*c/a^2/x^5+1/3*(-a*d+2*b*c)/a^3/x^3+(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x-1/2
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)-1/2*(-a^3*f+3*a^2*b*e-5*a*
b^2*d+7*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{c}{5a^2x^5}+\frac{2bc-ad}{3a^3x^3}+\frac{-3b^2c+2abd-a^2e}{a^4x} + \frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{2a^4\left(a+bx^2\right)} + \frac{\left(-7b^3c+5ab^2d-3a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{9/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

output
```
-1/5*c/(a^2*x^5) + (2*b*c - a*d)/(3*a^3*x^3) + (-3*b^2*c + 2*a*b*d - a^2*e
)/(a^4*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^4*(a + b*x^2))
 + ((-7*b^3*c + 5*a*b^2*d - 3*a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]
)/(2*a^(9/2)*Sqrt[b])
```

3.130.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6 (a+bx^2)^2} \, dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{-\frac{(-fa^3+bea^2-b^2da+b^3c)x^6}{a^3}+\frac{2(ea^2-bda+b^2c)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^6(bx^2+a)}dx}{2a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{-\frac{(-fa^3+bea^2-b^2da+b^3c)x^6}{a^3}+\frac{2(ea^2-bda+b^2c)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^6(bx^2+a)}dx}{2a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(\frac{2c}{ax^6}+\frac{fa^3-3bea^2+5b^2da-7b^3c}{a^3(bx^2+a)}+\frac{2(ea^2-2bda+3b^2c)}{a^3x^2}+\frac{2(ad-2bc)}{a^2x^4}\right)dx}{2a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{2(2bc-ad)}{3a^2x^3}-\frac{2(a^2e-2abd+3b^2c)}{a^3x}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3(-f)+3a^2be-5ab^2d+7b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{2c}{5ax^5}}{2a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^4\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x]
```

output
```
-1/2*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^4*(a + b*x^2)) + ((-2*c)/(
5*a*x^5) + (2*(2*b*c - a*d))/(3*a^2*x^3) - (2*(3*b^2*c - 2*a*b*d + a^2*e))
/(a^3*x) - ((7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*ArcTan[(Sqrt[b]*x)/S
qrt[a]])/(a^(7/2)*Sqrt[b]))/(2*a)
```

3.130.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.130.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5a^2x^5}-\frac{ad-2bc}{3a^3x^3}-\frac{a^2e-2abd+3b^2c}{a^4x}+\frac{\frac{\left(\frac{1}{2}fa^3-\frac{1}{2}a^2be+\frac{1}{2}ab^2d-\frac{1}{2}b^3c\right)x}{bx^2+a}+\frac{\left(fa^3-3a^2be+5ab^2d-7b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{a^4}$
risch	$\frac{\frac{\left(fa^3-3a^2be+5ab^2d-7b^3c\right)x^6}{2a^4}-\frac{\left(3a^2e-5abd+7b^2c\right)x^4}{3a^3}-\frac{(5ad-7bc)x^2}{15a^2}-\frac{c}{5a}}{x^5\left(bx^2+a\right)}+\frac{\left(\_R=\text{RootOf}\left(a^9\_Z^2b+a^6f^2-6a^5bef+10a^4b^2df+9a^4b^2e^2-\right.\right.}{}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/5*c/a^2/x^5-1/3*(a*d-2*b*c)/a^3/x^3-(a^2*e-2*a*b*d+3*b^2*c)/a^4/x+1/a^4
*((1/2*f*a^3-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(a^3*f-3*a
^2*b*e+5*a*b^2*d-7*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 438, normalized size of antiderivative = 2.88

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx$$

$$= \left[-\frac{30\,(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf)x^6+12\,a^4bc+20\,(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be)x^4-4\,(7\,a^3b^2c-5}{60\,(} \right.$$

$$-\frac{15\,(7\,ab^4c-5\,a^2b^3d+3\,a^3b^2e-a^4bf)x^6+6\,a^4bc+10\,(7\,a^2b^3c-5\,a^3b^2d+3\,a^4be)x^4-2\,(7\,a^3b^2c-5\,a^4}{30\,(a^5b^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/60*(30*(7*a*b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 12*a^4*
b*c + 20*(7*a^2*b^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 4*(7*a^3*b^2*c - 5*
a^4*b*d)*x^2 - 15*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*
a*b^3*c - 5*a^2*b^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(-a*b)*log((b*x^2 - 2*
sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^2*x^7 + a^6*b*x^5), -1/30*(15*(7*a*
b^4*c - 5*a^2*b^3*d + 3*a^3*b^2*e - a^4*b*f)*x^6 + 6*a^4*b*c + 10*(7*a^2*b
^3*c - 5*a^3*b^2*d + 3*a^4*b*e)*x^4 - 2*(7*a^3*b^2*c - 5*a^4*b*d)*x^2 + 15
*((7*b^4*c - 5*a*b^3*d + 3*a^2*b^2*e - a^3*b*f)*x^7 + (7*a*b^3*c - 5*a^2*b
^2*d + 3*a^3*b*e - a^4*f)*x^5)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^2*x
^7 + a^6*b*x^5)]
```

3.130.6 Sympy [A] (verification not implemented)

Time = 19.29 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.49

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)^2}\,dx = -\frac{\sqrt{-\frac{1}{a^9b}}(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(-a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{a^9b}}(a^3f-3a^2be+5ab^2d-7b^3c)\log\left(a^5\sqrt{-\frac{1}{a^9b}}+x\right)}{4} + \frac{-6a^3c+x^6\cdot(15a^3f-45a^2be+75ab^2d-105b^3c)+x^4(-30a^3e+50a^2bd-70ab^2c)+x^2(-10a^3d+14}{30a^5x^5+30a^4bx^7}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**2,x)
```

output
```
-sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b**2*d - 7*b**3*c)*log(-a**5
*sqrt(-1/(a**9*b)) + x)/4 + sqrt(-1/(a**9*b))*(a**3*f - 3*a**2*b*e + 5*a*b
**2*d - 7*b**3*c)*log(a**5*sqrt(-1/(a**9*b)) + x)/4 + (-6*a**3*c + x**6*(1
5*a**3*f - 45*a**2*b*e + 75*a*b**2*d - 105*b**3*c) + x**4*(-30*a**3*e + 50
*a**2*b*d - 70*a*b**2*c) + x**2*(-10*a**3*d + 14*a**2*b*c))/(30*a**5*x**5
+ 30*a**4*b*x**7)
```

3.130.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\,(a+bx^2)^2}\,dx = -\frac{15\,(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f)x^6+10\,(7\,ab^2c-5\,a^2bd+3\,a^3e)x^4+6\,a^3c-2\,(7\,a^2bc-5\,a^3d)x^2}{30\,(a^4bx^7+a^5x^5)} - \frac{(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/30*(15*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*x^6 + 10*(7*a*b^2*c -
5*a^2*b*d + 3*a^3*e)*x^4 + 6*a^3*c - 2*(7*a^2*b*c - 5*a^3*d)*x^2)/(a^4*b*x
^7 + a^5*x^5) - 1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^4)
```

3.130.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\left(7\,b^3c-5\,ab^2d+3\,a^2be-a^3f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^4} - \frac{b^3cx-ab^2dx+a^2bex-a^3fx}{2\,(bx^2+a)a^4} - \frac{45\,b^2cx^4-30\,abdx^4+15\,a^2ex^4-10\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^4x^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(7*b^3*c - 5*a*b^2*d + 3*a^2*b*e - a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt
(a*b)*a^4) - 1/2*(b^3*c*x - a*b^2*d*x + a^2*b*e*x - a^3*f*x)/((b*x^2 + a)*
a^4) - 1/15*(45*b^2*c*x^4 - 30*a*b*d*x^4 + 15*a^2*e*x^4 - 10*a*b*c*x^2 + 5
*a^2*d*x^2 + 3*a^2*c)/(a^4*x^5)
```

3.130.9 Mupad [B] (verification not implemented)

Time = 6.01 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^2}\,dx = -\frac{\frac{c}{5\,a}+\frac{x^6\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^4}+\frac{x^2\,\left(5\,a\,d-7\,b\,c\right)}{15\,a^2}+\frac{x^4\,\left(3\,e\,a^2-5\,d\,a\,b+7\,c\,b^2\right)}{3\,a^3}}{b\,x^7+a\,x^5} - \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-f\,a^3+3\,e\,a^2\,b-5\,d\,a\,b^2+7\,c\,b^3\right)}{2\,a^{9/2}\,\sqrt{b}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^2),x)
```

output
```
- (c/(5*a) + (x^6*(7*b^3*c - a^3*f - 5*a*b^2*d + 3*a^2*b*e))/(2*a^4) + (x^
2*(5*a*d - 7*b*c))/(15*a^2) + (x^4*(7*b^2*c + 3*a^2*e - 5*a*b*d))/(3*a^3))
/(a*x^5 + b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(7*b^3*c - a^3*f - 5*a*b^2*d
 + 3*a^2*b*e))/(2*a^(9/2)*b^(1/2))
```

3.131 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2} dx$

3.131.1 Optimal result

Integrand size = 30, antiderivative size = 189

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2} dx = &-\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}-\frac{3b^2c-2abd+a^2e}{3a^4x^3} \\ &+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)x}{2a^5(a+bx^2)} \\ &+\frac{\sqrt{b}(9b^3c-7ab^2d+5a^2be-3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}\end{aligned}$$

output

```
-1/7*c/a^2/x^7+1/5*(-a*d+2*b*c)/a^3/x^5+1/3*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x
^3+(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x+1/2*b*(-a^3*f+a^2*b*e-a*b^2*
d+b^3*c)*x/a^5/(b*x^2+a)+1/2*(-3*a^3*f+5*a^2*b*e-7*a*b^2*d+9*b^3*c)*arctan
(x*b^(1/2)/a^(1/2))*b^(1/2)/a^(11/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx = -\frac{c}{7a^2x^7}+\frac{2bc-ad}{5a^3x^5}+\frac{-3b^2c+2abd-a^2e}{3a^4x^3} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{a^5x} - \frac{b(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^5(a+bx^2)} - \frac{\sqrt{b}(-9b^3c+7ab^2d-5a^2be+3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{11/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

output
```
-1/7*c/(a^2*x^7) + (2*b*c - a*d)/(5*a^3*x^5) + (-3*b^2*c + 2*a*b*d - a^2*e
)/(3*a^4*x^3) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(a^5*x) - (b*(-(
b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^5*(a + b*x^2)) - (Sqrt[b]*(-9*
b^3*c + 7*a*b^2*d - 5*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a
^(11/2))
```

3.131.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx$$

$$\Big\downarrow \; 2336$$

$$\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{2a^5(a+bx^2)} - \frac{\int -\frac{\frac{b(-fa^3+bea^2-b^2da+b^3c)x^8}{a^4}-\frac{2(-fa^3+bea^2-b^2da+b^3c)x^6}{a^3}+\frac{2(ea^2-bda+b^2c)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^8(bx^2+a)}dx}{2a}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\frac{b(-fa^3+bea^2-b^2da+b^3c)x^8}{a^4}-\frac{2(-fa^3+bea^2-b^2da+b^3c)x^6}{a^3}+\frac{2(ea^2-bda+b^2c)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^8(bx^2+a)}dx}{2a}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2333$$

$$\frac{\int \left(\frac{2c}{ax^8}-\frac{b(3fa^3-5bea^2+7b^2da-9b^3c)}{a^4(bx^2+a)}+\frac{2(fa^3-2bea^2+3b^2da-4b^3c)}{a^4x^2}+\frac{2(ea^2-2bda+3b^2c)}{a^3x^4}+\frac{2(ad-2bc)}{a^2x^6}\right)dx}{2a}+\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}$$

$$\Big\downarrow \; 2009$$

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^5\left(a+bx^2\right)}+\frac{\frac{2(2bc-ad)}{5a^2x^5}-\frac{2(a^2e-2abd+3b^2c)}{3a^3x^3}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-3a^3f+5a^2be-7ab^2d+9b^3c)}{a^{9/2}}+\frac{2(a^3(-f)+2a^2be-3ab^2d+4b^3c)}{a^4x}-\frac{2c}{7ax^7}}{2a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x]
```

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(2*a^5*(a + b*x^2)) + ((-2*c)/(7
*a*x^7) + (2*(2*b*c - a*d))/(5*a^2*x^5) - (2*(3*b^2*c - 2*a*b*d + a^2*e))/
(3*a^3*x^3) + (2*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f))/(a^4*x) + (Sqr
t[b]*(9*b^3*c - 7*a*b^2*d + 5*a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a
]])/a^(9/2))/(2*a)
```

3.131.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.131.4 Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

method	result
default	$-\frac{c}{7a^2x^7}-\frac{ad-2bc}{5a^3x^5}-\frac{a^2e-2abd+3b^2c}{3a^4x^3}-\frac{f a^3-2a^2be+3a b^2d-4b^3c}{a^5x}-\frac{b\left(\frac{\left(\frac{1}{2}f a^3-\frac{1}{2}a^2be+\frac{1}{2}a b^2d-\frac{1}{2}b^3c\right)x}{b x^2+a}+\frac{\left(3f a^3-5a^2be+7a b^2 \cdots\right.}{2 \cdots}\right.}{a^5}$
risch	$\frac{-\frac{b\left(3f a^3-5a^2be+7a b^2d-9b^3c\right)x^8}{2a^5}-\frac{\left(3f a^3-5a^2be+7a b^2d-9b^3c\right)x^6}{3a^4}-\frac{\left(5a^2e-7abd+9b^2c\right)x^4}{15a^3}-\frac{(7ad-9bc)x^2}{35a^2}-\frac{c}{7a}}{x^7(b x^2+a)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(a^{11}\_Z^{\cdots}\right.}\right.}{\cdots}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a^2/x^7-1/5*(a*d-2*b*c)/a^3/x^5-1/3*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^3
-(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x-1/a^5*b*((1/2*f*a^3-1/2*a^2*b*e
+1/2*a*b^2*d-1/2*b^3*c)*x/(b*x^2+a)+1/2*(3*a^3*f-5*a^2*b*e+7*a*b^2*d-9*b^3
*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.58

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^2}\,dx$$

$$=\left[\frac{210\,(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)x^8+140\,(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f)x^6-60\,a^4c-28\,(9\,a^2 \cdots}{\cdots}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[1/420*(210*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 140*(9*a
*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 60*a^4*c - 28*(9*a^2*b^2
*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 12*(9*a^3*b*c - 7*a^4*d)*x^2 - 105*((9*b^4
*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^9 + (9*a*b^3*c - 7*a^2*b^2*d +
 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqrt(-b/a) - a)/(
b*x^2 + a)))/(a^5*b*x^9 + a^6*x^7), 1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^
2*b^2*e - 3*a^3*b*f)*x^8 + 70*(9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4
*f)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3
*b*c - 7*a^4*d)*x^2 + 105*((9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)
*x^9 + (9*a*b^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^7)*sqrt(b/a)*arct
an(x*sqrt(b/a)))/(a^5*b*x^9 + a^6*x^7)]
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.131.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx$$
$$= \frac{105\,(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)x^8+70\,(9\,ab^3c-7\,a^2b^2d+5\,a^3be-3\,a^4f)x^6-30\,a^4c-14\,(9\,a^2b^2c}{210\,(a^5bx^9+a^6x^7)}$$
$$+\frac{(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/210*(105*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*x^8 + 70*(9*a*b
^3*c - 7*a^2*b^2*d + 5*a^3*b*e - 3*a^4*f)*x^6 - 30*a^4*c - 14*(9*a^2*b^2*c
 - 7*a^3*b*d + 5*a^4*e)*x^4 + 6*(9*a^3*b*c - 7*a^4*d)*x^2)/(a^5*b*x^9 + a^
6*x^7) + 1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sq
rt(a*b))/(sqrt(a*b)*a^5)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8 (a+bx^2)^2}\,dx = \frac{(9\,b^4c-7\,ab^3d+5\,a^2b^2e-3\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^5}+\frac{b^4cx-ab^3dx+a^2b^2ex-a^3bfx}{2\,(bx^2+a)a^5} + \frac{420\,b^3cx^6-315\,ab^2dx^6+210\,a^2bex^6-105\,a^3fx^6-105\,ab^2cx^4+70\,a^2bdx^4-35\,a^3ex^4+42\,a^2bcx^2-2}{105\,a^5x^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
1/2*(9*b^4*c - 7*a*b^3*d + 5*a^2*b^2*e - 3*a^3*b*f)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*a^5) + 1/2*(b^4*c*x - a*b^3*d*x + a^2*b^2*e*x - a^3*b*f*x)/((b*
x^2 + a)*a^5) + 1/105*(420*b^3*c*x^6 - 315*a*b^2*d*x^6 + 210*a^2*b*e*x^6 -
 105*a^3*f*x^6 - 105*a*b^2*c*x^4 + 70*a^2*b*d*x^4 - 35*a^3*e*x^4 + 42*a^2*
b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^5*x^7)
```

3.131.9 Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8 (a+bx^2)^2}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^{11/2}} - \frac{\frac{c}{7\,a}-\frac{x^6\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{3\,a^4}+\frac{x^2\,\left(7\,a\,d-9\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(5\,e\,a^2-7\,d\,a\,b+9\,c\,b^2\right)}{15\,a^3}-\frac{b\,x^8\,\left(-3\,f\,a^3+5\,e\,a^2\,b-7\,d\,a\,b^2+9\,c\,b^3\right)}{2\,a^5}}{b\,x^9+a\,x^7}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^2),x)
```

output
```
(b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*
b*e))/(2*a^(11/2)) - (c/(7*a) - (x^6*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^
2*b*e))/(3*a^4) + (x^2*(7*a*d - 9*b*c))/(35*a^2) + (x^4*(9*b^2*c + 5*a^2*e
 - 7*a*b*d))/(15*a^3) - (b*x^8*(9*b^3*c - 3*a^3*f - 7*a*b^2*d + 5*a^2*b*e)
)/(2*a^5))/(a*x^7 + b*x^9)
```

3.131. $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^2}\,dx$

3.132 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$

3.132.1 Optimal result

Integrand size = 30, antiderivative size = 230

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = &-\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}-\frac{3b^2c-2abd+a^2e}{5a^4x^5}\\ &+\frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3}\\ &-\frac{b(5b^3c-4ab^2d+3a^2be-2a^3f)}{a^6x}\\ &-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{2a^6\left(a+bx^2\right)}\\ &-\frac{b^{3/2}(11b^3c-9ab^2d+7a^2be-5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}\end{aligned}$$

output
```
-1/9*c/a^2/x^9+1/7*(-a*d+2*b*c)/a^3/x^7+1/5*(-a^2*e+2*a*b*d-3*b^2*c)/a^4/x
^5+1/3*(-a^3*f+2*a^2*b*e-3*a*b^2*d+4*b^3*c)/a^5/x^3-b*(-2*a^3*f+3*a^2*b*e-
4*a*b^2*d+5*b^3*c)/a^6/x-1/2*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(b*x
^2+a)-1/2*b^(3/2)*(-5*a^3*f+7*a^2*b*e-9*a*b^2*d+11*b^3*c)*arctan(x*b^(1/2)
/a^(1/2))/a^(13/2)
```

3.132.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{c}{9a^2x^9}+\frac{2bc-ad}{7a^3x^7}+\frac{-3b^2c+2abd-a^2e}{5a^4x^5} + \frac{4b^3c-3ab^2d+2a^2be-a^3f}{3a^5x^3} + \frac{b(-5b^3c+4ab^2d-3a^2be+2a^3f)}{a^6x} + \frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{2a^6\left(a+bx^2\right)} + \frac{b^{3/2}(-11b^3c+9ab^2d-7a^2be+5a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{2a^{13/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

output
```
-1/9*c/(a^2*x^9) + (2*b*c - a*d)/(7*a^3*x^7) + (-3*b^2*c + 2*a*b*d - a^2*e
)/(5*a^4*x^5) + (4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f)/(3*a^5*x^3) + (b
*(-5*b^3*c + 4*a*b^2*d - 3*a^2*b*e + 2*a^3*f))/(a^6*x) + (b^2*(-(b^3*c) +
a*b^2*d - a^2*b*e + a^3*f)*x)/(2*a^6*(a + b*x^2)) + (b^(3/2)*(-11*b^3*c +
9*a*b^2*d - 7*a^2*b*e + 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(2*a^(13/2))
```

3.132.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}dx$$

$$\Big\downarrow \; 2336$$

$$-\frac{\int -\frac{-\frac{b^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^{10}}{a^5}+\frac{2b\left(-fa^3+bea^2-b^2da+b^3c\right)x^8}{a^4}-\frac{2\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{2\left(ea^2-bda+b^2c\right)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^{10}(bx^2+a)}dx}{2a}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{-\frac{b^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^{10}}{a^5}+\frac{2b\left(-fa^3+bea^2-b^2da+b^3c\right)x^8}{a^4}-\frac{2\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{2\left(ea^2-bda+b^2c\right)x^4}{a^2}-2\left(\frac{bc}{a}-d\right)x^2+2c}{x^{10}(bx^2+a)}dx}{2a}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 2333

$$\frac{\int \left(\frac{(5fa^3-7bea^2+9b^2da-11b^3c)b^2}{a^5(bx^2+a)}-\frac{2(2fa^3-3bea^2+4b^2da-5b^3c)b}{a^5x^2}+\frac{2(fa^3-2bea^2+3b^2da-4b^3c)}{a^4x^4}+\frac{2(ea^2-2bda+3b^2c)}{a^3x^6}+\frac{2(ad-2bc)}{a^2x^8}+\right.}{2a}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}$$

$\Big\downarrow$ 2009

$$\frac{\frac{2(2bc-ad)}{7a^2x^7}-\frac{2(a^2e-2abd+3b^2c)}{5a^3x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-5a^3f+7a^2be-9ab^2d+11b^3c)}{a^{11/2}}-\frac{2b(-2a^3f+3a^2be-4ab^2d+5b^3c)}{a^5x}+\frac{2(a^3(-f)+2a^2b}{3a^4}}{2a}-\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{2a^6\left(a+bx^2\right)}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x]
```

output
```
-1/2*(b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^6*(a + b*x^2)) + ((-2*
c)/(9*a*x^9) + (2*(2*b*c - a*d))/(7*a^2*x^7) - (2*(3*b^2*c - 2*a*b*d + a^2
*e))/(5*a^3*x^5) + (2*(4*b^3*c - 3*a*b^2*d + 2*a^2*b*e - a^3*f))/(3*a^4*x^
3) - (2*b*(5*b^3*c - 4*a*b^2*d + 3*a^2*b*e - 2*a^3*f))/(a^5*x) - (b^(3/2)*
(11*b^3*c - 9*a*b^2*d + 7*a^2*b*e - 5*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/
a^(11/2))/(2*a)
```

3.132. $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx$

3.132.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.132.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.91

method	result
default	$-\frac{c}{9a^2x^9}-\frac{ad-2bc}{7a^3x^7}-\frac{a^2e-2abd+3b^2c}{5a^4x^5}-\frac{f\,a^3-2a^2be+3a\,b^2d-4b^3c}{3a^5x^3}+\frac{b(2f\,a^3-3a^2be+4a\,b^2d-5b^3c)}{a^6x}+\frac{b^2\left(\frac{\left(\frac{1}{2}f\,a^3-\frac{1}{2}a^2be+\ldots\right.}{b\,x^2\ldots}\right.}{}$
risch	$\frac{\frac{b^2\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^{10}}{2a^6}+\frac{b\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^8}{3a^5}-\frac{\left(5f\,a^3-7a^2be+9a\,b^2d-11b^3c\right)x^6}{15a^4}-\frac{\left(7a^2e-9abd+11b^2c\right)x^4}{35a^3}-\frac{(9ad-1\ldots}{63\ldots}}{x^9(b\,x^2+a)}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a^2/x^9-1/7*(a*d-2*b*c)/a^3/x^7-1/5*(a^2*e-2*a*b*d+3*b^2*c)/a^4/x^5
-1/3*(a^3*f-2*a^2*b*e+3*a*b^2*d-4*b^3*c)/a^5/x^3+b*(2*a^3*f-3*a^2*b*e+4*a*
b^2*d-5*b^3*c)/a^6/x+b^2/a^6*((1/2*f*a^3-1/2*a^2*b*e+1/2*a*b^2*d-1/2*b^3*c
)*x/(b*x^2+a)+1/2*(5*a^3*f-7*a^2*b*e+9*a*b^2*d-11*b^3*c)/(a*b)^(1/2)*arcta
n(b*x/(a*b)^(1/2)))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 582, normalized size of antiderivative = 2.53

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx$$

$$= \left[-\frac{630\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+420\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-84\,(11\,a^2\right.$$

$$-\frac{315\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-42\,(11\,a^2b}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
[-1/1260*(630*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 42
0*(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 84*(11*a^2*b^
3*c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 140*a^5*c + 36*(11*a^3*b^2*
c - 9*a^4*b*d + 7*a^5*e)*x^4 - 20*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^
5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^
3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^9)*sqrt(-b/a)*log((b*x^2 + 2*a*x*sqrt(-b/
a) - a)/(b*x^2 + a)))/(a^6*b*x^11 + a^7*x^9), -1/630*(315*(11*b^5*c - 9*a*
b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*(11*a*b^4*c - 9*a^2*b^3*d +
7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*c - 9*a^3*b^2*d + 7*a^4*b*e
- 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c - 9*a^4*b*d + 7*a^5*e)*x^4 -
10*(11*a^4*b*c - 9*a^5*d)*x^2 + 315*((11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e -
 5*a^3*b^2*f)*x^11 + (11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*
x^9)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b*x^11 + a^7*x^9)]
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**2,x)
```

output
```
Timed out
```

3.132.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx = -\frac{315\,(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)x^{10}+210\,(11\,ab^4c-9\,a^2b^3d+7\,a^3b^2e-5\,a^4bf)x^8-42\,(11\,a^2b}{630\,(a^6bx^{11}} - \frac{(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
-1/630*(315*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*x^10 + 210*
(11*a*b^4*c - 9*a^2*b^3*d + 7*a^3*b^2*e - 5*a^4*b*f)*x^8 - 42*(11*a^2*b^3*
c - 9*a^3*b^2*d + 7*a^4*b*e - 5*a^5*f)*x^6 + 70*a^5*c + 18*(11*a^3*b^2*c -
 9*a^4*b*d + 7*a^5*e)*x^4 - 10*(11*a^4*b*c - 9*a^5*d)*x^2)/(a^6*b*x^11 + a
^7*x^9) - 1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*
x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

3.132.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^2}\,dx = -\frac{(11\,b^5c-9\,ab^4d+7\,a^2b^3e-5\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{2\,\sqrt{ab}a^6} - \frac{b^5cx-ab^4dx+a^2b^3ex-a^3b^2fx}{2\,(bx^2+a)a^6} - \frac{1575\,b^4cx^8-1260\,ab^3dx^8+945\,a^2b^2ex^8-630\,a^3bfx^8-420\,ab^3cx^6+315\,a^2b^2dx^6-210\,a^3bex^6+105\,a}{315\,a^6x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
-1/2*(11*b^5*c - 9*a*b^4*d + 7*a^2*b^3*e - 5*a^3*b^2*f)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^6) - 1/2*(b^5*c*x - a*b^4*d*x + a^2*b^3*e*x - a^3*b^2*f*x
)/((b*x^2 + a)*a^6) - 1/315*(1575*b^4*c*x^8 - 1260*a*b^3*d*x^8 + 945*a^2*b
^2*e*x^8 - 630*a^3*b*f*x^8 - 420*a*b^3*c*x^6 + 315*a^2*b^2*d*x^6 - 210*a^3
*b*e*x^6 + 105*a^4*f*x^6 + 189*a^2*b^2*c*x^4 - 126*a^3*b*d*x^4 + 63*a^4*e*
x^4 - 90*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^6*x^9)
```

3.132.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.95

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^2}\,dx = -\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,\left(9\,a\,d-11\,b\,c\right)}{63\,a^2}+\frac{x^4\,\left(7\,e\,a^2-9\,d\,a\,b+11\,c\,b^2\right)}{35\,a^3}+\frac{b\,x^8\,\left(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3\right)}{3\,a^5}}{b\,x^{11}+a\,x^9} - \frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-5\,f\,a^3+7\,e\,a^2\,b-9\,d\,a\,b^2+11\,c\,b^3\right)}{2\,a^{13/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^2),x)
```

output
```
- (c/(9*a) - (x^6*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(15*a^4) +
 (x^2*(9*a*d - 11*b*c))/(63*a^2) + (x^4*(11*b^2*c + 7*a^2*e - 9*a*b*d))/(3
5*a^3) + (b*x^8*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(3*a^5) + (b
^2*x^10*(11*b^3*c - 5*a^3*f - 9*a*b^2*d + 7*a^2*b*e))/(2*a^6))/(a*x^9 + b*
x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2))*(11*b^3*c - 5*a^3*f - 9*a*b^2*d
 + 7*a^2*b*e))/(2*a^(13/2))
```

3.133 $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.133.1 Optimal result

Integrand size = 30, antiderivative size = 287

$$\begin{aligned}\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx =& -\frac{a(15b^3c-27ab^2d+43a^2be-63a^3f)\,x}{4b^7} \\ &+\frac{(5b^3c-9ab^2d+15a^2be-23a^3f)\,x^3}{6b^6} \\ &-\frac{(5b^3c-9ab^2d+17a^2be-29a^3f)\,x^5}{20ab^5} \\ &+\frac{(be-3af)x^7}{7b^4}+\frac{fx^9}{9b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^9}{4a\,(a+bx^2)^2} \\ &-\frac{a^2(5b^3c-9ab^2d+13a^2be-17a^3f)\,x}{8b^7\,(a+bx^2)} \\ &+\frac{a^{3/2}(35b^3c-63ab^2d+99a^2be-143a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}\end{aligned}$$

output
```
-1/4*a*(-63*a^3*f+43*a^2*b*e-27*a*b^2*d+15*b^3*c)*x/b^7+1/6*(-23*a^3*f+15*
a^2*b*e-9*a*b^2*d+5*b^3*c)*x^3/b^6-1/20*(-29*a^3*f+17*a^2*b*e-9*a*b^2*d+5*
b^3*c)*x^5/a/b^5+1/7*(-3*a*f+b*e)*x^7/b^4+1/9*f*x^9/b^3+1/4*(c-a*(a^2*f-a*
b*e+b^2*d)/b^3)*x^9/a/(b*x^2+a)^2-1/8*a^2*(-17*a^3*f+13*a^2*b*e-9*a*b^2*d+
5*b^3*c)*x/b^7/(b*x^2+a)+1/8*a^(3/2)*(-143*a^3*f+99*a^2*b*e-63*a*b^2*d+35*
b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(15/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{a(-3b^3c+6ab^2d-10a^2be+15a^3f)\,x}{b^7} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x^3}{3b^6} + \frac{(b^2d-3abe+6a^2f)\,x^5}{5b^5} + \frac{(be-3af)x^7}{7b^4} + \frac{fx^9}{9b^3} + \frac{a^3(b^3c-ab^2d+a^2be-a^3f)\,x}{4b^7\,(a+bx^2)^2} + \frac{a^2(-13b^3c+17ab^2d-21a^2be+25a^3f)\,x}{8b^7\,(a+bx^2)} - \frac{a^{3/2}(-35b^3c+63ab^2d-99a^2be+143a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{15/2}}$$

input
```
Integrate[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
(a*(-3*b^3*c + 6*a*b^2*d - 10*a^2*b*e + 15*a^3*f)*x)/b^7 + ((b^3*c - 3*a*b
^2*d + 6*a^2*b*e - 10*a^3*f)*x^3)/(3*b^6) + ((b^2*d - 3*a*b*e + 6*a^2*f)*x
^5)/(5*b^5) + ((b*e - 3*a*f)*x^7)/(7*b^4) + (f*x^9)/(9*b^3) + (a^3*(b^3*c
- a*b^2*d + a^2*b*e - a^3*f)*x)/(4*b^7*(a + b*x^2)^2) + (a^2*(-13*b^3*c +
17*a*b^2*d - 21*a^2*b*e + 25*a^3*f)*x)/(8*b^7*(a + b*x^2)) - (a^(3/2)*(-35
*b^3*c + 63*a*b^2*d - 99*a^2*b*e + 143*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*b^(15/2))
```

3.133.3 Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2335, 9, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^7\left(-4afx^5-4a\left(e-\frac{af}{b}\right)x^3+\left(-\frac{9fa^3}{b^2}+\frac{9ea^2}{b}-9da+5bc\right)x\right)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^8\left(-4afx^4-4a\left(e-\frac{af}{b}\right)x^2+5bc-9ad+\frac{9a^2e}{b}-\frac{9a^3f}{b^2}\right)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \; 1580$$

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x(-17a^3f+13a^2be-9ab^2d+5b^3c)}{2b^6(a+bx^2)}-\frac{\int \frac{8ab^5fx^{10}+8ab^4(be-2af)x^8-2b^3\left(-17fa^3+13bea^2-9b^2da+5b^3c\right)x^6+2ab^2\left(-17fa^3+13bea^2-9b^2da+5b^3c\right)x^4-2a}{bx^2+a}}{2b^6}}{4ab}$$

$$\Big\downarrow \; 2341$$

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x(-17a^3f+13a^2be-9ab^2d+5b^3c)}{2b^6(a+bx^2)}-\frac{\int\Big(8ab^4fx^8+8ab^3(be-3af)x^6-2b^2(-29fa^3+17bea^2-9b^2da+5b^3c)x^4+4ab(-23fa^3+15bea^2-9b^2da+5b}{2b^6}}{4ab}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^9\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-$$

$$\frac{\frac{a^3x(-17a^3f+13a^2be-9ab^2d+5b^3c)}{2b^6(a+bx^2)}-\frac{-\frac{2}{5}b^2x^5(-29a^3f+17a^2be-9ab^2d+5b^3c)+\frac{4}{3}abx^3(-23a^3f+15a^2be-9ab^2d+5b^3c)-2a^2x(-63a^3f+43a^2b}{2b^6}}{4ab}$$

input
```
Int[(x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^9)/(4*a*(a + b*x^2)^2) - ((a^3*(5
*b^3*c - 9*a*b^2*d + 13*a^2*b*e - 17*a^3*f)*x)/(2*b^6*(a + b*x^2)) - (-2*a
^2*(15*b^3*c - 27*a*b^2*d + 43*a^2*b*e - 63*a^3*f)*x + (4*a*b*(5*b^3*c - 9
*a*b^2*d + 15*a^2*b*e - 23*a^3*f)*x^3)/3 - (2*b^2*(5*b^3*c - 9*a*b^2*d + 1
7*a^2*b*e - 29*a^3*f)*x^5)/5 + (8*a*b^3*(b*e - 3*a*f)*x^7)/7 + (8*a*b^4*f*
x^9)/9 + (a^(5/2)*(35*b^3*c - 63*a*b^2*d + 99*a^2*b*e - 143*a^3*f)*ArcTan[
(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^6))/(4*a*b)
```

3.133. $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.133.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.133. $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.133.4 Maple [A] (verified)

Time = 3.62 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93

method	result
default	$\frac{\frac{1}{9}fx^9b^4-\frac{3}{7}ab^3fx^7+\frac{1}{7}b^4ex^7+\frac{6}{5}a^2b^2fx^5-\frac{3}{5}ab^3ex^5+\frac{1}{5}b^4dx^5-\frac{10}{3}a^3bfx^3+2a^2b^2ex^3-ab^3dx^3+\frac{1}{3}b^4cx^3+15a^4fx-10a^3bex+6a^2b^2}{b^7}$
risch	$\frac{fx^9}{9b^3}-\frac{3afx^7}{7b^4}+\frac{ex^7}{7b^3}+\frac{6a^2fx^5}{5b^5}-\frac{3aex^5}{5b^4}+\frac{dx^5}{5b^3}-\frac{10a^3fx^3}{3b^6}+\frac{2a^2ex^3}{b^5}-\frac{adx^3}{b^4}+\frac{cx^3}{3b^3}+\frac{15a^4fx}{b^7}-\frac{10a^3ex}{b^6}+6a$

input
```
int(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^7*(1/9*f*x^9*b^4-3/7*a*b^3*f*x^7+1/7*b^4*e*x^7+6/5*a^2*b^2*f*x^5-3/5*a
*b^3*e*x^5+1/5*b^4*d*x^5-10/3*a^3*b*f*x^3+2*a^2*b^2*e*x^3-a*b^3*d*x^3+1/3*
b^4*c*x^3+15*a^4*f*x-10*a^3*b*e*x+6*a^2*b^2*d*x-3*a*b^3*c*x)-a^2/b^7*(((-2
5/8*a^3*b*f+21/8*a^2*e*b^2-17/8*a*b^3*d+13/8*b^4*c)*x^3-1/8*a*(23*a^3*f-19
*a^2*b*e+15*a*b^2*d-11*b^3*c)*x)/(b*x^2+a)^2+1/8*(143*a^3*f-99*a^2*b*e+63*
a*b^2*d-35*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.66

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$

$$= \left[\frac{560\,b^6fx^{13}+80\,(9\,b^6e-13\,ab^5f)x^{11}+16\,(63\,b^6d-99\,ab^5e+143\,a^2b^4f)x^9+48\,(35\,b^6c-63\,ab^5d+99\,a}{}\right.$$

input
```
integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/5040*(560*b^6*f*x^13 + 80*(9*b^6*e - 13*a*b^5*f)*x^11 + 16*(63*b^6*d -
99*a*b^5*e + 143*a^2*b^4*f)*x^9 + 48*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e
 - 143*a^3*b^3*f)*x^7 - 336*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 14
3*a^4*b^2*f)*x^5 - 1050*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*
a^5*b*f)*x^3 - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f +
 (35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^
2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(-a/b)*log((
b*x^2 - 2*b*x*sqrt(-a/b) - a)/(b*x^2 + a)) - 630*(35*a^3*b^3*c - 63*a^4*b^
2*d + 99*a^5*b*e - 143*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7), 1/2520
*(280*b^6*f*x^13 + 40*(9*b^6*e - 13*a*b^5*f)*x^11 + 8*(63*b^6*d - 99*a*b^5
*e + 143*a^2*b^4*f)*x^9 + 24*(35*b^6*c - 63*a*b^5*d + 99*a^2*b^4*e - 143*a
^3*b^3*f)*x^7 - 168*(35*a*b^5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^
2*f)*x^5 - 525*(35*a^2*b^4*c - 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*
x^3 + 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f + (35*a*b^
5*c - 63*a^2*b^4*d + 99*a^3*b^3*e - 143*a^4*b^2*f)*x^4 + 2*(35*a^2*b^4*c -
 63*a^3*b^3*d + 99*a^4*b^2*e - 143*a^5*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt
(a/b)/a) - 315*(35*a^3*b^3*c - 63*a^4*b^2*d + 99*a^5*b*e - 143*a^6*f)*x)/(
b^9*x^4 + 2*a*b^8*x^2 + a^2*b^7)]
```

3.133.6 Sympy [A] (verification not implemented)

Time = 19.39 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.75

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x^7\left(-\frac{3af}{7b^4}+\frac{e}{7b^3}\right)+x^5\cdot\left(\frac{6a^2f}{5b^5}-\frac{3ae}{5b^4}+\frac{d}{5b^3}\right) + x^3\left(-\frac{10a^3f}{3b^6}+\frac{2a^2e}{b^5}-\frac{ad}{b^4}+\frac{c}{3b^3}\right)+x\left(\frac{15a^4f}{b^7}-\frac{10a^3e}{b^6}+\frac{6a^2d}{b^5}-\frac{3ac}{b^4}\right) + \frac{\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)\log\left(-\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16} - \frac{\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)\log\left(\frac{b^7\sqrt{-\frac{a^3}{b^{15}}}\cdot(143a^3f-99a^2be+63ab^2d-35b^3c)}{143a^4f-99a^3be+63a^2b^2d-35ab^3c}+x\right)}{16} + \frac{x^3\cdot(25a^5bf-21a^4b^2e+17a^3b^3d-13a^2b^4c)+x(23a^6f-19a^5be+15a^4b^2d-11a^3b^3c)}{8a^2b^7+16ab^8x^2+8b^9x^4} + \frac{fx^9}{9b^3}$$

input
```
integrate(x**8*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
x**7*(-3*a*f/(7*b**4) + e/(7*b**3)) + x**5*(6*a**2*f/(5*b**5) - 3*a*e/(5*b
**4) + d/(5*b**3)) + x**3*(-10*a**3*f/(3*b**6) + 2*a**2*e/b**5 - a*d/b**4
+ c/(3*b**3)) + x*(15*a**4*f/b**7 - 10*a**3*e/b**6 + 6*a**2*d/b**5 - 3*a*c
/b**4) + sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b*
*3*c)*log(-b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d
- 35*b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x
)/16 - sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 35*b**3
*c)*log(b**7*sqrt(-a**3/b**15)*(143*a**3*f - 99*a**2*b*e + 63*a*b**2*d - 3
5*b**3*c)/(143*a**4*f - 99*a**3*b*e + 63*a**2*b**2*d - 35*a*b**3*c) + x)/1
6 + (x**3*(25*a**5*b*f - 21*a**4*b**2*e + 17*a**3*b**3*d - 13*a**2*b**4*c)
 + x*(23*a**6*f - 19*a**5*b*e + 15*a**4*b**2*d - 11*a**3*b**3*c))/(8*a**2*
b**7 + 16*a*b**8*x**2 + 8*b**9*x**4) + f*x**9/(9*b**3)
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.98

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(13\,a^2b^4c-17\,a^3b^3d+21\,a^4b^2e-25\,a^5bf)x^3+(11\,a^3b^3c-15\,a^4b^2d+19\,a^5be-23\,a^6f)x}{8\,(b^9x^4+2\,ab^8x^2+a^2b^7)} + \frac{(35\,a^2b^3c-63\,a^3b^2d+99\,a^4be-143\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} + \frac{35\,b^4fx^9+45\,(b^4e-3\,ab^3f)x^7+63\,(b^4d-3\,ab^3e+6\,a^2b^2f)x^5+105\,(b^4c-3\,ab^3d+6\,a^2b^2e-10\,a^3bf)}{315\,b^7}$$

input
```
integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((13*a^2*b^4*c - 17*a^3*b^3*d + 21*a^4*b^2*e - 25*a^5*b*f)*x^3 + (11*
a^3*b^3*c - 15*a^4*b^2*d + 19*a^5*b*e - 23*a^6*f)*x)/(b^9*x^4 + 2*a*b^8*x^
2 + a^2*b^7) + 1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*
arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^7) + 1/315*(35*b^4*f*x^9 + 45*(b^4*e -
3*a*b^3*f)*x^7 + 63*(b^4*d - 3*a*b^3*e + 6*a^2*b^2*f)*x^5 + 105*(b^4*c - 3
*a*b^3*d + 6*a^2*b^2*e - 10*a^3*b*f)*x^3 - 315*(3*a*b^3*c - 6*a^2*b^2*d +
10*a^3*b*e - 15*a^4*f)*x)/b^7
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.02

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(35\,a^2b^3c-63\,a^3b^2d+99\,a^4be-143\,a^5f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^7} - \frac{13\,a^2b^4cx^3-17\,a^3b^3dx^3+21\,a^4b^2ex^3-25\,a^5bfx^3+11\,a^3b^3cx-15\,a^4b^2dx+19\,a^5bex-23\,a^6fx}{8\,(bx^2+a)^2b^7} + \frac{35\,b^{24}fx^9+45\,b^{24}ex^7-135\,ab^{23}fx^7+63\,b^{24}dx^5-189\,ab^{23}ex^5+378\,a^2b^{22}fx^5+105\,b^{24}cx^3-315\,ab^{23}d}{315\,b^{27}}$$

input
```
integrate(x^8*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(35*a^2*b^3*c - 63*a^3*b^2*d + 99*a^4*b*e - 143*a^5*f)*arctan(b*x/sqrt
(a*b))/(sqrt(a*b)*b^7) - 1/8*(13*a^2*b^4*c*x^3 - 17*a^3*b^3*d*x^3 + 21*a^4
*b^2*e*x^3 - 25*a^5*b*f*x^3 + 11*a^3*b^3*c*x - 15*a^4*b^2*d*x + 19*a^5*b*e
*x - 23*a^6*f*x)/((b*x^2 + a)^2*b^7) + 1/315*(35*b^24*f*x^9 + 45*b^24*e*x^
7 - 135*a*b^23*f*x^7 + 63*b^24*d*x^5 - 189*a*b^23*e*x^5 + 378*a^2*b^22*f*x
^5 + 105*b^24*c*x^3 - 315*a*b^23*d*x^3 + 630*a^2*b^22*e*x^3 - 1050*a^3*b^2
1*f*x^3 - 945*a*b^23*c*x + 1890*a^2*b^22*d*x - 3150*a^3*b^21*e*x + 4725*a^
4*b^20*f*x)/b^27
```

3.133.9 Mupad [B] (verification not implemented)

Time = 5.59 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.76

$$\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^7\left(\frac{e}{7\,b^3}-\frac{3\,a\,f}{7\,b^4}\right)+x^3\left(\frac{c}{3\,b^3}-\frac{a^3\,f}{3\,b^6}-\frac{a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)$$
$$+\frac{x\,\left(\frac{23\,f\,a^6}{8}-\frac{19\,e\,a^5\,b}{8}+\frac{15\,d\,a^4\,b^2}{8}-\frac{11\,c\,a^3\,b^3}{8}\right)-x^3\,\left(-\frac{25\,f\,a^5\,b}{8}+\frac{21\,e\,a^4\,b^2}{8}-\frac{17\,d\,a^3\,b^3}{8}+\frac{13\,c\,a^2\,b^4}{8}\right)}{a^2\,b^7+2\,a\,b^8\,x^2+b^9\,x^4}$$
$$-x\left(\frac{3\,a\,\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right)}{b}-\frac{3\,a^2\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b^2}+\frac{a^3\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^3}\right)$$
$$-x^5\left(\frac{3\,a^2\,f}{5\,b^5}-\frac{d}{5\,b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{5\,b}\right)+\frac{f\,x^9}{9\,b^3}$$
$$-\frac{a^{3/2}\,\mathrm{atan}\left(\frac{a^{3/2}\,\sqrt{b}\,x\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{143\,f\,a^5-99\,e\,a^4\,b+63\,d\,a^3\,b^2-35\,c\,a^2\,b^3}\right)\,\left(-143\,f\,a^3+99\,e\,a^2\,b-63\,d\,a\,b^2+35\,c\,b^3\right)}{8\,b^{15/2}}$$

input
```
int((x^8*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

output
```
x^7*(e/(7*b^3) - (3*a*f)/(7*b^4)) + x^3*(c/(3*b^3) - (a^3*f)/(3*b^6) - (a^
2*(e/b^3 - (3*a*f)/b^4))/b^2 + (a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (
3*a*f)/b^4))/b))/b) + (x*((23*a^6*f)/8 - (11*a^3*b^3*c)/8 + (15*a^4*b^2*d)
/8 - (19*a^5*b*e)/8) - x^3*((13*a^2*b^4*c)/8 - (17*a^3*b^3*d)/8 + (21*a^4*
b^2*e)/8 - (25*a^5*b*f)/8))/(a^2*b^7 + b^9*x^4 + 2*a*b^8*x^2) - x*((3*a*(c
/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b
^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b))/b - (3*a^2*((3*a^2*f)/b^5
 - d/b^3 + (3*a*(e/b^3 - (3*a*f)/b^4))/b))/b^2 + (a^3*(e/b^3 - (3*a*f)/b^4
))/b^3) - x^5*((3*a^2*f)/(5*b^5) - d/(5*b^3) + (3*a*(e/b^3 - (3*a*f)/b^4))
/(5*b)) + (f*x^9)/(9*b^3) - (a^(3/2)*atan((a^(3/2)*b^(1/2)*x*(35*b^3*c - 1
43*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(143*a^5*f - 35*a^2*b^3*c + 63*a^3*b^
2*d - 99*a^4*b*e))*(35*b^3*c - 143*a^3*f - 63*a*b^2*d + 99*a^2*b*e))/(8*b^
(15/2))
```

3.133. $\int \frac{x^8(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.134 $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx$

3.134.1 Optimal result

Integrand size = 30, antiderivative size = 247

$$\begin{aligned}\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx = & \frac{(3b^3c-7ab^2d+13a^2be-21a^3f)x}{2b^6} \\ & - \frac{(3b^3c-7ab^2d+15a^2be-27a^3f)x^3}{12ab^5} \\ & + \frac{(be-3af)x^5}{5b^4} + \frac{fx^7}{7b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^7}{4a(a+bx^2)^2} \\ & + \frac{a(3b^3c-7ab^2d+11a^2be-15a^3f)x}{8b^6(a+bx^2)} \\ & - \frac{\sqrt{a}(15b^3c-35ab^2d+63a^2be-99a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}\end{aligned}$$

output
```
1/2*(-21*a^3*f+13*a^2*b*e-7*a*b^2*d+3*b^3*c)*x/b^6-1/12*(-27*a^3*f+15*a^2*
b*e-7*a*b^2*d+3*b^3*c)*x^3/a/b^5+1/5*(-3*a*f+b*e)*x^5/b^4+1/7*f*x^7/b^3+1/
4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^7/a/(b*x^2+a)^2+1/8*a*(-15*a^3*f+11*a^2*
b*e-7*a*b^2*d+3*b^3*c)*x/b^6/(b*x^2+a)-1/8*(-99*a^3*f+63*a^2*b*e-35*a*b^2*
d+15*b^3*c)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(13/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\,x}{b^6}+\frac{(b^2d-3abe+6a^2f)\,x^3}{3b^5}+\frac{(be-3af)x^5}{5b^4}+\frac{fx^7}{7b^3}+\frac{a^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{4b^6\,(a+bx^2)^2}+\frac{a(9b^3c-13ab^2d+17a^2be-21a^3f)\,x}{8b^6\,(a+bx^2)}+\frac{\sqrt{a}(-15b^3c+35ab^2d-63a^2be+99a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8b^{13/2}}$$

input
```
Integrate[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
((b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*x)/b^6 + ((b^2*d - 3*a*b*e + 6
*a^2*f)*x^3)/(3*b^5) + ((b*e - 3*a*f)*x^5)/(5*b^4) + (f*x^7)/(7*b^3) + (a^
2*(-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*b^6*(a + b*x^2)^2) + (a*(9*
b^3*c - 13*a*b^2*d + 17*a^2*b*e - 21*a^3*f)*x)/(8*b^6*(a + b*x^2)) + (Sqrt
[a]*(-15*b^3*c + 35*a*b^2*d - 63*a^2*b*e + 99*a^3*f)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*b^(13/2))
```

3.134.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2335, 9, 1580, 25, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6\left(c+dx^2+ex^4+fx^6\right)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2335}$$

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\,(a+bx^2)^2}-\frac{\int \frac{x^5\left(-4afx^5-4a\left(e-\frac{af}{b}\right)x^3+\left(-\frac{7fa^3}{b^2}+\frac{7ea^2}{b}-7da+3bc\right)x\right)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \text{ 9}$$

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\int \frac{x^6\left(-4afx^4-4a\left(e-\frac{af}{b}\right)x^2+3bc-7ad+\frac{7a^2e}{b}-\frac{7a^3f}{b^2}\right)}{(bx^2+a)^2}dx}{4ab}$$

$\Big\downarrow$ 1580

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{-\frac{\int -\frac{-8ab^4fx^8-8ab^3(be-2af)x^6+2b^2\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)x^4-2ab\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)x^2+a^2\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)}{bx^2+a}dx}{2b^5}-}{4ab}$$

$\Big\downarrow$ 25

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\frac{\int \frac{-8ab^4fx^8-8ab^3(be-2af)x^6+2b^2\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)x^4-2ab\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)x^2+a^2\left(-15fa^3+11bea^2-7b^2da+3b^3c\right)}{bx^2+a}dx}{2b^5}-\frac{a^2x}{}}{4ab}$$

$\Big\downarrow$ 2341

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\frac{\int \left(-8ab^3fx^6-8ab^2(be-3af)x^4+2b(-27fa^3+15bea^2-7b^2da+3b^3c)x^2-4a(-21fa^3+13bea^2-7b^2da+3b^3c)+\frac{-99fa^5+63bea^4-35b^2da^3+15b^3ca^2}{bx^2+a}\right)d}{2b^5}}{4ab}$$

$\Big\downarrow$ 2009

$$\frac{x^7\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a(a+bx^2)^2}-\frac{\frac{\frac{2}{3}bx^3(-27a^3f+15a^2be-7ab^2d+3b^3c)-4ax(-21a^3f+13a^2be-7ab^2d+3b^3c)+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-99a^3f+63a^2be-35ab^2d+15b^3c)}{\sqrt{b}}-\frac{8}{7}ab^3fx^7-\frac{8}{5}ab^2}{2b^5}}{4ab}$$

input
```
Int[(x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

3.134. $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^7)/(4*a*(a + b*x^2)^2) - (-1/2*(a
^2*(3*b^3*c - 7*a*b^2*d + 11*a^2*b*e - 15*a^3*f)*x)/(b^5*(a + b*x^2)) + (-
4*a*(3*b^3*c - 7*a*b^2*d + 13*a^2*b*e - 21*a^3*f)*x + (2*b*(3*b^3*c - 7*a*
b^2*d + 15*a^2*b*e - 27*a^3*f)*x^3)/3 - (8*a*b^2*(b*e - 3*a*f)*x^5)/5 - (8
*a*b^3*f*x^7)/7 + (a^(3/2)*(15*b^3*c - 35*a*b^2*d + 63*a^2*b*e - 99*a^3*f)
*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^5))/(4*a*b)
```

3.134.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.134. $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3} dx$

3.134.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.89

method	result
default	$-\frac{-\frac{1}{7} f\,x^{7} b^{3}+\frac{3}{5} a\,b^{2} f\,x^{5}-\frac{1}{5} b^{3} e\,x^{5}-2 a^{2} b f\,x^{3}+a\,b^{2} e\,x^{3}-\frac{1}{3} b^{3} d\,x^{3}+10 f\,a^{3} x-6 a^{2} b e x+3 a\,b^{2} d x-b^{3} c x}{b^{6}}+\frac{a\left(\frac{\left(-\frac{21}{8} a^{3} b f+\frac{17}{8} a^{2} e\,b^{2}-\frac{13}{8} a \cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{f\,x^{7}}{7 b^{3}}-\frac{3 a f\,x^{5}}{5 b^{4}}+\frac{e\,x^{5}}{5 b^{3}}+\frac{2 a^{2} f\,x^{3}}{b^{5}}-\frac{a e\,x^{3}}{b^{4}}+\frac{d\,x^{3}}{3 b^{3}}-\frac{10 f\,a^{3} x}{b^{6}}+\frac{6 a^{2} e x}{b^{5}}-\frac{3 a d x}{b^{4}}+\frac{c x}{b^{3}}+\frac{\left(-\frac{21}{8} a^{4} b f+\frac{17}{8} a^{3} b^{2} e-\frac{13}{8} a^{2} b^{3} d-\cdots\right.}{\cdots}$

input
```
int(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^6*(-1/7*f*x^7*b^3+3/5*a*b^2*f*x^5-1/5*b^3*e*x^5-2*a^2*b*f*x^3+a*b^2*e
*x^3-1/3*b^3*d*x^3+10*f*a^3*x-6*a^2*b*e*x+3*a*b^2*d*x-b^3*c*x)+a/b^6*(((-2
1/8*a^3*b*f+17/8*a^2*e*b^2-13/8*a*b^3*d+9/8*b^4*c)*x^3-1/8*a*(19*a^3*f-15*
a^2*b*e+11*a*b^2*d-7*b^3*c)*x)/(b*x^2+a)^2+1/8*(99*a^3*f-63*a^2*b*e+35*a*b
^2*d-15*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 668, normalized size of antiderivative = 2.70

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$=\left[\frac{240\,b^5fx^{11}+48\,(7\,b^5e-11\,ab^4f)x^9+16\,(35\,b^5d-63\,ab^4e+99\,a^2b^3f)x^7+112\,(15\,b^5c-35\,ab^4d+63\,a^2\cdots}{\cdots}\right.$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/1680*(240*b^5*f*x^11 + 48*(7*b^5*e - 11*a*b^4*f)*x^9 + 16*(35*b^5*d - 6
3*a*b^4*e + 99*a^2*b^3*f)*x^7 + 112*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e
- 99*a^3*b^2*f)*x^5 + 350*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a
^4*b*f)*x^3 - 105*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (
15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c -
 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(-a/b)*log((b*x^2 + 2*
b*x*sqrt(-a/b) - a)/(b*x^2 + a)) + 210*(15*a^2*b^3*c - 35*a^3*b^2*d + 63*a
^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a^2*b^6), 1/840*(120*b^5*f*
x^11 + 24*(7*b^5*e - 11*a*b^4*f)*x^9 + 8*(35*b^5*d - 63*a*b^4*e + 99*a^2*b
^3*f)*x^7 + 56*(15*b^5*c - 35*a*b^4*d + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^5 +
 175*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^3*b^2*e - 99*a^4*b*f)*x^3 - 105*(15
*a^2*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f + (15*b^5*c - 35*a*b^4*d
 + 63*a^2*b^3*e - 99*a^3*b^2*f)*x^4 + 2*(15*a*b^4*c - 35*a^2*b^3*d + 63*a^
3*b^2*e - 99*a^4*b*f)*x^2)*sqrt(a/b)*arctan(b*x*sqrt(a/b)/a) + 105*(15*a^2
*b^3*c - 35*a^3*b^2*d + 63*a^4*b*e - 99*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 +
 a^2*b^6)]
```

3.134.6 Sympy [A] (verification not implemented)

Time = 17.92 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.28

$$\begin{aligned}
&\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx \\
&= x^5\left(-\frac{3af}{5b^4}+\frac{e}{5b^3}\right)+x^3\cdot\left(\frac{2a^2f}{b^5}-\frac{ae}{b^4}+\frac{d}{3b^3}\right)+x\left(-\frac{10a^3f}{b^6}+\frac{6a^2e}{b^5}-\frac{3ad}{b^4}+\frac{c}{b^3}\right) \\
&\quad-\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^3f-63a^2be+35ab^2d-15b^3c)\log\left(-b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16} \\
&\quad+\frac{\sqrt{-\frac{a}{b^{13}}}\cdot(99a^3f-63a^2be+35ab^2d-15b^3c)\log\left(b^6\sqrt{-\frac{a}{b^{13}}}+x\right)}{16} \\
&\quad+\frac{x^3(-21a^4bf+17a^3b^2e-13a^2b^3d+9ab^4c)+x(-19a^5f+15a^4be-11a^3b^2d+7a^2b^3c)}{8a^2b^6+16ab^7x^2+8b^8x^4} \\
&\quad+\frac{fx^7}{7b^3}
\end{aligned}$$

input
```
integrate(x**6*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
x**5*(-3*a*f/(5*b**4) + e/(5*b**3)) + x**3*(2*a**2*f/b**5 - a*e/b**4 + d/(
3*b**3)) + x*(-10*a**3*f/b**6 + 6*a**2*e/b**5 - 3*a*d/b**4 + c/b**3) - sqr
t(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b**2*d - 15*b**3*c)*log(-b**6*
sqrt(-a/b**13) + x)/16 + sqrt(-a/b**13)*(99*a**3*f - 63*a**2*b*e + 35*a*b*
*2*d - 15*b**3*c)*log(b**6*sqrt(-a/b**13) + x)/16 + (x**3*(-21*a**4*b*f +
17*a**3*b**2*e - 13*a**2*b**3*d + 9*a*b**4*c) + x*(-19*a**5*f + 15*a**4*b*
e - 11*a**3*b**2*d + 7*a**2*b**3*c))/(8*a**2*b**6 + 16*a*b**7*x**2 + 8*b**
8*x**4) + f*x**7/(7*b**3)
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.96

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{(9\,ab^4c-13\,a^2b^3d+17\,a^3b^2e-21\,a^4bf)x^3+(7\,a^2b^3c-11\,a^3b^2d+15\,a^4be-19\,a^5f)x}{8\,(b^8x^4+2\,ab^7x^2+a^2b^6)}$$
$$-\frac{(15\,ab^3c-35\,a^2b^2d+63\,a^3be-99\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6}$$
$$+\frac{15\,b^3fx^7+21\,(b^3e-3\,ab^2f)x^5+35\,(b^3d-3\,ab^2e+6\,a^2bf)x^3+105\,(b^3c-3\,ab^2d+6\,a^2be-10\,a^3f)x}{105\,b^6}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((9*a*b^4*c - 13*a^2*b^3*d + 17*a^3*b^2*e - 21*a^4*b*f)*x^3 + (7*a^2*b
^3*c - 11*a^3*b^2*d + 15*a^4*b*e - 19*a^5*f)*x)/(b^8*x^4 + 2*a*b^7*x^2 + a
^2*b^6) - 1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b
*x/sqrt(a*b))/(sqrt(a*b)*b^6) + 1/105*(15*b^3*f*x^7 + 21*(b^3*e - 3*a*b^2*
f)*x^5 + 35*(b^3*d - 3*a*b^2*e + 6*a^2*b*f)*x^3 + 105*(b^3*c - 3*a*b^2*d +
 6*a^2*b*e - 10*a^3*f)*x)/b^6
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.99

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(15\,ab^3c-35\,a^2b^2d+63\,a^3be-99\,a^4f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^6} + \frac{9\,ab^4cx^3-13\,a^2b^3dx^3+17\,a^3b^2ex^3-21\,a^4bfx^3+7\,a^2b^3cx-11\,a^3b^2dx+15\,a^4bex-19\,a^5fx}{8\,(bx^2+a)^2b^6} + \frac{15\,b^{18}fx^7+21\,b^{18}ex^5-63\,ab^{17}fx^5+35\,b^{18}dx^3-105\,ab^{17}ex^3+210\,a^2b^{16}fx^3+105\,b^{18}cx-315\,ab^{17}dx}{105\,b^{21}}$$

input
```
integrate(x^6*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(15*a*b^3*c - 35*a^2*b^2*d + 63*a^3*b*e - 99*a^4*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*b^6) + 1/8*(9*a*b^4*c*x^3 - 13*a^2*b^3*d*x^3 + 17*a^3*b^2*
e*x^3 - 21*a^4*b*f*x^3 + 7*a^2*b^3*c*x - 11*a^3*b^2*d*x + 15*a^4*b*e*x - 1
9*a^5*f*x)/((b*x^2 + a)^2*b^6) + 1/105*(15*b^18*f*x^7 + 21*b^18*e*x^5 - 63
*a*b^17*f*x^5 + 35*b^18*d*x^3 - 105*a*b^17*e*x^3 + 210*a^2*b^16*f*x^3 + 10
5*b^18*c*x - 315*a*b^17*d*x + 630*a^2*b^16*e*x - 1050*a^3*b^15*f*x)/b^21
```

3.134.9 Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.41

$$\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x^5\left(\frac{e}{5\,b^3}-\frac{3\,a\,f}{5\,b^4}\right)+x\left(\frac{c}{b^3}-\frac{a^3\,f}{b^6}-\frac{3\,a^2\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b^2}+\frac{3\,a\,\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)}{b}\right) - x^3\left(\frac{a^2\,f}{b^5}-\frac{d}{3\,b^3}+\frac{a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right) - \frac{\left(\frac{21\,f\,a^4\,b}{8}-\frac{17\,e\,a^3\,b^2}{8}+\frac{13\,d\,a^2\,b^3}{8}-\frac{9\,c\,a\,b^4}{8}\right)\,x^3+\left(\frac{19\,f\,a^5}{8}-\frac{15\,e\,a^4\,b}{8}+\frac{11\,d\,a^3\,b^2}{8}-\frac{7\,c\,a^2\,b^3}{8}\right)\,x}{a^2\,b^6+2\,a\,b^7\,x^2+b^8\,x^4} + \frac{f\,x^7}{7\,b^3} + \frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\sqrt{a}\,\sqrt{b}\,x\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{99\,f\,a^4-63\,e\,a^3\,b+35\,d\,a^2\,b^2-15\,c\,a\,b^3}\right)\,\left(-99\,f\,a^3+63\,e\,a^2\,b-35\,d\,a\,b^2+15\,c\,b^3\right)}{8\,b^{13/2}}$$

input
```
int((x^6*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

output
```
x^5*(e/(5*b^3) - (3*a*f)/(5*b^4)) + x*(c/b^3 - (a^3*f)/b^6 - (3*a^2*(e/b^3
 - (3*a*f)/b^4))/b^2 + (3*a*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*a*f)
/b^4))/b))/b) - x^3*((a^2*f)/b^5 - d/(3*b^3) + (a*(e/b^3 - (3*a*f)/b^4))/b
) - (x*((19*a^5*f)/8 - (7*a^2*b^3*c)/8 + (11*a^3*b^2*d)/8 - (15*a^4*b*e)/8
) + x^3*((13*a^2*b^3*d)/8 - (17*a^3*b^2*e)/8 - (9*a*b^4*c)/8 + (21*a^4*b*f
)/8))/(a^2*b^6 + b^8*x^4 + 2*a*b^7*x^2) + (f*x^7)/(7*b^3) + (a^(1/2)*atan(
(a^(1/2)*b^(1/2)*x*(15*b^3*c - 99*a^3*f - 35*a*b^2*d + 63*a^2*b*e))/(99*a^
4*f + 35*a^2*b^2*d - 15*a*b^3*c - 63*a^3*b*e))*(15*b^3*c - 99*a^3*f - 35*a
*b^2*d + 63*a^2*b*e))/(8*b^(13/2))
```

3.134. $\int \frac{x^6(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.135 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.135.1 Optimal result

Integrand size = 30, antiderivative size = 207

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = -\frac{(b^3c-5ab^2d+13a^2be-25a^3f)\,x}{4ab^5} + \frac{(be-3af)x^3}{3b^4} + \frac{fx^5}{5b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^5}{4a\,(a+bx^2)^2} - \frac{(b^3c-5ab^2d+9a^2be-13a^3f)\,x}{8b^5\,(a+bx^2)} + \frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

output
```
-1/4*(-25*a^3*f+13*a^2*b*e-5*a*b^2*d+b^3*c)*x/a/b^5+1/3*(-3*a*f+b*e)*x^3/b
^4+1/5*f*x^5/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^5/a/(b*x^2+a)^2-1/8*(
-13*a^3*f+9*a^2*b*e-5*a*b^2*d+b^3*c)*x/b^5/(b*x^2+a)+1/8*(-63*a^3*f+35*a^2
*b*e-15*a*b^2*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/b^(11/2)/a^(1/2)
```

3.135.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.85

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \frac{x(945a^4f-525a^3b(e-3fx^2)+a^2b^2(225d-875ex^2+504fx^4)-ab^3(45c-375dx^2+280ex^4+72fx^6)-}{120b^5\,(a+bx^2)^2}$$
$$+\frac{(3b^3c-15ab^2d+35a^2be-63a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8\sqrt{a}b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
(x*(945*a^4*f - 525*a^3*b*(e - 3*f*x^2) + a^2*b^2*(225*d - 875*e*x^2 + 504
*f*x^4) - a*b^3*(45*c - 375*d*x^2 + 280*e*x^4 + 72*f*x^6) + b^4*x^2*(-75*c
 + 8*(15*d*x^2 + 5*e*x^4 + 3*f*x^6))))/(120*b^5*(a + b*x^2)^2) + ((3*b^3*c
 - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*Sqr
t[a]*b^(11/2))
```

3.135.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2335, 9, 1580, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^5\left(c-\frac{a\left(a^2f-abe+b^2d\right)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int \frac{x^3\left(-4afx^5-4a\left(e-\frac{af}{b}\right)x^3+\left(-\frac{5fa^3}{b^2}+\frac{5ea^2}{b}-5da+bc\right)x\right)}{\left(bx^2+a\right)^2}dx}{4ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int\frac{x^4\left(-4afx^4-4a\left(e-\frac{af}{b}\right)x^2+bc-5ad+\frac{5a^2e}{b}-\frac{5a^3f}{b^2}\right)}{(bx^2+a)^2}dx}{4ab}$$

$$\Big\downarrow \; 1580$$

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\frac{ax(-13a^3f+9a^2be-5ab^2d+b^3c)}{2b^4(a+bx^2)}-\frac{\int\frac{8ab^3fx^6+8ab^2(be-2af)x^4-2b\left(-13fa^3+9bea^2-5b^2da+b^3c\right)x^2+a\left(-13fa^3+9bea^2-5b^2da+b^3c\right)}{bx^2+a}dx}{2b^4}}{4ab}$$

$$\Big\downarrow \; 2341$$

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\frac{ax(-13a^3f+9a^2be-5ab^2d+b^3c)}{2b^4(a+bx^2)}-\frac{\int\left(8ab^2fx^4+8ab(be-3af)x^2-2(-25fa^3+13bea^2-5b^2da+b^3c)+\frac{-63fa^4+35bea^3-15b^2da^2+3b^3ca}{bx^2+a}\right)dx}{2b^4}}{4ab}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^5\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\frac{ax(-13a^3f+9a^2be-5ab^2d+b^3c)}{2b^4(a+bx^2)}-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-63a^3f+35a^2be-15ab^2d+3b^3c\right)}{\sqrt{b}}-2x(-25a^3f+13a^2be-5ab^2d+b^3c)+\frac{8}{5}ab^2fx^5+\frac{8}{3}abx^3(be-3a}{2b^4}}{4ab}$$

input
```
Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^5)/(4*a*(a + b*x^2)^2) - ((a*(b^3
*c - 5*a*b^2*d + 9*a^2*b*e - 13*a^3*f)*x)/(2*b^4*(a + b*x^2)) - (-2*(b^3*c
 - 5*a*b^2*d + 13*a^2*b*e - 25*a^3*f)*x + (8*a*b*(b*e - 3*a*f)*x^3)/3 + (8
*a*b^2*f*x^5)/5 + (Sqrt[a]*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*
ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[b])/(2*b^4))/(4*a*b)
```

3.135. $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\left(a+bx^2\right)^3}\,dx$

3.135.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.135. $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.135.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.86

method	result
default	$\frac{\frac{1}{5} f\,x^5 b^2 - abf\,x^3 + \frac{1}{3} b^2 e\,x^3 + 6a^2 fx - 3abex + b^2 dx}{b^5} - \frac{\frac{\left(-\frac{17}{8} a^3 bf + \frac{13}{8} a^2 e\,b^2 - \frac{9}{8} a\,b^3 d + \frac{5}{8} b^4 c\right) x^3 - \frac{a\left(15 f\,a^3 - 11 a^2 be + 7 a\,b^2 d - 3 b^3 c\right) x}{8}}{\left(b\,x^2 + a\right)^2} + \frac{\left(63 f\,a\right.}{}}{b^5}$
risch	$\frac{f\,x^5}{5b^3} - \frac{af\,x^3}{b^4} + \frac{e\,x^3}{3b^3} + \frac{6a^2 fx}{b^5} - \frac{3aex}{b^4} + \frac{dx}{b^3} + \frac{\left(\frac{17}{8} a^3 bf - \frac{13}{8} a^2 e\,b^2 + \frac{9}{8} a\,b^3 d - \frac{5}{8} b^4 c\right) x^3 + \frac{a\left(15 f\,a^3 - 11 a^2 be + 7 a\,b^2 d - 3 b^3 c\right) x}{8}}{b^5 \left(b\,x^2 + a\right)^2} -$

input
```
int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b^5*(1/5*f*x^5*b^2-a*b*f*x^3+1/3*b^2*e*x^3+6*a^2*f*x-3*a*b*e*x+b^2*d*x)-
1/b^5*(((-17/8*a^3*b*f+13/8*a^2*e*b^2-9/8*a*b^3*d+5/8*b^4*c)*x^3-1/8*a*(15
*a^3*f-11*a^2*b*e+7*a*b^2*d-3*b^3*c)*x)/(b*x^2+a)^2+1/8*(63*a^3*f-35*a^2*b
*e+15*a*b^2*d-3*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 614, normalized size of antiderivative = 2.97

$$\int \frac{x^4(c + dx^2 + ex^4 + fx^6)}{(a + bx^2)^3}\,dx$$

$$= \left[\frac{48\,ab^5fx^9 + 16\,(5\,ab^5e - 9\,a^2b^4f)x^7 + 16\,(15\,ab^5d - 35\,a^2b^4e + 63\,a^3b^3f)x^5 - 50\,(3\,ab^5c - 15\,a^2b^4d + 35}{}\right.$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/240*(48*a*b^5*f*x^9 + 16*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 16*(15*a*b^5*d
 - 35*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 50*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^
3*b^3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e
- 63*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*
(3*a*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(-a*b)*log
((b*x^2 + 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 30*(3*a^2*b^4*c - 15*a^3*b^3*
d + 35*a^4*b^2*e - 63*a^5*b*f)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6), 1
/120*(24*a*b^5*f*x^9 + 8*(5*a*b^5*e - 9*a^2*b^4*f)*x^7 + 8*(15*a*b^5*d - 3
5*a^2*b^4*e + 63*a^3*b^3*f)*x^5 - 25*(3*a*b^5*c - 15*a^2*b^4*d + 35*a^3*b^
3*e - 63*a^4*b^2*f)*x^3 + 15*(3*a^2*b^3*c - 15*a^3*b^2*d + 35*a^4*b*e - 63
*a^5*f + (3*b^5*c - 15*a*b^4*d + 35*a^2*b^3*e - 63*a^3*b^2*f)*x^4 + 2*(3*a
*b^4*c - 15*a^2*b^3*d + 35*a^3*b^2*e - 63*a^4*b*f)*x^2)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) - 15*(3*a^2*b^4*c - 15*a^3*b^3*d + 35*a^4*b^2*e - 63*a^5*b*f
)*x)/(a*b^8*x^4 + 2*a^2*b^7*x^2 + a^3*b^6)]
```

3.135.6 Sympy [A] (verification not implemented)

Time = 15.94 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.35

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^3\left(-\frac{af}{b^4}+\frac{e}{3b^3}\right)+x\left(\frac{6a^2f}{b^5}-\frac{3ae}{b^4}+\frac{d}{b^3}\right)$$
$$+\frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^3f-35a^2be+15ab^2d-3b^3c)\log\left(-ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}$$
$$-\frac{\sqrt{-\frac{1}{ab^{11}}}\cdot(63a^3f-35a^2be+15ab^2d-3b^3c)\log\left(ab^5\sqrt{-\frac{1}{ab^{11}}}+x\right)}{16}$$
$$+\frac{x^3\cdot(17a^3bf-13a^2b^2e+9ab^3d-5b^4c)+x(15a^4f-11a^3be+7a^2b^2d-3ab^3c)}{8a^2b^5+16ab^6x^2+8b^7x^4}+\frac{fx^5}{5b^3}$$

input
```
integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
x**3*(-a*f/b**4 + e/(3*b**3)) + x*(6*a**2*f/b**5 - 3*a*e/b**4 + d/b**3) +
sqrt(-1/(a*b**11))*(63*a**3*f - 35*a**2*b*e + 15*a*b**2*d - 3*b**3*c)*log(
-a*b**5*sqrt(-1/(a*b**11)) + x)/16 - sqrt(-1/(a*b**11))*(63*a**3*f - 35*a*
*2*b*e + 15*a*b**2*d - 3*b**3*c)*log(a*b**5*sqrt(-1/(a*b**11)) + x)/16 + (
x**3*(17*a**3*b*f - 13*a**2*b**2*e + 9*a*b**3*d - 5*b**4*c) + x*(15*a**4*f
 - 11*a**3*b*e + 7*a**2*b**2*d - 3*a*b**3*c))/(8*a**2*b**5 + 16*a*b**6*x**
2 + 8*b**7*x**4) + f*x**5/(5*b**3)
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.93

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= -\frac{(5\,b^4c-9\,ab^3d+13\,a^2b^2e-17\,a^3bf)x^3+(3\,ab^3c-7\,a^2b^2d+11\,a^3be-15\,a^4f)x}{8\,(b^7x^4+2\,ab^6x^2+a^2b^5)}$$
$$+\frac{(3\,b^3c-15\,ab^2d+35\,a^2be-63\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}$$
$$+\frac{3\,b^2fx^5+5\,(b^2e-3\,abf)x^3+15\,(b^2d-3\,abe+6\,a^2f)x}{15\,b^5}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*((5*b^4*c - 9*a*b^3*d + 13*a^2*b^2*e - 17*a^3*b*f)*x^3 + (3*a*b^3*c -
 7*a^2*b^2*d + 11*a^3*b*e - 15*a^4*f)*x)/(b^7*x^4 + 2*a*b^6*x^2 + a^2*b^5)
 + 1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b)
)/(sqrt(a*b)*b^5) + 1/15*(3*b^2*f*x^5 + 5*(b^2*e - 3*a*b*f)*x^3 + 15*(b^2*
d - 3*a*b*e + 6*a^2*f)*x)/b^5
```

3.135.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$=\frac{(3\,b^3c-15\,ab^2d+35\,a^2be-63\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}b^5}$$
$$-\frac{5\,b^4cx^3-9\,ab^3dx^3+13\,a^2b^2ex^3-17\,a^3bfx^3+3\,ab^3cx-7\,a^2b^2dx+11\,a^3bex-15\,a^4fx}{8\,(bx^2+a)^2b^5}$$
$$+\frac{3\,b^{12}fx^5+5\,b^{12}ex^3-15\,ab^{11}fx^3+15\,b^{12}dx-45\,ab^{11}ex+90\,a^2b^{10}fx}{15\,b^{15}}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(3*b^3*c - 15*a*b^2*d + 35*a^2*b*e - 63*a^3*f)*arctan(b*x/sqrt(a*b))/(
sqrt(a*b)*b^5) - 1/8*(5*b^4*c*x^3 - 9*a*b^3*d*x^3 + 13*a^2*b^2*e*x^3 - 17*
a^3*b*f*x^3 + 3*a*b^3*c*x - 7*a^2*b^2*d*x + 11*a^3*b*e*x - 15*a^4*f*x)/((b
*x^2 + a)^2*b^5) + 1/15*(3*b^12*f*x^5 + 5*b^12*e*x^3 - 15*a*b^11*f*x^3 + 1
5*b^12*d*x - 45*a*b^11*e*x + 90*a^2*b^10*f*x)/b^15
```

3.135.9 Mupad [B] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x^3\left(\frac{e}{3\,b^3}-\frac{a\,f}{b^4}\right)-x\left(\frac{3\,a^2\,f}{b^5}-\frac{d}{b^3}+\frac{3\,a\,\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right)}{b}\right)$$
$$-\frac{x^3\left(-\frac{17\,f\,a^3\,b}{8}+\frac{13\,e\,a^2\,b^2}{8}-\frac{9\,d\,a\,b^3}{8}+\frac{5\,c\,b^4}{8}\right)-x\left(\frac{15\,f\,a^4}{8}-\frac{11\,e\,a^3\,b}{8}+\frac{7\,d\,a^2\,b^2}{8}-\frac{3\,c\,a\,b^3}{8}\right)}{a^2\,b^5+2\,a\,b^6\,x^2+b^7\,x^4}$$
$$+\frac{f\,x^5}{5\,b^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-63\,f\,a^3+35\,e\,a^2\,b-15\,d\,a\,b^2+3\,c\,b^3\right)}{8\,\sqrt{a}\,b^{11/2}}$$

input
```
int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

output
```
x^3*(e/(3*b^3) - (a*f)/b^4) - x*((3*a^2*f)/b^5 - d/b^3 + (3*a*(e/b^3 - (3*
a*f)/b^4))/b) - (x^3*((5*b^4*c)/8 + (13*a^2*b^2*e)/8 - (9*a*b^3*d)/8 - (17
*a^3*b*f)/8) - x*((15*a^4*f)/8 + (7*a^2*b^2*d)/8 - (3*a*b^3*c)/8 - (11*a^3
*b*e)/8))/(a^2*b^5 + b^7*x^4 + 2*a*b^6*x^2) + (f*x^5)/(5*b^3) + (atan((b^(
1/2)*x)/a^(1/2))*(3*b^3*c - 63*a^3*f - 15*a*b^2*d + 35*a^2*b*e))/(8*a^(1/2
)*b^(11/2))
```

3.136 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$

3.136.1 Optimal result

Integrand size = 30, antiderivative size = 167

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(be-3af)x}{b^4}+\frac{fx^3}{3b^3}+\frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x^3}{4a(a+bx^2)^2} - \frac{(b^3c+3ab^2d-7a^2be+11a^3f)\,x}{8ab^4(a+bx^2)} + \frac{(b^3c+3ab^2d-15a^2be+35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

output
```
(-3*a*f+b*e)*x/b^4+1/3*f*x^3/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x^3/a/(
b*x^2+a)^2-1/8*(11*a^3*f-7*a^2*b*e+3*a*b^2*d+b^3*c)*x/a/b^4/(b*x^2+a)+1/8*
(35*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(3/2)/b^
(9/2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.93

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{x(-105a^4f+3b^4cx^2+5a^3b(9e-35fx^2)+a^2b^2(-9d+75ex^2-56fx^4)+ab^3(-3c-15dx^2+24ex^4+8}{24ab^4(a+bx^2)^2} + \frac{(b^3c+3ab^2d-15a^2be+35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{3/2}b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
(x*(-105*a^4*f + 3*b^4*c*x^2 + 5*a^3*b*(9*e - 35*f*x^2) + a^2*b^2*(-9*d +
75*e*x^2 - 56*f*x^4) + a*b^3*(-3*c - 15*d*x^2 + 24*e*x^4 + 8*f*x^6)))/(24*
a*b^4*(a + b*x^2)^2) + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*ArcTan
[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(9/2))
```

3.136.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2335, 9, 25, 1580, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \left(c+d x^2+e x^4+f x^6\right)}{\left(a+b x^2\right)^3} \, dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{4 a \left(a+b x^2\right)^2}-\frac{\int -\frac{x \left(4 a f x^5+4 a \left(e-\frac{a f}{b}\right) x^3+\left(\frac{3 f a^3}{b^2}-\frac{3 e a^2}{b}+3 d a+b c\right) x\right)}{\left(b x^2+a\right)^2}dx}{4 a b}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{4 a \left(a+b x^2\right)^2}-\frac{\int -\frac{x^2 \left(4 a f x^4+4 a \left(e-\frac{a f}{b}\right) x^2+b c+3 a d-\frac{3 a^2 e}{b}+\frac{3 a^3 f}{b^2}\right)}{\left(b x^2+a\right)^2}dx}{4 a b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2 \left(4 a f x^4+4 a \left(e-\frac{a f}{b}\right) x^2+b c+3 a d-\frac{3 a^2 e}{b}+\frac{3 a^3 f}{b^2}\right)}{\left(b x^2+a\right)^2}dx}{4 a b}+\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{4 a \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{1580}$$

$$\frac{-\frac{\int -\frac{8 a b^2 f x^4+8 a b (b e-2 a f) x^2+b^3 c+3 a b^2 d-7 a^2 b e+11 a^3 f}{b x^2+a}dx}{2 b^3}-\frac{x \left(11 a^3 f-7 a^2 b e+3 a b^2 d+b^3 c\right)}{2 b^3 \left(a+b x^2\right)}}{4 a b}+\frac{x^3 \left(c-\frac{a \left(a^2 f-a b e+b^2 d\right)}{b^3}\right)}{4 a \left(a+b x^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8ab^2fx^4+8ab(be-2af)x^2+b^3c+3ab^2d-7a^2be+11a^3f}{bx^2+a}dx}{2b^3}-\frac{x(11a^3f-7a^2be+3ab^2d+b^3c)}{2b^3(a+bx^2)}}{4ab}+\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 1467$$

$$\frac{\frac{\int \left(8abfx^2+8a(be-3af)+\frac{35fa^3-15bea^2+3b^2da+b^3c}{bx^2+a}\right)dx}{2b^3}-\frac{x(11a^3f-7a^2be+3ab^2d+b^3c)}{2b^3(a+bx^2)}}{4ab}+\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{x^3\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\frac{\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(35a^3f-15a^2be+3ab^2d+b^3c)}{\sqrt{a}\sqrt{b}}+8ax(be-3af)+\frac{8}{3}abfx^3}{2b^3}-\frac{x(11a^3f-7a^2be+3ab^2d+b^3c)}{2b^3(a+bx^2)}}{4ab}$$

input
```
Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x^3)/(4*a*(a + b*x^2)^2) + (-1/2*((
b^3*c + 3*a*b^2*d - 7*a^2*b*e + 11*a^3*f)*x))/(b^3*(a + b*x^2)) + (8*a*(b*e
 - 3*a*f)*x + (8*a*b*f*x^3)/3 + ((b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*
f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*b^3))/(4*a*b)
```

3.136.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.136.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
default	$-\frac{-\frac{1}{3} f x^3 b+3 a f x-b e x}{b^4}+\frac{\frac{-\frac{b\left(13 f a^3-9 a^2 b e+5 a b^2 d-b^3 c\right) x^3}{8a}+\left(-\frac{11}{8} f a^3+\frac{7}{8} a^2 b e-\frac{3}{8} a b^2 d-\frac{1}{8} b^3 c\right) x}{\left(b x^2+a\right)^2}+\frac{\left(35 f a^3-15 a^2 b e+3 a b^2 d+b^3 c\right) \arctan}{8a\sqrt{ab}}}{b^4}$
risch	$\frac{f x^3}{3b^3}-\frac{3afx}{b^4}+\frac{ex}{b^3}+\frac{-\frac{b\left(13 f a^3-9 a^2 b e+5 a b^2 d-b^3 c\right) x^3}{8a}+\left(-\frac{11}{8} f a^3+\frac{7}{8} a^2 b e-\frac{3}{8} a b^2 d-\frac{1}{8} b^3 c\right) x}{b^4 (b x^2+a)^2}-\frac{35 a^2 \ln\left(bx+\sqrt{-ab}\right) f}{16 b^4 \sqrt{-ab}}+\frac{15 a \ln}{16}$

input
```
int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/3*f*x^3*b+3*a*f*x-b*e*x)+1/b^4*((-1/8*b*(13*a^3*f-9*a^2*b*e+5*a
*b^2*d-b^3*c)/a*x^3+(-11/8*f*a^3+7/8*a^2*b*e-3/8*a*b^2*d-1/8*b^3*c)*x)/(b*
x^2+a)^2+1/8*(35*a^3*f-15*a^2*b*e+3*a*b^2*d+b^3*c)/a/(a*b)^(1/2)*arctan(b*
x/(a*b)^(1/2)))
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.32

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,a^2b^4fx^7+16\,(3\,a^2b^4e-7\,a^3b^3f)x^5+2\,(3\,ab^5c-15\,a^2b^4d+75\,a^3b^3e-175\,a^4b^2f)x^3-3\,(a^2b^3c+3\,a^3}{}\right.$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/48*(16*a^2*b^4*f*x^7 + 16*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + 2*(3*a*b^5*
c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 - 3*(a^2*b^3*c + 3*a^
3*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e + 35*a
^3*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*f)*x^2)
*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 6*(a^2*b^4*c +
 3*a^3*b^3*d - 15*a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2
+ a^4*b^5), 1/24*(8*a^2*b^4*f*x^7 + 8*(3*a^2*b^4*e - 7*a^3*b^3*f)*x^5 + (3
*a*b^5*c - 15*a^2*b^4*d + 75*a^3*b^3*e - 175*a^4*b^2*f)*x^3 + 3*(a^2*b^3*c
 + 3*a^3*b^2*d - 15*a^4*b*e + 35*a^5*f + (b^5*c + 3*a*b^4*d - 15*a^2*b^3*e
 + 35*a^3*b^2*f)*x^4 + 2*(a*b^4*c + 3*a^2*b^3*d - 15*a^3*b^2*e + 35*a^4*b*
f)*x^2)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) - 3*(a^2*b^4*c + 3*a^3*b^3*d - 15*
a^4*b^2*e + 35*a^5*b*f)*x)/(a^2*b^7*x^4 + 2*a^3*b^6*x^2 + a^4*b^5)]
```

3.136.6 Sympy [A] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.56

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx$$
$$= x\left(-\frac{3af}{b^4}+\frac{e}{b^3}\right)-\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(-a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^3b^9}}\cdot(35a^3f-15a^2be+3ab^2d+b^3c)\log\left(a^2b^4\sqrt{-\frac{1}{a^3b^9}}+x\right)}{16}$$
$$+\frac{x^3(-13a^3bf+9a^2b^2e-5ab^3d+b^4c)+x(-11a^4f+7a^3be-3a^2b^2d-ab^3c)}{8a^3b^4+16a^2b^5x^2+8ab^6x^4}+\frac{fx^3}{3b^3}$$

input
```
integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
x*(-3*a*f/b**4 + e/b**3) - sqrt(-1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e +
 3*a*b**2*d + b**3*c)*log(-a**2*b**4*sqrt(-1/(a**3*b**9)) + x)/16 + sqrt(-
1/(a**3*b**9))*(35*a**3*f - 15*a**2*b*e + 3*a*b**2*d + b**3*c)*log(a**2*b*
*4*sqrt(-1/(a**3*b**9)) + x)/16 + (x**3*(-13*a**3*b*f + 9*a**2*b**2*e - 5*
a*b**3*d + b**4*c) + x*(-11*a**4*f + 7*a**3*b*e - 3*a**2*b**2*d - a*b**3*c
))/(8*a**3*b**4 + 16*a**2*b**5*x**2 + 8*a*b**6*x**4) + f*x**3/(3*b**3)
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\begin{aligned}\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx &= \frac{(b^4c-5\,ab^3d+9\,a^2b^2e-13\,a^3bf)x^3-(ab^3c+3\,a^2b^2d-7\,a^3be+11\,a^4f)x}{8\,(ab^6x^4+2\,a^2b^5x^2+a^3b^4)} \\ &\quad + \frac{bfx^3+3\,(be-3\,af)x}{3\,b^4} + \frac{(b^3c+3\,ab^2d-15\,a^2be+35\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4}\end{aligned}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((b^4*c - 5*a*b^3*d + 9*a^2*b^2*e - 13*a^3*b*f)*x^3 - (a*b^3*c + 3*a^2
*b^2*d - 7*a^3*b*e + 11*a^4*f)*x)/(a*b^6*x^4 + 2*a^2*b^5*x^2 + a^3*b^4) +
1/3*(b*f*x^3 + 3*(b*e - 3*a*f)*x)/b^4 + 1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*
e + 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^4)
```

3.136.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = \frac{(b^3c+3\,ab^2d-15\,a^2be+35\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}ab^4} + \frac{b^4cx^3-5\,ab^3dx^3+9\,a^2b^2ex^3-13\,a^3bfx^3-ab^3cx-3\,a^2b^2dx+7\,a^3bex-11\,a^4fx}{8\,(bx^2+a)^2ab^4} + \frac{b^6fx^3+3\,b^6ex-9\,ab^5fx}{3\,b^9}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(b^3*c + 3*a*b^2*d - 15*a^2*b*e + 35*a^3*f)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a*b^4) + 1/8*(b^4*c*x^3 - 5*a*b^3*d*x^3 + 9*a^2*b^2*e*x^3 - 13*a^3*
b*f*x^3 - a*b^3*c*x - 3*a^2*b^2*d*x + 7*a^3*b*e*x - 11*a^4*f*x)/((b*x^2 +
a)^2*a*b^4) + 1/3*(b^6*f*x^3 + 3*b^6*e*x - 9*a*b^5*f*x)/b^9
```

3.136.9 Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{(a+bx^2)^3}\,dx = x\left(\frac{e}{b^3}-\frac{3\,a\,f}{b^4}\right) - \frac{x\left(\frac{11\,f\,a^3}{8}-\frac{7\,e\,a^2\,b}{8}+\frac{3\,d\,a\,b^2}{8}+\frac{c\,b^3}{8}\right)-\frac{x^3\,\left(-13\,f\,a^3\,b+9\,e\,a^2\,b^2-5\,d\,a\,b^3+c\,b^4\right)}{8\,a}}{a^2\,b^4+2\,a\,b^5\,x^2+b^6\,x^4} + \frac{f\,x^3}{3\,b^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(35\,f\,a^3-15\,e\,a^2\,b+3\,d\,a\,b^2+c\,b^3\right)}{8\,a^{3/2}\,b^{9/2}}$$

input
```
int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^3,x)
```

output
```
x*(e/b^3 - (3*a*f)/b^4) - (x*((b^3*c)/8 + (11*a^3*f)/8 + (3*a*b^2*d)/8 - (
7*a^2*b*e)/8) - (x^3*(b^4*c + 9*a^2*b^2*e - 5*a*b^3*d - 13*a^3*b*f))/(8*a)
)/(a^2*b^4 + b^6*x^4 + 2*a*b^5*x^2) + (f*x^3)/(3*b^3) + (atan((b^(1/2)*x)/
a^(1/2))*(b^3*c + 35*a^3*f + 3*a*b^2*d - 15*a^2*b*e))/(8*a^(3/2)*b^(9/2))
```

3.137 $\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3} dx$

3.137.1 Optimal result

Integrand size = 27, antiderivative size = 147

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3} dx = \frac{fx}{b^3} + \frac{\left(c-\frac{a(b^2d-abe+a^2f)}{b^3}\right)x}{4a(a+bx^2)^2} + \frac{(3b^3c+ab^2d-5a^2be+9a^3f)x}{8a^2b^3(a+bx^2)} + \frac{(3b^3c+ab^2d+3a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

output
```
f*x/b^3+1/4*(c-a*(a^2*f-a*b*e+b^2*d)/b^3)*x/a/(b*x^2+a)^2+1/8*(9*a^3*f-5*a
^2*b*e+a*b^2*d+3*b^3*c)*x/a^2/b^3/(b*x^2+a)+1/8*(-15*a^3*f+3*a^2*b*e+a*b^2
*d+3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(5/2)/b^(7/2)
```

3.137.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3} dx = \frac{x(15a^4f+3b^4cx^2+ab^3(5c+dx^2)+a^3b(-3e+25fx^2)-a^2b^2(d+5ex^2-8fx^4))}{8a^2b^3(a+bx^2)^2} + \frac{(3b^3c+ab^2d+3a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{5/2}b^{7/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

output
```
(x*(15*a^4*f + 3*b^4*c*x^2 + a*b^3*(5*c + d*x^2) + a^3*b*(-3*e + 25*f*x^2)
 - a^2*b^2*(d + 5*e*x^2 - 8*f*x^4)))/(8*a^2*b^3*(a + b*x^2)^2) + ((3*b^3*c
 + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)
*b^(7/2))
```

3.137.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2345, 25, 1471, 25, 27, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}-\frac{\int -\frac{\frac{4afx^4}{b}+\frac{4a(be-af)x^2}{b^2}+\frac{fa^3-bea^2+b^2da+3b^3c}{b^3}}{(bx^2+a)^2}dx}{4a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4afx^4}{b}+\frac{4a(be-af)x^2}{b^2}+\frac{fa^3-bea^2+b^2da+3b^3c}{b^3}}{(bx^2+a)^2}dx}{4a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{2ab^3(a+bx^2)}-\frac{\int -\frac{-7fa^3+8bfx^2a^2+3bea^2+b^2da+3b^3c}{b^3(bx^2+a)}dx}{2a}}{4a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{-7fa^3+8bfx^2a^2+3bea^2+b^2da+3b^3c}{b^3(bx^2+a)}dx}{2a}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{2ab^3(a+bx^2)}}{4a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{-7fa^3+8bfx^2a^2+3bea^2+b^2da+3b^3c}{bx^2+a}dx}{2ab^3}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{2ab^3(a+bx^2)}}{4a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(-15a^3f+3a^2be+ab^2d+3b^3c)\int \frac{1}{bx^2+a}dx+8a^2fx}{2ab^3}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{2ab^3(a+bx^2)}}{4a}+\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{x\left(c-\frac{a(a^2f-abe+b^2d)}{b^3}\right)}{4a\left(a+bx^2\right)^2}+\frac{\frac{8a^2fx+\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-15a^3f+3a^2be+ab^2d+3b^3c\right)}{\sqrt{a}\sqrt{b}}}{2ab^3}+\frac{x(9a^3f-5a^2be+ab^2d+3b^3c)}{2ab^3(a+bx^2)}}{4a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x]
```

output
```
((c - (a*(b^2*d - a*b*e + a^2*f))/b^3)*x)/(4*a*(a + b*x^2)^2) + (((3*b^3*c
 + a*b^2*d - 5*a^2*b*e + 9*a^3*f)*x)/(2*a*b^3*(a + b*x^2)) + (8*a^2*f*x +
((3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
Sqrt[a]*Sqrt[b]))/(2*a*b^3))/(4*a)
```

3.137.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.137.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

method	result
default	$\frac{fx}{b^3}-\frac{\frac{-\frac{b\left(9f\,a^3-5a^2be+a\,b^2d+3b^3c\right)x^3}{8a^2}-\frac{\left(7f\,a^3-3a^2be-a\,b^2d+5b^3c\right)x}{8a}}{\left(b\,x^2+a\right)^2}+\frac{\left(15f\,a^3-3a^2be-a\,b^2d-3b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}}{b^3}$
risch	$\frac{fx}{b^3}+\frac{\frac{b\left(9f\,a^3-5a^2be+a\,b^2d+3b^3c\right)x^3}{8a^2}+\frac{\left(7f\,a^3-3a^2be-a\,b^2d+5b^3c\right)x}{8a}}{b^3\left(b\,x^2+a\right)^2}-\frac{15a\ln\left(bx-\sqrt{-ab}\right)f}{16b^3\sqrt{-ab}}+\frac{3\ln\left(bx-\sqrt{-ab}\right)e}{16b^2\sqrt{-ab}}+\frac{\ln\left(bx-\sqrt{-ab}\right)d}{16b\sqrt{-ab}\,a}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
f*x/b^3-1/b^3*((-1/8*b*(9*a^3*f-5*a^2*b*e+a*b^2*d+3*b^3*c)/a^2*x^3-1/8*(7*
a^3*f-3*a^2*b*e-a*b^2*d+5*b^3*c)/a*x)/(b*x^2+a)^2+1/8*(15*a^3*f-3*a^2*b*e-
a*b^2*d-3*b^3*c)/a^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 504, normalized size of antiderivative = 3.43

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$$
$$= \left[\frac{16\,a^3b^3fx^5 + 2\,(3\,ab^5c + a^2b^4d - 5\,a^3b^3e + 25\,a^4b^2f)x^3 + (3\,a^2b^3c + a^3b^2d + 3\,a^4be - 15\,a^5f + (3\,b^5c + a\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/16*(16*a^3*b^3*f*x^5 + 2*(3*a*b^5*c + a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*
b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3*a^4*b*e - 15*a^5*f + (3*b^5*c +
a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a
^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(-a*b)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(b
*x^2 + a)) + 2*(5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*x)/(a^
3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4), 1/8*(8*a^3*b^3*f*x^5 + (3*a*b^5*c +
a^2*b^4*d - 5*a^3*b^3*e + 25*a^4*b^2*f)*x^3 + (3*a^2*b^3*c + a^3*b^2*d + 3
*a^4*b*e - 15*a^5*f + (3*b^5*c + a*b^4*d + 3*a^2*b^3*e - 15*a^3*b^2*f)*x^4
 + 2*(3*a*b^4*c + a^2*b^3*d + 3*a^3*b^2*e - 15*a^4*b*f)*x^2)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + (5*a^2*b^4*c - a^3*b^3*d - 3*a^4*b^2*e + 15*a^5*b*f)*
x)/(a^3*b^6*x^4 + 2*a^4*b^5*x^2 + a^5*b^4)]
```

3.137.6 Sympy [A] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.65

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx$$
$$= \frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(-a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16} - \frac{\sqrt{-\frac{1}{a^5b^7}}\cdot(15a^3f-3a^2be-ab^2d-3b^3c)\log\left(a^3b^3\sqrt{-\frac{1}{a^5b^7}}+x\right)}{16} + \frac{x^3\cdot(9a^3bf-5a^2b^2e+ab^3d+3b^4c)+x(7a^4f-3a^3be-a^2b^2d+5ab^3c)}{8a^4b^3+16a^3b^4x^2+8a^2b^5x^4} + \frac{fx}{b^3}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**3,x)
```

output
```
sqrt(-1/(a**5*b**7))*(15*a**3*f - 3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(-a
**3*b**3*sqrt(-1/(a**5*b**7)) + x)/16 - sqrt(-1/(a**5*b**7))*(15*a**3*f -
3*a**2*b*e - a*b**2*d - 3*b**3*c)*log(a**3*b**3*sqrt(-1/(a**5*b**7)) + x)/
16 + (x**3*(9*a**3*b*f - 5*a**2*b**2*e + a*b**3*d + 3*b**4*c) + x*(7*a**4*
f - 3*a**3*b*e - a**2*b**2*d + 5*a*b**3*c))/(8*a**4*b**3 + 16*a**3*b**4*x*
*2 + 8*a**2*b**5*x**4) + f*x/b**3
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx = \frac{(3\,b^4c+ab^3d-5\,a^2b^2e+9\,a^3bf)x^3+(5\,ab^3c-a^2b^2d-3\,a^3be+7\,a^4f)x}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)} + \frac{fx}{b^3} + \frac{(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/8*((3*b^4*c + a*b^3*d - 5*a^2*b^2*e + 9*a^3*b*f)*x^3 + (5*a*b^3*c - a^2*
b^2*d - 3*a^3*b*e + 7*a^4*f)*x)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2 + a^4*b^3) +
f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx = \frac{fx}{b^3} + \frac{(3\,b^3c+ab^2d+3\,a^2be-15\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^2b^3} + \frac{3\,b^4cx^3+ab^3dx^3-5\,a^2b^2ex^3+9\,a^3bfx^3+5\,ab^3cx-a^2b^2dx-3\,a^3bex+7\,a^4fx}{8\,(bx^2+a)^2a^2b^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
f*x/b^3 + 1/8*(3*b^3*c + a*b^2*d + 3*a^2*b*e - 15*a^3*f)*arctan(b*x/sqrt(a
*b))/(sqrt(a*b)*a^2*b^3) + 1/8*(3*b^4*c*x^3 + a*b^3*d*x^3 - 5*a^2*b^2*e*x^
3 + 9*a^3*b*f*x^3 + 5*a*b^3*c*x - a^2*b^2*d*x - 3*a^3*b*e*x + 7*a^4*f*x)/(
(b*x^2 + a)^2*a^2*b^3)
```

3.137.9 Mupad [B] (verification not implemented)

Time = 5.54 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{(a+bx^2)^3}\,dx = \frac{\frac{x\,(7\,f\,a^3-3\,e\,a^2\,b-d\,a\,b^2+5\,c\,b^3)}{8\,a}+\frac{x^3\,(9\,f\,a^3\,b-5\,e\,a^2\,b^2+d\,a\,b^3+3\,c\,b^4)}{8\,a^2}}{a^2\,b^3+2\,a\,b^4\,x^2+b^5\,x^4} + \frac{f\,x}{b^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(-15\,f\,a^3+3\,e\,a^2\,b+d\,a\,b^2+3\,c\,b^3)}{8\,a^{5/2}\,b^{7/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^3,x)
```

output
```
((x*(5*b^3*c + 7*a^3*f - a*b^2*d - 3*a^2*b*e))/(8*a) + (x^3*(3*b^4*c - 5*a
^2*b^2*e + a*b^3*d + 9*a^3*b*f))/(8*a^2))/(a^2*b^3 + b^5*x^4 + 2*a*b^4*x^2
) + (f*x)/b^3 + (atan((b^(1/2)*x)/a^(1/2))*(3*b^3*c - 15*a^3*f + a*b^2*d +
 3*a^2*b*e))/(8*a^(5/2)*b^(7/2))
```

3.138 $\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx$

3.138.1 Optimal result

Integrand size = 30, antiderivative size = 153

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx = -\frac{c}{a^3x} - \frac{\left(\frac{bc}{a} - d + \frac{ae}{b} - \frac{a^2f}{b^2}\right)x}{4a(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} - \frac{(15b^3c - 3ab^2d - a^2be - 3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

output
```
-c/a^3/x-1/4*(b*c/a-d+a*e/b-a^2*f/b^2)*x/a/(b*x^2+a)^2-1/8*(5*a^3*f-a^2*b*
e-3*a*b^2*d+7*b^3*c)*x/a^3/b^2/(b*x^2+a)-1/8*(-3*a^3*f-a^2*b*e-3*a*b^2*d+1
5*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(5/2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3} dx = -\frac{c}{a^3x} + \frac{(-b^3c + ab^2d - a^2be + a^3f)x}{4a^2b^2(a+bx^2)^2} - \frac{(7b^3c - 3ab^2d - a^2be + 5a^3f)x}{8a^3b^2(a+bx^2)} + \frac{(-15b^3c + 3ab^2d + a^2be + 3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{7/2}b^{5/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

output
```
-(c/(a^3*x)) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^2*b^2*(a +
b*x^2)^2) - ((7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(8*a^3*b^2*(a +
b*x^2)) + ((-15*b^3*c + 3*a*b^2*d + a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(8*a^(7/2)*b^(5/2))
```

3.138.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2336, 25, 1582, 25, 359, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3}dx$$

$$\Big\downarrow \text{2336}$$

$$-\frac{\int -\frac{\frac{4afx^4}{b}-\left(\frac{fa^2}{b^2}-\frac{ea}{b}-3d+\frac{3bc}{a}\right)x^2+4c}{x^2(bx^2+a)^2}dx}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{4afx^4}{b}-\left(\frac{fa^2}{b^2}-\frac{ea}{b}-3d+\frac{3bc}{a}\right)x^2+4c}{x^2(bx^2+a)^2}dx}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{1582}$$

$$\frac{-\frac{\int -\frac{8ab^2c-\left(-3fa^3-bea^2-3b^2da+7b^3c\right)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2b^2}-\frac{x(5a^3f-a^2be-3ab^2d+7b^3c)}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{8ab^2c-\left(-3fa^3-bea^2-3b^2da+7b^3c\right)x^2}{x^2\left(bx^2+a\right)}dx}{2a^2b^2}-\frac{x(5a^3f-a^2be-3ab^2d+7b^3c)}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{-(-3a^3f-a^2be-3ab^2d+15b^3c)\int\frac{1}{bx^2+a}dx-\frac{8b^2c}{x}}{2a^2b^2}-\frac{x(5a^3f-a^2be-3ab^2d+7b^3c)}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-3a^3f-a^2be-3ab^2d+15b^3c)}{\sqrt{a}\sqrt{b}}-\frac{8b^2c}{x}}{2a^2b^2}-\frac{x(5a^3f-a^2be-3ab^2d+7b^3c)}{2a^2b^2(a+bx^2)}}{4a}-\frac{x\left(-\frac{a^2f}{b^2}+\frac{bc}{a}+\frac{ae}{b}-d\right)}{4a(a+bx^2)^2}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x]
```

output
```
-1/4*(((b*c)/a - d + (a*e)/b - (a^2*f)/b^2)*x)/(a*(a + b*x^2)^2) + (-1/2*(
(7*b^3*c - 3*a*b^2*d - a^2*b*e + 5*a^3*f)*x)/(a^2*b^2*(a + b*x^2)) + ((-8*
b^2*c)/x - ((15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]])/(Sqrt[a]*Sqrt[b]))/(2*a^2*b^2))/(4*a)
```

3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.138.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.92

method	result
default	$-\frac{c}{a^{3}x}+\frac{\frac{-\frac{\left(5f\,a^{3}-a^{2}be-3a\,b^{2}d+7b^{3}c\right)x^{3}}{8b}-\frac{a\left(3f\,a^{3}+a^{2}be-5a\,b^{2}d+9b^{3}c\right)x}{8b^{2}}}{\left(b\,x^{2}+a\right)^{2}}+\frac{\left(3f\,a^{3}+a^{2}be+3a\,b^{2}d-15b^{3}c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8b^{2}\sqrt{ab}}}{a^{3}}$
risch	$\frac{-\frac{\left(5f\,a^{3}-a^{2}be-3a\,b^{2}d+15b^{3}c\right)x^{4}}{8a^{3}b}-\frac{\left(3f\,a^{3}+a^{2}be-5a\,b^{2}d+25b^{3}c\right)x^{2}}{8a^{2}b^{2}}-\frac{c}{a}}{x(b\,x^{2}+a)^{2}}+\frac{\Big(\sum_{\_R=\mathrm{RootOf}\left(a^{7}b^{5}\_Z^{2}+9a^{6}f^{2}+6a^{5}bef+18a^{4}b^{2}df+a^{4}b^{2}e^{2}-90\right.}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-c/a^3/x+1/a^3*((-1/8*(5*a^3*f-a^2*b*e-3*a*b^2*d+7*b^3*c)/b*x^3-1/8*a*(3*a
^3*f+a^2*b*e-5*a*b^2*d+9*b^3*c)/b^2*x)/(b*x^2+a)^2+1/8*(3*a^3*f+a^2*b*e+3*
a*b^2*d-15*b^3*c)/b^2/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 517, normalized size of antiderivative = 3.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$

$$=\Bigg[-\frac{16\,a^3b^3c+2\,(15\,ab^5c-3\,a^2b^4d-a^3b^3e+5\,a^4b^2f)x^4+2\,(25\,a^2b^4c-5\,a^3b^3d+a^4b^2e+3\,a^5bf)x^2-((1}{}$$

$$-\frac{8\,a^3b^3c+(15\,ab^5c-3\,a^2b^4d-a^3b^3e+5\,a^4b^2f)x^4+(25\,a^2b^4c-5\,a^3b^3d+a^4b^2e+3\,a^5bf)x^2+((15\,b^5c}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/16*(16*a^3*b^3*c + 2*(15*a*b^5*c - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2
*f)*x^4 + 2*(25*a^2*b^4*c - 5*a^3*b^3*d + a^4*b^2*e + 3*a^5*b*f)*x^2 - ((1
5*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2
*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 + (15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*
e - 3*a^5*f)*x)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/
(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x), -1/8*(8*a^3*b^3*c + (15*a*b^5*c
 - 3*a^2*b^4*d - a^3*b^3*e + 5*a^4*b^2*f)*x^4 + (25*a^2*b^4*c - 5*a^3*b^3*
d + a^4*b^2*e + 3*a^5*b*f)*x^2 + ((15*b^5*c - 3*a*b^4*d - a^2*b^3*e - 3*a^
3*b^2*f)*x^5 + 2*(15*a*b^4*c - 3*a^2*b^3*d - a^3*b^2*e - 3*a^4*b*f)*x^3 +
(15*a^2*b^3*c - 3*a^3*b^2*d - a^4*b*e - 3*a^5*f)*x)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a))/(a^4*b^5*x^5 + 2*a^5*b^4*x^3 + a^6*b^3*x)]
```

3.138.6 Sympy [A] (verification not implemented)

Time = 11.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.63

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(-a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}$$
$$+\frac{\sqrt{-\frac{1}{a^7b^5}}\cdot\left(3a^3f+a^2be+3ab^2d-15b^3c\right)\log\left(a^4b^2\sqrt{-\frac{1}{a^7b^5}}+x\right)}{16}$$
$$+\frac{-8a^2b^2c+x^4(-5a^3bf+a^2b^2e+3ab^3d-15b^4c)+x^2(-3a^4f-a^3be+5a^2b^2d-25ab^3c)}{8a^5b^2x+16a^4b^3x^3+8a^3b^4x^5}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**3,x)
```

output
```
-sqrt(-1/(a**7*b**5))*(3*a**3*f + a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(-
a**4*b**2*sqrt(-1/(a**7*b**5)) + x)/16 + sqrt(-1/(a**7*b**5))*(3*a**3*f +
a**2*b*e + 3*a*b**2*d - 15*b**3*c)*log(a**4*b**2*sqrt(-1/(a**7*b**5)) + x)
/16 + (-8*a**2*b**2*c + x**4*(-5*a**3*b*f + a**2*b**2*e + 3*a*b**3*d - 15*
b**4*c) + x**2*(-3*a**4*f - a**3*b*e + 5*a**2*b**2*d - 25*a*b**3*c))/(8*a*
*5*b**2*x + 16*a**4*b**3*x**3 + 8*a**3*b**4*x**5)
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{8\,a^2b^2c+(15\,b^4c-3\,ab^3d-a^2b^2e+5\,a^3bf)x^4+(25\,ab^3c-5\,a^2b^2d+a^3be+3\,a^4f)x^2}{8\,(a^3b^4x^5+2\,a^4b^3x^3+a^5b^2x)}$$
$$-\frac{(15\,b^3c-3\,ab^2d-a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(8*a^2*b^2*c + (15*b^4*c - 3*a*b^3*d - a^2*b^2*e + 5*a^3*b*f)*x^4 + (
25*a*b^3*c - 5*a^2*b^2*d + a^3*b*e + 3*a^4*f)*x^2)/(a^3*b^4*x^5 + 2*a^4*b^
3*x^3 + a^5*b^2*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan
(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*b^2)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\left(a+bx^2\right)^3}\,dx$$
$$= -\frac{c}{a^3x}-\frac{(15\,b^3c-3\,ab^2d-a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^3b^2}$$
$$-\frac{7\,b^4cx^3-3\,ab^3dx^3-a^2b^2ex^3+5\,a^3bfx^3+9\,ab^3cx-5\,a^2b^2dx+a^3bex+3\,a^4fx}{8\,(bx^2+a)^2a^3b^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-c/(a^3*x) - 1/8*(15*b^3*c - 3*a*b^2*d - a^2*b*e - 3*a^3*f)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^3*b^2) - 1/8*(7*b^4*c*x^3 - 3*a*b^3*d*x^3 - a^2*b^2*e
*x^3 + 5*a^3*b*f*x^3 + 9*a*b^3*c*x - 5*a^2*b^2*d*x + a^3*b*e*x + 3*a^4*f*x
)/((b*x^2 + a)^2*a^3*b^2)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2(a+bx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,(3\,f\,a^3+e\,a^2\,b+3\,d\,a\,b^2-15\,c\,b^3)}{8\,a^{7/2}\,b^{5/2}} - \frac{\frac{c}{a}+\frac{x^4\,(5\,f\,a^3-e\,a^2\,b-3\,d\,a\,b^2+15\,c\,b^3)}{8\,a^3\,b}+\frac{x^2\,(3\,f\,a^3+e\,a^2\,b-5\,d\,a\,b^2+25\,c\,b^3)}{8\,a^2\,b^2}}{a^2\,x+2\,a\,b\,x^3+b^2\,x^5}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^3),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(3*a^3*f - 15*b^3*c + 3*a*b^2*d + a^2*b*e))/(8*
a^(7/2)*b^(5/2)) - (c/a + (x^4*(15*b^3*c + 5*a^3*f - 3*a*b^2*d - a^2*b*e))
/(8*a^3*b) + (x^2*(25*b^3*c + 3*a^3*f - 5*a*b^2*d + a^2*b*e))/(8*a^2*b^2))
/(a^2*x + b^2*x^5 + 2*a*b*x^3)
```

3.139 $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx$

3.139.1 Optimal result

Integrand size = 30, antiderivative size = 168

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = -\frac{c}{3a^3x^3}+\frac{3bc-ad}{a^4x}+\frac{\left(\frac{b^2c}{a^2}-\frac{bd}{a}+e-\frac{af}{b}\right)x}{4a\left(a+bx^2\right)^2} + \frac{\left(11b^3c-7ab^2d+3a^2be+a^3f\right)x}{8a^4b\left(a+bx^2\right)} + \frac{\left(35b^3c-15ab^2d+3a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

output
```
-1/3*c/a^3/x^3+(-a*d+3*b*c)/a^4/x+1/4*(b^2*c/a^2-b*d/a+e-a*f/b)*x/a/(b*x^2
+a)^2+1/8*(a^3*f+3*a^2*b*e-7*a*b^2*d+11*b^3*c)*x/a^4/b/(b*x^2+a)+1/8*(a^3*
f+3*a^2*b*e-15*a*b^2*d+35*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(9/2)/b^(3/2)
```

3.139.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{-3a^4fx^4+105b^4cx^6+5ab^3x^4(35c-9dx^2)+a^2b^2x^2(56c-75dx^2+9ex^4)+a^3b(-8c+3x^2(-8d+5ex^2}{24a^4bx^3\left(a+bx^2\right)^2} + \frac{\left(35b^3c-15ab^2d+3a^2be+a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{9/2}b^{3/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

output
```
(-3*a^4*f*x^4 + 105*b^4*c*x^6 + 5*a*b^3*x^4*(35*c - 9*d*x^2) + a^2*b^2*x^2
*(56*c - 75*d*x^2 + 9*e*x^4) + a^3*b*(-8*c + 3*x^2*(-8*d + 5*e*x^2 + f*x^4
)))/(24*a^4*b*x^3*(a + b*x^2)^2) + ((35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a
^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(9/2)*b^(3/2))
```

3.139.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2336, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4 \left(a+bx^2\right)^3} \, dx$$

$$\Big\downarrow \ 2336$$

$$\frac{x \left(\frac{b^2 c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a \left(a+bx^2\right)^2}-\frac{\int -\frac{\left(\frac{3cb^2}{a^2}-\frac{3db}{a}+3e+\frac{af}{b}\right)x^4-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^4\left(bx^2+a\right)^2}dx}{4a}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\left(\frac{3cb^2}{a^2}-\frac{3db}{a}+3e+\frac{af}{b}\right)x^4-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^4\left(bx^2+a\right)^2}dx}{4a}+\frac{x \left(\frac{b^2 c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1582$$

$$\frac{\frac{\int \frac{b\left(fa^3+3bea^2-7b^2da+11b^3c\right)x^4-8ab^2(2bc-ad)x^2+8a^2b^2c}{x^4\left(bx^2+a\right)}dx}{2a^3b^2}+\frac{x(a^3f+3a^2be-7ab^2d+11b^3c)}{2a^3b(a+bx^2)}}{4a}+\frac{x \left(\frac{b^2 c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 1584$$

$$\frac{\frac{\int \left(-\frac{8(3bc-ad)b^2}{x^2}+\frac{8acb^2}{x^4}+\frac{\left(fa^3+3bea^2-15b^2da+35b^3c\right)b}{bx^2+a}\right)dx}{2a^3b^2}+\frac{x(a^3f+3a^2be-7ab^2d+11b^3c)}{2a^3b(a+bx^2)}}{4a}+\frac{x \left(\frac{b^2 c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a \left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{x\left(\frac{b^2c}{a^2}-\frac{bd}{a}-\frac{af}{b}+e\right)}{4a\left(a+bx^2\right)^2}+\frac{\frac{\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(a^3f+3a^2be-15ab^2d+35b^3c\right)}{\sqrt{a}}+\frac{8b^2(3bc-ad)}{x}-\frac{8ab^2c}{3x^3}}{2a^3b^2}+\frac{x(a^3f+3a^2be-7ab^2d+11b^3c)}{2a^3b(a+bx^2)}}{4a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x]
```

output
```
(((b^2*c)/a^2 - (b*d)/a + e - (a*f)/b)*x)/(4*a*(a + b*x^2)^2) + (((11*b^3*
c - 7*a*b^2*d + 3*a^2*b*e + a^3*f)*x)/(2*a^3*b*(a + b*x^2)) + ((-8*a*b^2*c
)/(3*x^3) + (8*b^2*(3*b*c - a*d))/x + (Sqrt[b]*(35*b^3*c - 15*a*b^2*d + 3*
a^2*b*e + a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/Sqrt[a])/(2*a^3*b^2))/(4*a)
```

3.139.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.139.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{3a^3x^3}-\frac{ad-3bc}{a^4x}+\frac{\frac{\left(\frac{1}{8}fa^3+\frac{3}{8}a^2be-\frac{7}{8}ab^2d+\frac{11}{8}b^3c\right)x^3-\frac{a\left(fa^3-5a^2be+9ab^2d-13b^3c\right)x}{8b}}{\left(bx^2+a\right)^2}+\frac{\left(fa^3+3a^2be-15ab^2d+35b^3c\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right.}{8b\sqrt{ab}}}{a^4}$
risch	$\frac{\frac{\left(fa^3+3a^2be-15ab^2d+35b^3c\right)x^6}{8a^4}-\frac{\left(3fa^3-15a^2be+75ab^2d-175b^3c\right)x^4}{24a^3b}-\frac{(3ad-7bc)x^2}{3a^2}-\frac{c}{3a}}{x^3\left(bx^2+a\right)^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a^9b^3\_Z^2+a^6f^2+6a^5bef-30a\right.}\right.}{}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*c/a^3/x^3-(a*d-3*b*c)/a^4/x+1/a^4*(((1/8*f*a^3+3/8*a^2*b*e-7/8*a*b^2*
d+11/8*b^3*c)*x^3-1/8*a*(a^3*f-5*a^2*b*e+9*a*b^2*d-13*b^3*c)/b*x)/(b*x^2+a
)^2+1/8*(a^3*f+3*a^2*b*e-15*a*b^2*d+35*b^3*c)/b/(a*b)^(1/2)*arctan(b*x/(a*
b)^(1/2)))
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 570, normalized size of antiderivative = 3.39

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx$$

$$=\left[-\frac{16\,a^4b^2c-6\,(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f)x^6-2\,(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf)x^4}{}\right.$$

$$-\frac{8\,a^4b^2c-3\,(35\,ab^5c-15\,a^2b^4d+3\,a^3b^3e+a^4b^2f)x^6-(175\,a^2b^4c-75\,a^3b^3d+15\,a^4b^2e-3\,a^5bf)x^4-}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/48*(16*a^4*b^2*c - 6*(35*a*b^5*c - 15*a^2*b^4*d + 3*a^3*b^3*e + a^4*b^
2*f)*x^6 - 2*(175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*x^4
 - 16*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 + 3*((35*b^5*c - 15*a*b^4*d + 3*a^2*
b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*a^3*b^2*e + a^4*
b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*f)*x^3)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^5*b^4*x^7 + 2*a^6*b^3
*x^5 + a^7*b^2*x^3), -1/24*(8*a^4*b^2*c - 3*(35*a*b^5*c - 15*a^2*b^4*d + 3
*a^3*b^3*e + a^4*b^2*f)*x^6 - (175*a^2*b^4*c - 75*a^3*b^3*d + 15*a^4*b^2*e
 - 3*a^5*b*f)*x^4 - 8*(7*a^3*b^3*c - 3*a^4*b^2*d)*x^2 - 3*((35*b^5*c - 15*
a*b^4*d + 3*a^2*b^3*e + a^3*b^2*f)*x^7 + 2*(35*a*b^4*c - 15*a^2*b^3*d + 3*
a^3*b^2*e + a^4*b*f)*x^5 + (35*a^2*b^3*c - 15*a^3*b^2*d + 3*a^4*b*e + a^5*
f)*x^3)*sqrt(a*b)*arctan(sqrt(a*b)*x/a))/(a^5*b^4*x^7 + 2*a^6*b^3*x^5 + a^
7*b^2*x^3)]
```

3.139.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\left(a+bx^2\right)^3}\,dx = \frac{3\,(35\,b^4c-15\,ab^3d+3\,a^2b^2e+a^3bf)x^6-8\,a^3bc+(175\,ab^3c-75\,a^2b^2d+15\,a^3be-3\,a^4f)x^4+8\,(7\,a^2b^2c}{24\,(a^4b^3x^7+2\,a^5b^2x^5+a^6bx^3)} + \frac{(35\,b^3c-15\,ab^2d+3\,a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/24*(3*(35*b^4*c - 15*a*b^3*d + 3*a^2*b^2*e + a^3*b*f)*x^6 - 8*a^3*b*c +
(175*a*b^3*c - 75*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^4 + 8*(7*a^2*b^2*c -
 3*a^3*b*d)*x^2)/(a^4*b^3*x^7 + 2*a^5*b^2*x^5 + a^6*b*x^3) + 1/8*(35*b^3*c
 - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^4*b)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx = \frac{(35\,b^3c-15\,ab^2d+3\,a^2be+a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4b} + \frac{11\,b^4cx^3-7\,ab^3dx^3+3\,a^2b^2ex^3+a^3bfx^3+13\,ab^3cx-9\,a^2b^2dx+5\,a^3bex-a^4fx}{8\,(bx^2+a)^2a^4b} + \frac{9\,bcx^2-3\,adx^2-ac}{3\,a^4x^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(35*b^3*c - 15*a*b^2*d + 3*a^2*b*e + a^3*f)*arctan(b*x/sqrt(a*b))/(sqr
t(a*b)*a^4*b) + 1/8*(11*b^4*c*x^3 - 7*a*b^3*d*x^3 + 3*a^2*b^2*e*x^3 + a^3*
b*f*x^3 + 13*a*b^3*c*x - 9*a^2*b^2*d*x + 5*a^3*b*e*x - a^4*f*x)/((b*x^2 +
a)^2*a^4*b) + 1/3*(9*b*c*x^2 - 3*a*d*x^2 - a*c)/(a^4*x^3)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^{9/2}\,b^{3/2}} - \frac{\frac{c}{3\,a}-\frac{x^6\,\left(f\,a^3+3\,e\,a^2\,b-15\,d\,a\,b^2+35\,c\,b^3\right)}{8\,a^4}+\frac{x^2\,\left(3\,a\,d-7\,b\,c\right)}{3\,a^2}-\frac{x^4\,\left(-3\,f\,a^3+15\,e\,a^2\,b-75\,d\,a\,b^2+175\,c\,b^3\right)}{24\,a^3\,b}}{a^2\,x^3+2\,a\,b\,x^5+b^2\,x^7}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^3),x)
```

output
```
(atan((b^(1/2)*x)/a^(1/2))*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2*b*e))/(8
*a^(9/2)*b^(3/2)) - (c/(3*a) - (x^6*(35*b^3*c + a^3*f - 15*a*b^2*d + 3*a^2
*b*e))/(8*a^4) + (x^2*(3*a*d - 7*b*c))/(3*a^2) - (x^4*(175*b^3*c - 3*a^3*f
 - 75*a*b^2*d + 15*a^2*b*e))/(24*a^3*b))/(a^2*x^3 + b^2*x^7 + 2*a*b*x^5)
```

3.139. $\int \frac{c+dx^2+ex^4+fx^6}{x^4(a+bx^2)^3}\,dx$

3.140 $\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3}\,dx$

3.140.1 Optimal result

Integrand size = 30, antiderivative size = 196

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3}\,dx = -\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}-\frac{6b^2c-3abd+a^2e}{a^5x} - \frac{(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^4(a+bx^2)^2} - \frac{(15b^3c-11ab^2d+7a^2be-3a^3f)\,x}{8a^5(a+bx^2)} - \frac{(63b^3c-35ab^2d+15a^2be-3a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

output
```
-1/5*c/a^3/x^5+1/3*(-a*d+3*b*c)/a^4/x^3+(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x-1/4
*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^4/(b*x^2+a)^2-1/8*(-3*a^3*f+7*a^2*b*e-
11*a*b^2*d+15*b^3*c)*x/a^5/(b*x^2+a)-1/8*(-3*a^3*f+15*a^2*b*e-35*a*b^2*d+6
3*b^3*c)*arctan(x*b^(1/2)/a^(1/2))/a^(11/2)/b^(1/2)
```

3.140.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx=-\frac{c}{5a^3x^5}+\frac{3bc-ad}{3a^4x^3}+\frac{-6b^2c+3abd-a^2e}{a^5x}+\frac{\left(-b^3c+ab^2d-a^2be+a^3f\right)x}{4a^4\left(a+bx^2\right)^2}+\frac{\left(-15b^3c+11ab^2d-7a^2be+3a^3f\right)x}{8a^5\left(a+bx^2\right)}+\frac{\left(-63b^3c+35ab^2d-15a^2be+3a^3f\right)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{11/2}\sqrt{b}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/5*c/(a^3*x^5) + (3*b*c - a*d)/(3*a^4*x^3) + (-6*b^2*c + 3*a*b*d - a^2*e
)/(a^5*x) + ((-(b^3*c) + a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^4*(a + b*x^2)^
2) + ((-15*b^3*c + 11*a*b^2*d - 7*a^2*b*e + 3*a^3*f)*x)/(8*a^5*(a + b*x^2)
) + ((-63*b^3*c + 35*a*b^2*d - 15*a^2*b*e + 3*a^3*f)*ArcTan[(Sqrt[b]*x)/Sq
rt[a]])/(8*a^(11/2)*Sqrt[b])
```

3.140.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \ 2336$$

$$-\frac{\int -\frac{-\frac{3\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{4\left(ea^2-bda+b^2c\right)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^6\left(bx^2+a\right)^2}dx}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{-\frac{3\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{4\left(ea^2-bda+b^2c\right)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^6\left(bx^2+a\right)^2}dx}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2336$$

$$\frac{-\frac{\int -\frac{-\frac{\left(-3fa^3+7bea^2-11b^2da+15b^3c\right)x^6}{a^3}+\frac{8\left(ea^2-2bda+3b^2c\right)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)x^2+8c}{x^6\left(bx^2+a\right)}dx}{2a}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-\frac{\left(-3fa^3+7bea^2-11b^2da+15b^3c\right)x^6}{a^3}+\frac{8\left(ea^2-2bda+3b^2c\right)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)x^2+8c}{x^6\left(bx^2+a\right)}dx}{2a}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2333$$

$$\frac{\frac{\int \left(\frac{8c}{ax^6}+\frac{3fa^3-15bea^2+35b^2da-63b^3c}{a^3\left(bx^2+a\right)}+\frac{8\left(ea^2-3bda+6b^2c\right)}{a^3x^2}+\frac{8(ad-3bc)}{a^2x^4}\right)dx}{2a}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{\frac{8(3bc-ad)}{3a^2x^3}-\frac{8\left(a^2e-3abd+6b^2c\right)}{a^3x}-\frac{\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-3a^3f+15a^2be-35ab^2d+63b^3c\right)}{a^{7/2}\sqrt{b}}-\frac{8c}{5ax^5}}{2a}-\frac{x\left(-3a^3f+7a^2be-11ab^2d+15b^3c\right)}{2a^4\left(a+bx^2\right)}}{4a}-\frac{x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^4\left(a+bx^2\right)^2}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x]
```

output
```
-1/4*((b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^4*(a + b*x^2)^2) + (-1/2*(
(15*b^3*c - 11*a*b^2*d + 7*a^2*b*e - 3*a^3*f)*x)/(a^4*(a + b*x^2)) + ((-8*
c)/(5*a*x^5) + (8*(3*b*c - a*d))/(3*a^2*x^3) - (8*(6*b^2*c - 3*a*b*d + a^2
*e))/(a^3*x) - ((63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*ArcTan[(Sqr
t[b]*x)/Sqrt[a]])/(a^(7/2)*Sqrt[b]))/(2*a))/(4*a)
```

3.140. $\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.140.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{5a^3x^5}-\frac{ad-3bc}{3a^4x^3}-\frac{a^2e-3abd+6b^2c}{a^5x}+\frac{\frac{\left(\frac{3}{8}a^3bf-\frac{7}{8}a^2e\,b^2+\frac{11}{8}a\,b^3d-\frac{15}{8}b^4c\right)x^3+\frac{a\left(5f\,a^3-9a^2be+13a\,b^2d-17b^3c\right)x}{8}}{\left(b\,x^2+a\right)^2}+\frac{\left(3f\,a^3-15a^2be\cdots\right.}{\cdots}}{a^5}$
risch	$\frac{\frac{b\left(3f\,a^3-15a^2be+35a\,b^2d-63b^3c\right)x^8}{8a^5}+\frac{5\left(3f\,a^3-15a^2be+35a\,b^2d-63b^3c\right)x^6}{24a^4}-\frac{\left(15a^2e-35abd+63b^2c\right)x^4}{15a^3}-\frac{(5ad-9bc)x^2}{15a^2}-\frac{c}{5a}}{x^5(b\,x^2+a)^2}-\frac{3\ln\left(-\sqrt{-ab}\,x\cdots\right.}{16\sqrt{-ab}\cdots}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/5*c/a^3/x^5-1/3*(a*d-3*b*c)/a^4/x^3-(a^2*e-3*a*b*d+6*b^2*c)/a^5/x+1/a^5
*(((3/8*a^3*b*f-7/8*a^2*e*b^2+11/8*a*b^3*d-15/8*b^4*c)*x^3+1/8*a*(5*a^3*f-
9*a^2*b*e+13*a*b^2*d-17*b^3*c)*x)/(b*x^2+a)^2+1/8*(3*a^3*f-15*a^2*b*e+35*a
*b^2*d-63*b^3*c)/(a*b)^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 628, normalized size of antiderivative = 3.20

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^6 (a + bx^2)^3} \, dx$$

$$= \left[-\frac{30 (63 ab^5c - 35 a^2b^4d + 15 a^3b^3e - 3 a^4b^2f)x^8 + 48 a^5bc + 50 (63 a^2b^4c - 35 a^3b^3d + 15 a^4b^2e - 3 a^5bf}{} \right.$$

$$- \frac{15 (63 ab^5c - 35 a^2b^4d + 15 a^3b^3e - 3 a^4b^2f)x^8 + 24 a^5bc + 25 (63 a^2b^4c - 35 a^3b^3d + 15 a^4b^2e - 3 a^5bf)}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/240*(30*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f)*x^8 +
 48*a^5*b*c + 50*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^5*b*f)*
x^6 + 16*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 16*(9*a^4*b^2*c
 - 5*a^5*b*d)*x^2 - 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*b^2*f
)*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7 + (63
*a^2*b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(-a*b)*log((b*x
^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*
b*x^5), -1/120*(15*(63*a*b^5*c - 35*a^2*b^4*d + 15*a^3*b^3*e - 3*a^4*b^2*f
)*x^8 + 24*a^5*b*c + 25*(63*a^2*b^4*c - 35*a^3*b^3*d + 15*a^4*b^2*e - 3*a^
5*b*f)*x^6 + 8*(63*a^3*b^3*c - 35*a^4*b^2*d + 15*a^5*b*e)*x^4 - 8*(9*a^4*b
^2*c - 5*a^5*b*d)*x^2 + 15*((63*b^5*c - 35*a*b^4*d + 15*a^2*b^3*e - 3*a^3*
b^2*f)*x^9 + 2*(63*a*b^4*c - 35*a^2*b^3*d + 15*a^3*b^2*e - 3*a^4*b*f)*x^7
+ (63*a^2*b^3*c - 35*a^3*b^2*d + 15*a^4*b*e - 3*a^5*f)*x^5)*sqrt(a*b)*arct
an(sqrt(a*b)*x/a))/(a^6*b^3*x^9 + 2*a^7*b^2*x^7 + a^8*b*x^5)]
```

3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^6 (a + bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.03

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx =$$
$$-\frac{15\,(63\,b^4c-35\,ab^3d+15\,a^2b^2e-3\,a^3bf)x^8+25\,(63\,ab^3c-35\,a^2b^2d+15\,a^3be-3\,a^4f)x^6+24\,a^4c+8\,(}{120\,(a^5b^2x^9+2\,a^6bx^7+a^7x^5)}$$
$$-\frac{(63\,b^3c-35\,ab^2d+15\,a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/120*(15*(63*b^4*c - 35*a*b^3*d + 15*a^2*b^2*e - 3*a^3*b*f)*x^8 + 25*(63
*a*b^3*c - 35*a^2*b^2*d + 15*a^3*b*e - 3*a^4*f)*x^6 + 24*a^4*c + 8*(63*a^2
*b^2*c - 35*a^3*b*d + 15*a^4*e)*x^4 - 8*(9*a^3*b*c - 5*a^4*d)*x^2)/(a^5*b^
2*x^9 + 2*a^6*b*x^7 + a^7*x^5) - 1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e -
 3*a^3*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^5)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\left(a+bx^2\right)^3}\,dx$$
$$=-\frac{(63\,b^3c-35\,ab^2d+15\,a^2be-3\,a^3f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^5}$$
$$-\frac{15\,b^4cx^3-11\,ab^3dx^3+7\,a^2b^2ex^3-3\,a^3bfx^3+17\,ab^3cx-13\,a^2b^2dx+9\,a^3bex-5\,a^4fx}{8\,(bx^2+a)^2a^5}$$
$$-\frac{90\,b^2cx^4-45\,abdx^4+15\,a^2ex^4-15\,abcx^2+5\,a^2dx^2+3\,a^2c}{15\,a^5x^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(63*b^3*c - 35*a*b^2*d + 15*a^2*b*e - 3*a^3*f)*arctan(b*x/sqrt(a*b))/
(sqrt(a*b)*a^5) - 1/8*(15*b^4*c*x^3 - 11*a*b^3*d*x^3 + 7*a^2*b^2*e*x^3 - 3
*a^3*b*f*x^3 + 17*a*b^3*c*x - 13*a^2*b^2*d*x + 9*a^3*b*e*x - 5*a^4*f*x)/((
b*x^2 + a)^2*a^5) - 1/15*(90*b^2*c*x^4 - 45*a*b*d*x^4 + 15*a^2*e*x^4 - 15*
a*b*c*x^2 + 5*a^2*d*x^2 + 3*a^2*c)/(a^5*x^5)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6(a+bx^2)^3}\,dx = -\frac{\frac{c}{5a}+\frac{5x^6(-3fa^3+15ea^2b-35dab^2+63cb^3)}{24a^4}+\frac{x^2(5ad-9bc)}{15a^2}+\frac{x^4(15ea^2-35dab+63cb^2)}{15a^3}+\frac{bx^8(-3fa^3+15ea^2b-35dab^2+}{8a^5}}{a^2x^5+2abx^7+b^2x^9} - \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(-3fa^3+15ea^2b-35dab^2+63cb^3)}{8a^{11/2}\sqrt{b}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^3),x)
```

output
```
- (c/(5*a) + (5*x^6*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(24*a^
4) + (x^2*(5*a*d - 9*b*c))/(15*a^2) + (x^4*(63*b^2*c + 15*a^2*e - 35*a*b*d
))/(15*a^3) + (b*x^8*(63*b^3*c - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^
5))/(a^2*x^5 + b^2*x^9 + 2*a*b*x^7) - (atan((b^(1/2)*x)/a^(1/2))*(63*b^3*c
 - 3*a^3*f - 35*a*b^2*d + 15*a^2*b*e))/(8*a^(11/2)*b^(1/2))
```

3.141 $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx$

3.141.1 Optimal result

Integrand size = 30, antiderivative size = 234

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx = -\frac{c}{7a^3x^7}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{3a^5x^3} + \frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5(a+bx^2)^2} + \frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6(a+bx^2)} + \frac{\sqrt{b}(99b^3c-63ab^2d+35a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

output
```
-1/7*c/a^3/x^7+1/5*(-a*d+3*b*c)/a^4/x^5+1/3*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x
^3+(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x+1/4*b*(-a^3*f+a^2*b*e-a*b^2
*d+b^3*c)*x/a^5/(b*x^2+a)^2+1/8*b*(-7*a^3*f+11*a^2*b*e-15*a*b^2*d+19*b^3*c
)*x/a^6/(b*x^2+a)+1/8*(-15*a^3*f+35*a^2*b*e-63*a*b^2*d+99*b^3*c)*arctan(x*
b^(1/2)/a^(1/2))*b^(1/2)/a^(13/2)
```

3.141.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx=-\frac{c}{7a^3x^7}+\frac{3bc-ad}{5a^4x^5}-\frac{6b^2c-3abd+a^2e}{3a^5x^3}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{a^6x}+\frac{b(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^5\left(a+bx^2\right)^2}+\frac{b(19b^3c-15ab^2d+11a^2be-7a^3f)\,x}{8a^6\left(a+bx^2\right)}+\frac{\sqrt{b}(99b^3c-63ab^2d+35a^2be-15a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{13/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

output
```
-1/7*c/(a^3*x^7) + (3*b*c - a*d)/(5*a^4*x^5) - (6*b^2*c - 3*a*b*d + a^2*e)
/(3*a^5*x^3) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(a^6*x) + (b*(b^
3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + (b*(19*b^3*c -
 15*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(8*a^6*(a + b*x^2)) + (Sqrt[b]*(99*
b^3*c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(
8*a^(13/2))
```

3.141.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \ 2336$$

$$\frac{bx\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^5\left(a+bx^2\right)^2}-\frac{\int -\frac{-\frac{3b\left(-fa^3+bea^2-b^2da+b^3c\right)x^8}{a^4}-\frac{4\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{4\left(ea^2-bda+b^2c\right)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^8\left(bx^2+a\right)^2}dx}{4a}$$

$\downarrow$ 25

$$\frac{\int \frac{\frac{3b(-fa^3+bea^2-b^2da+b^3c)x^8}{a^4}-\frac{4(-fa^3+bea^2-b^2da+b^3c)x^6}{a^3}+\frac{4(ea^2-bda+b^2c)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^8(bx^2+a)^2}dx}{4a}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}$$

$\downarrow$ 2336

$$\frac{\frac{bx(-7a^3f+11a^2be-15ab^2d+19b^3c)}{2a^5(a+bx^2)}-\frac{\int -\frac{\frac{b(-7fa^3+11bea^2-15b^2da+19b^3c)x^8}{a^4}-\frac{8(-fa^3+2bea^2-3b^2da+4b^3c)x^6}{a^3}+\frac{8(ea^2-2bda+3b^2c)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)x}{x^8(bx^2+a)}}{2a}}{4a}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\frac{b(-7fa^3+11bea^2-15b^2da+19b^3c)x^8}{a^4}-\frac{8(-fa^3+2bea^2-3b^2da+4b^3c)x^6}{a^3}+\frac{8(ea^2-2bda+3b^2c)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)x^2+8c}{x^8(bx^2+a)}dx}{2a}+\frac{bx(-7a^3f+11a^2be-15ab^2d+1}{2a^5(a+bx^2)}}{4a}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}$$

$\downarrow$ 2333

$$\frac{\frac{\int \left(\frac{8c}{ax^8}-\frac{b(15fa^3-35bea^2+63b^2da-99b^3c)}{a^4(bx^2+a)}+\frac{8(fa^3-3bea^2+6b^2da-10b^3c)}{a^4x^2}+\frac{8(ea^2-3bda+6b^2c)}{a^3x^4}+\frac{8(ad-3bc)}{a^2x^6}\right)dx}{2a}+\frac{bx(-7a^3f+11a^2be-15ab^2d+19b^3c}{2a^5(a+bx^2)}}{4a}+\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}$$

$\downarrow$ 2009

$$\frac{bx(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^5(a+bx^2)^2}+\frac{\frac{bx(-7a^3f+11a^2be-15ab^2d+19b^3c)}{2a^5(a+bx^2)}+\frac{\frac{8(3bc-ad)}{5a^2x^5}-\frac{8(a^2e-3abd+6b^2c)}{3a^3x^3}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)(-15a^3f+35a^2be-63ab^2d+99b^3c)}{a^{9/2}}+\frac{8(a^3(-f)+3a^2be-6ab^2d-}{a^4x}}{2a}}{4a}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x]
```

3.141. $\int \frac{c+dx^2+ex^4+fx^6}{x^8(a+bx^2)^3}\,dx$

output
```
(b*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(4*a^5*(a + b*x^2)^2) + ((b*(19*
b^3*c - 15*a*b^2*d + 11*a^2*b*e - 7*a^3*f)*x)/(2*a^5*(a + b*x^2)) + ((-8*c
)/(7*a*x^7) + (8*(3*b*c - a*d))/(5*a^2*x^5) - (8*(6*b^2*c - 3*a*b*d + a^2*
e))/(3*a^3*x^3) + (8*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f))/(a^4*x) +
 (Sqrt[b]*(99*b^3*c - 63*a*b^2*d + 35*a^2*b*e - 15*a^3*f)*ArcTan[(Sqrt[b]*
x)/Sqrt[a]])/a^(9/2))/(2*a))/(4*a)
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.141.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.91

method	result
default	$-\frac{c}{7a^3x^7}-\frac{ad-3bc}{5a^4x^5}-\frac{a^2e-3abd+6b^2c}{3a^5x^3}-\frac{f a^3-3a^2be+6a b^2d-10b^3c}{a^6x}-\frac{b\left(\frac{\left(\frac{7}{8}a^3bf-\frac{11}{8}a^2eb^2+\frac{15}{8}ab^3d-\frac{19}{8}b^4c\right)x^3+\frac{a\left(9fa^3-13a\right.}{}}{\left(bx^2+a\right)^2}\right.}{}$
risch	$\frac{-\frac{b^2\left(15fa^3-35a^2be+63ab^2d-99b^3c\right)x^{10}}{8a^6}-\frac{5b\left(15fa^3-35a^2be+63ab^2d-99b^3c\right)x^8}{24a^5}-\frac{\left(15fa^3-35a^2be+63ab^2d-99b^3c\right)x^6}{15a^4}-\frac{\left(35a^2e-63abd+99b\right.}{105a^3}}{x^7\left(bx^2+a\right)^2}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/7*c/a^3/x^7-1/5*(a*d-3*b*c)/a^4/x^5-1/3*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^3
-(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x-b/a^6*(((7/8*a^3*b*f-11/8*a^2*
e*b^2+15/8*a*b^3*d-19/8*b^4*c)*x^3+1/8*a*(9*a^3*f-13*a^2*b*e+17*a*b^2*d-21
*b^3*c)*x)/(b*x^2+a)^2+1/8*(15*a^3*f-35*a^2*b*e+63*a*b^2*d-99*b^3*c)/(a*b)
^(1/2)*arctan(b*x/(a*b)^(1/2)))
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$

$$= \left[\frac{210\,(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f)x^{10}+350\,(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf)x^8+112}{}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[1/1680*(210*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 +
350*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 112*(99*
a^2*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 240*a^5*c - 16*(99
*a^3*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 48*(11*a^4*b*c - 7*a^5*d)*x^2 -
105*((99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*
b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63
*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(-b/a)*log((b*x^2 - 2*a*x*sqr
t(-b/a) - a)/(b*x^2 + a)))/(a^6*b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7), 1/840*(
105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 175*(99*a
*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2*b^3*c
- 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3*b^2*c
- 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2 + 105*((99*b^
5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^11 + 2*(99*a*b^4*c - 63*
a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^9 + (99*a^2*b^3*c - 63*a^3*b^2*d
+ 35*a^4*b*e - 15*a^5*f)*x^7)*sqrt(b/a)*arctan(x*sqrt(b/a)))/(a^6*b^2*x^11
 + 2*a^7*b*x^9 + a^8*x^7)]
```

3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\left(a+bx^2\right)^3}\,dx$$
$$= \frac{105\,(99\,b^5c-63\,ab^4d+35\,a^2b^3e-15\,a^3b^2f)x^{10}+175\,(99\,ab^4c-63\,a^2b^3d+35\,a^3b^2e-15\,a^4bf)x^8+56\,(9}{840\,(a^6b^2x^{11}}$$
$$+\frac{(99\,b^4c-63\,ab^3d+35\,a^2b^2e-15\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/840*(105*(99*b^5*c - 63*a*b^4*d + 35*a^2*b^3*e - 15*a^3*b^2*f)*x^10 + 17
5*(99*a*b^4*c - 63*a^2*b^3*d + 35*a^3*b^2*e - 15*a^4*b*f)*x^8 + 56*(99*a^2
*b^3*c - 63*a^3*b^2*d + 35*a^4*b*e - 15*a^5*f)*x^6 - 120*a^5*c - 8*(99*a^3
*b^2*c - 63*a^4*b*d + 35*a^5*e)*x^4 + 24*(11*a^4*b*c - 7*a^5*d)*x^2)/(a^6*
b^2*x^11 + 2*a^7*b*x^9 + a^8*x^7) + 1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^
2*e - 15*a^3*b*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^6)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8 (a+bx^2)^3}\,dx = \frac{(99\,b^4c-63\,ab^3d+35\,a^2b^2e-15\,a^3bf)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^6} + \frac{19\,b^5cx^3-15\,ab^4dx^3+11\,a^2b^3ex^3-7\,a^3b^2fx^3+21\,ab^4cx-17\,a^2b^3dx+13\,a^3b^2ex-9\,a^4bfx}{8\,(bx^2+a)^2a^6} + \frac{1050\,b^3cx^6-630\,ab^2dx^6+315\,a^2bex^6-105\,a^3fx^6-210\,ab^2cx^4+105\,a^2bdx^4-35\,a^3ex^4+63\,a^2bcx^2-}{105\,a^6x^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
1/8*(99*b^4*c - 63*a*b^3*d + 35*a^2*b^2*e - 15*a^3*b*f)*arctan(b*x/sqrt(a*
b))/(sqrt(a*b)*a^6) + 1/8*(19*b^5*c*x^3 - 15*a*b^4*d*x^3 + 11*a^2*b^3*e*x^
3 - 7*a^3*b^2*f*x^3 + 21*a*b^4*c*x - 17*a^2*b^3*d*x + 13*a^3*b^2*e*x - 9*a
^4*b*f*x)/((b*x^2 + a)^2*a^6) + 1/105*(1050*b^3*c*x^6 - 630*a*b^2*d*x^6 +
315*a^2*b*e*x^6 - 105*a^3*f*x^6 - 210*a*b^2*c*x^4 + 105*a^2*b*d*x^4 - 35*a
^3*e*x^4 + 63*a^2*b*c*x^2 - 21*a^3*d*x^2 - 15*a^3*c)/(a^6*x^7)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 5.61 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.98

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8 (a+bx^2)^3}\,dx = \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{8\,a^{13/2}} - \frac{\frac{c}{7\,a}-\frac{x^6\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a\,b^2+99\,c\,b^3\right)}{15\,a^4}+\frac{x^2\,\left(7\,a\,d-11\,b\,c\right)}{35\,a^2}+\frac{x^4\,\left(35\,e\,a^2-63\,d\,a\,b+99\,c\,b^2\right)}{105\,a^3}-\frac{5\,b\,x^8\,\left(-15\,f\,a^3+35\,e\,a^2\,b-63\,d\,a}{24\,a^5}}{a^2\,x^7+2\,a\,b\,x^9+b^2\,x^{11}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^3),x)
```

output
```
(b^(1/2)*atan((b^(1/2)*x)/a^(1/2))*(99*b^3*c - 15*a^3*f - 63*a*b^2*d + 35*
a^2*b*e))/(8*a^(13/2)) - (c/(7*a) - (x^6*(99*b^3*c - 15*a^3*f - 63*a*b^2*d
 + 35*a^2*b*e))/(15*a^4) + (x^2*(7*a*d - 11*b*c))/(35*a^2) + (x^4*(99*b^2*
c + 35*a^2*e - 63*a*b*d))/(105*a^3) - (5*b*x^8*(99*b^3*c - 15*a^3*f - 63*a
*b^2*d + 35*a^2*b*e))/(24*a^5) - (b^2*x^10*(99*b^3*c - 15*a^3*f - 63*a*b^2
*d + 35*a^2*b*e))/(8*a^6))/(a^2*x^7 + b^2*x^11 + 2*a*b*x^9)
```

3.142 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx$

3.142.1 Optimal result

Integrand size = 30, antiderivative size = 277

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx =& -\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5} \\ &+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3} \\ &-\frac{b(15b^3c-10ab^2d+6a^2be-3a^3f)}{a^7x} \\ &-\frac{b^2(b^3c-ab^2d+a^2be-a^3f)\,x}{4a^6(a+bx^2)^2} \\ &-\frac{b^2(23b^3c-19ab^2d+15a^2be-11a^3f)\,x}{8a^7(a+bx^2)} \\ &-\frac{b^{3/2}(143b^3c-99ab^2d+63a^2be-35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}\end{aligned}$$

output
```
-1/9*c/a^3/x^9+1/7*(-a*d+3*b*c)/a^4/x^7+1/5*(-a^2*e+3*a*b*d-6*b^2*c)/a^5/x
^5+1/3*(-a^3*f+3*a^2*b*e-6*a*b^2*d+10*b^3*c)/a^6/x^3-b*(-3*a^3*f+6*a^2*b*e
-10*a*b^2*d+15*b^3*c)/a^7/x-1/4*b^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*x/a^6/(
b*x^2+a)^2-1/8*b^2*(-11*a^3*f+15*a^2*b*e-19*a*b^2*d+23*b^3*c)*x/a^7/(b*x^2
+a)-1/8*b^(3/2)*(-35*a^3*f+63*a^2*b*e-99*a*b^2*d+143*b^3*c)*arctan(x*b^(1/
2)/a^(1/2))/a^(15/2)
```

3.142.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.00

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx = -\frac{c}{9a^3x^9}+\frac{3bc-ad}{7a^4x^7}-\frac{6b^2c-3abd+a^2e}{5a^5x^5}+\frac{10b^3c-6ab^2d+3a^2be-a^3f}{3a^6x^3}+\frac{b(-15b^3c+10ab^2d-6a^2be+3a^3f)}{a^7x}+\frac{b^2(-b^3c+ab^2d-a^2be+a^3f)\,x}{4a^6(a+bx^2)^2}+\frac{b^2(-23b^3c+19ab^2d-15a^2be+11a^3f)\,x}{8a^7(a+bx^2)}+\frac{b^{3/2}(-143b^3c+99ab^2d-63a^2be+35a^3f)\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{8a^{15/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

output
```
-1/9*c/(a^3*x^9) + (3*b*c - a*d)/(7*a^4*x^7) - (6*b^2*c - 3*a*b*d + a^2*e)
/(5*a^5*x^5) + (10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)/(3*a^6*x^3) + (b
*(-15*b^3*c + 10*a*b^2*d - 6*a^2*b*e + 3*a^3*f))/(a^7*x) + (b^2*(-(b^3*c)
+ a*b^2*d - a^2*b*e + a^3*f)*x)/(4*a^6*(a + b*x^2)^2) + (b^2*(-23*b^3*c +
19*a*b^2*d - 15*a^2*b*e + 11*a^3*f)*x)/(8*a^7*(a + b*x^2)) + (b^(3/2)*(-14
3*b^3*c + 99*a*b^2*d - 63*a^2*b*e + 35*a^3*f)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])
/(8*a^(15/2))
```

3.142.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2336, 25, 2336, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 2336}$$

$$-\frac{\int -\frac{-\frac{3b^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^{10}}{a^5}+\frac{4b\left(-fa^3+bea^2-b^2da+b^3c\right)x^8}{a^4}-\frac{4\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{4\left(ea^2-bda+b^2c\right)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^{10}(bx^2+a)^2}dx}{4a}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{-\frac{3b^2\left(-fa^3+bea^2-b^2da+b^3c\right)x^{10}}{a^5}+\frac{4b\left(-fa^3+bea^2-b^2da+b^3c\right)x^8}{a^4}-\frac{4\left(-fa^3+bea^2-b^2da+b^3c\right)x^6}{a^3}+\frac{4\left(ea^2-bda+b^2c\right)x^4}{a^2}-4\left(\frac{bc}{a}-d\right)x^2+4c}{x^{10}(bx^2+a)^2}dx}{4a}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2336$$

$$-\frac{\frac{\int -\frac{-\frac{b^2\left(-11fa^3+15bea^2-19b^2da+23b^3c\right)x^{10}}{a^5}+\frac{8b\left(-2fa^3+3bea^2-4b^2da+5b^3c\right)x^8}{a^4}-\frac{8\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^6}{a^3}+\frac{8\left(ea^2-2bda+3b^2c\right)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)\cdots}{x^{10}\left(bx^2+a\right)}\cdots}{2a}\cdots}{4a}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{-\frac{b^2\left(-11fa^3+15bea^2-19b^2da+23b^3c\right)x^{10}}{a^5}+\frac{8b\left(-2fa^3+3bea^2-4b^2da+5b^3c\right)x^8}{a^4}-\frac{8\left(-fa^3+2bea^2-3b^2da+4b^3c\right)x^6}{a^3}+\frac{8\left(ea^2-2bda+3b^2c\right)x^4}{a^2}-8\left(\frac{2bc}{a}-d\right)x^2+8\cdots}{x^{10}\left(bx^2+a\right)}\cdots}{2a}\cdots}{4a}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2333$$

$$\frac{\frac{\int\left(\frac{\left(35fa^3-63bea^2+99b^2da-143b^3c\right)b^2}{a^5\left(bx^2+a\right)}-\frac{8\left(3fa^3-6bea^2+10b^2da-15b^3c\right)b}{a^5x^2}+\frac{8\left(fa^3-3bea^2+6b^2da-10b^3c\right)}{a^4x^4}+\frac{8\left(ea^2-3bda+6b^2c\right)}{a^3x^6}+\frac{8(ad-3bc)}{a^2x^8}+\frac{8c}{ax^{10}}\right)dx}{2a}-\cdots}{4a}-\frac{b^2x(a^3(-f)+a^2be-ab^2d+b^3c)}{4a^6\left(a+bx^2\right)^2}$$

$$\Big\downarrow \; 2009$$

3.142. $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$

$$\frac{\frac{8(3bc-ad)}{7a^2x^7}-\frac{8\left(a^2e-3abd+6b^2c\right)}{5a^3x^5}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)\left(-35a^3f+63a^2be-99ab^2d+143b^3c\right)}{a^{11/2}}-\frac{8b\left(-3a^3f+6a^2be-10ab^2d+15b^3c\right)}{a^5x}+\frac{8\left(a^3(-f)+3a^2be-6ab^2d+10b}{3a^4x^3}}{2a}$$

$$4a$$

$$\frac{b^2x\left(a^3(-f)+a^2be-ab^2d+b^3c\right)}{4a^6\left(a+bx^2\right)^2}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x]
```

output
```
-1/4*(b^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*x)/(a^6*(a + b*x^2)^2) + (-1
/2*(b^2*(23*b^3*c - 19*a*b^2*d + 15*a^2*b*e - 11*a^3*f)*x)/(a^6*(a + b*x^2
)) + ((-8*c)/(9*a*x^9) + (8*(3*b*c - a*d))/(7*a^2*x^7) - (8*(6*b^2*c - 3*a
*b*d + a^2*e))/(5*a^3*x^5) + (8*(10*b^3*c - 6*a*b^2*d + 3*a^2*b*e - a^3*f)
)/(3*a^4*x^3) - (8*b*(15*b^3*c - 10*a*b^2*d + 6*a^2*b*e - 3*a^3*f))/(a^5*x
) - (b^(3/2)*(143*b^3*c - 99*a*b^2*d + 63*a^2*b*e - 35*a^3*f)*ArcTan[(Sqrt
[b]*x)/Sqrt[a]])/a^(11/2))/(2*a))/(4*a)
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2336
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; F
reeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

3.142.4 Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.90

method	result
default	$-\frac{c}{9a^3x^9}-\frac{ad-3bc}{7a^4x^7}-\frac{a^2e-3abd+6b^2c}{5a^5x^5}-\frac{f\,a^3-3a^2be+6a\,b^2d-10b^3c}{3a^6x^3}+\frac{b(3f\,a^3-6a^2be+10a\,b^2d-15b^3c)}{a^7x}+\frac{b^2\left(\frac{\left(\frac{11}{8}a^3bf-\frac{15}{8}\right.}{}\right.}{}$
risch	$\frac{\frac{b^3\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^{12}}{8a^7}+\frac{5b^2\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^{10}}{24a^6}+\frac{b\left(35f\,a^3-63a^2be+99a\,b^2d-143b^3c\right)x^8}{15a^5}-\frac{\left(35f\,a^3-63a^2b\right.}{}}{x^9(b\,x^2+a)^2}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/9*c/a^3/x^9-1/7*(a*d-3*b*c)/a^4/x^7-1/5*(a^2*e-3*a*b*d+6*b^2*c)/a^5/x^5
-1/3*(a^3*f-3*a^2*b*e+6*a*b^2*d-10*b^3*c)/a^6/x^3+b*(3*a^3*f-6*a^2*b*e+10*
a*b^2*d-15*b^3*c)/a^7/x+b^2/a^7*(((11/8*a^3*b*f-15/8*a^2*e*b^2+19/8*a*b^3*
d-23/8*b^4*c)*x^3+1/8*a*(13*a^3*f-17*a^2*b*e+21*a*b^2*d-25*b^3*c)*x)/(b*x^
2+a)^2+1/8*(35*a^3*f-63*a^2*b*e+99*a*b^2*d-143*b^3*c)/(a*b)^(1/2)*arctan(b
*x/(a*b)^(1/2)))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.79

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx$$

$$= \left[-\frac{630\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+1050\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{1}\cdots}{\cdots}\right.$$

$$-\frac{315\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+525\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{10}\cdots}{\cdots}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
[-1/5040*(630*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12
+ 1050*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 3
36*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 560*a^
6*c - 48*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 16*(
143*a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 80*(13*a^5*b*c - 9*a^6*d)*x^2
 + 315*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 + 2*(1
43*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a^2*b
^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(-b/a)*log((b*x^
2 + 2*a*x*sqrt(-b/a) - a)/(b*x^2 + a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9
*x^9), -1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)
*x^12 + 525*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^1
0 + 168*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 2
80*a^6*c - 24*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 +
 8*(143*a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)
*x^2 + 315*((143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^13 +
2*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^11 + (143*a
^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^9)*sqrt(b/a)*arctan
(x*sqrt(b/a)))/(a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9)]
```

3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**3,x)
```

output
```
Timed out
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.05

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{315\,(143\,b^6c-99\,ab^5d+63\,a^2b^4e-35\,a^3b^3f)x^{12}+525\,(143\,ab^5c-99\,a^2b^4d+63\,a^3b^3e-35\,a^4b^2f)x^{10}}{}$$

$$-\frac{(143\,b^5c-99\,ab^4d+63\,a^2b^3e-35\,a^3b^2f)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
-1/2520*(315*(143*b^6*c - 99*a*b^5*d + 63*a^2*b^4*e - 35*a^3*b^3*f)*x^12 +
 525*(143*a*b^5*c - 99*a^2*b^4*d + 63*a^3*b^3*e - 35*a^4*b^2*f)*x^10 + 168
*(143*a^2*b^4*c - 99*a^3*b^3*d + 63*a^4*b^2*e - 35*a^5*b*f)*x^8 + 280*a^6*
c - 24*(143*a^3*b^3*c - 99*a^4*b^2*d + 63*a^5*b*e - 35*a^6*f)*x^6 + 8*(143
*a^4*b^2*c - 99*a^5*b*d + 63*a^6*e)*x^4 - 40*(13*a^5*b*c - 9*a^6*d)*x^2)/(
a^7*b^2*x^13 + 2*a^8*b*x^11 + a^9*x^9) - 1/8*(143*b^5*c - 99*a*b^4*d + 63*
a^2*b^3*e - 35*a^3*b^2*f)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^7)
```

3.142.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.06

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx = -\frac{\left(143\,b^5c-99\,ab^4d+63\,a^2b^3e-35\,a^3b^2f\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}a^7} - \frac{23\,b^6cx^3-19\,ab^5dx^3+15\,a^2b^4ex^3-11\,a^3b^3fx^3+25\,ab^5cx-21\,a^2b^4dx+17\,a^3b^3ex-13\,a^4b^2fx}{8\,\left(bx^2+a\right)^2a^7} - \frac{4725\,b^4cx^8-3150\,ab^3dx^8+1890\,a^2b^2ex^8-945\,a^3bfx^8-1050\,ab^3cx^6+630\,a^2b^2dx^6-315\,a^3bex^6+105}{315\,a^7x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
-1/8*(143*b^5*c - 99*a*b^4*d + 63*a^2*b^3*e - 35*a^3*b^2*f)*arctan(b*x/sqr
t(a*b))/(sqrt(a*b)*a^7) - 1/8*(23*b^6*c*x^3 - 19*a*b^5*d*x^3 + 15*a^2*b^4*
e*x^3 - 11*a^3*b^3*f*x^3 + 25*a*b^5*c*x - 21*a^2*b^4*d*x + 17*a^3*b^3*e*x
- 13*a^4*b^2*f*x)/((b*x^2 + a)^2*a^7) - 1/315*(4725*b^4*c*x^8 - 3150*a*b^3
*d*x^8 + 1890*a^2*b^2*e*x^8 - 945*a^3*b*f*x^8 - 1050*a*b^3*c*x^6 + 630*a^2
*b^2*d*x^6 - 315*a^3*b*e*x^6 + 105*a^4*f*x^6 + 378*a^2*b^2*c*x^4 - 189*a^3
*b*d*x^4 + 63*a^4*e*x^4 - 135*a^3*b*c*x^2 + 45*a^4*d*x^2 + 35*a^4*c)/(a^7*
x^9)
```

3.142.9 Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\left(a+bx^2\right)^3}\,dx =$$

$$-\frac{\frac{c}{9\,a}-\frac{x^6\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{105\,a^4}+\frac{x^2\,\left(9\,a\,d-13\,b\,c\right)}{63\,a^2}+\frac{x^4\,\left(63\,e\,a^2-99\,d\,a\,b+143\,c\,b^2\right)}{315\,a^3}+\frac{b\,x^8\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\right.}{15\,a^5}}{a^2\,x^9+2\,a\,b\,x^{11}+b^2}$$

$$-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\,\left(-35\,f\,a^3+63\,e\,a^2\,b-99\,d\,a\,b^2+143\,c\,b^3\right)}{8\,a^{15/2}}$$

input

```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^3),x)
```

output

```
- (c/(9*a) - (x^6*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(105*a
^4) + (x^2*(9*a*d - 13*b*c))/(63*a^2) + (x^4*(143*b^2*c + 63*a^2*e - 99*a*
b*d))/(315*a^3) + (b*x^8*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))
/(15*a^5) + (5*b^2*x^10*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/
(24*a^6) + (b^3*x^12*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*
a^7))/(a^2*x^9 + b^2*x^13 + 2*a*b*x^11) - (b^(3/2)*atan((b^(1/2)*x)/a^(1/2
))*(143*b^3*c - 35*a^3*f - 99*a*b^2*d + 63*a^2*b*e))/(8*a^(15/2))
```

3.143 $\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.143.1 Optimal result

Integrand size = 32, antiderivative size = 214

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6} - \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)(a+bx^2)^{3/2}}{3b^6} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)(a+bx^2)^{5/2}}{5b^6} + \frac{(b^2d-4abe+10a^2f)(a+bx^2)^{7/2}}{7b^6} + \frac{(be-5af)(a+bx^2)^{9/2}}{9b^6} + \frac{f(a+bx^2)^{11/2}}{11b^6}$$

output
```
-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10
*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*
e+b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(
b*x^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

3.143.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-1280a^5f+128a^4b(11e+5fx^2)-16a^3b^2(99d+44ex^2+30fx^4)+8a^2b^3(231c+99dx^2+66e\ldots}{3465\ldots}$$

input
```
Integrate[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(9
9*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f
*x^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*
(693*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

3.143.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{2}\int \frac{x^4\left(fx^6+ex^4+dx^2+c\right)}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \; 2123$$

$$\frac{1}{2}\int \left(\frac{f\left(bx^2+a\right)^{9/2}}{b^5}+\frac{(be-5af)\left(bx^2+a\right)^{7/2}}{b^5}+\frac{\left(10fa^2-4bea+b^2d\right)\left(bx^2+a\right)^{5/2}}{b^5}+\frac{\left(-10fa^3+6bea^2-3b^2d\ldots}{b^5}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{2\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{2a\left(a+bx^2\right)^{3/2}\left(-5a\ldots\right.}{}\right.$$

input
```
Int[(x^5*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (2*a*(2
*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + (2*
(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + (2
*(b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + (2*(b*e - 5*a*f
)*(a + b*x^2)^(9/2))/(9*b^6) + (2*f*(a + b*x^2)^(11/2))/(11*b^6))/2
```

3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.143.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{256\left(-\frac{693\left(\frac{5}{11}f\,x^{6}+\frac{5}{9}e\,x^{4}+\frac{5}{7}d\,x^{2}+c\right)x^{4}b^{5}}{1280}+\frac{231\left(\frac{25}{66}f\,x^{6}+\frac{10}{21}e\,x^{4}+\frac{9}{14}d\,x^{2}+c\right)x^{2}a\,b^{4}}{320}-\frac{231\left(\frac{50}{231}f\,x^{6}+\frac{2}{7}e\,x^{4}+\frac{3}{7}d\,x^{2}+c\right)a^{2}b^{3}}{160}+\frac{99(}{}\right.}{693b^{6}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594}{346}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594}{346}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-315f\,x^{10}b^{5}+350a\,b^{4}f\,x^{8}-385b^{5}e\,x^{8}-400a^{2}b^{3}f\,x^{6}+440a\,b^{4}e\,x^{6}-495b^{5}d\,x^{6}+480a^{3}b^{2}f\,x^{4}-528a^{2}b^{3}e\,x^{4}+594}{346}$
default	$e\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+d\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}}{}\right.}{}\right.$

input
```
int(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-256/693*(-693/1280*(5/11*f*x^6+5/9*e*x^4+5/7*d*x^2+c)*x^4*b^5+231/320*(25
/66*f*x^6+10/21*e*x^4+9/14*d*x^2+c)*x^2*a*b^4-231/160*(50/231*f*x^6+2/7*e*
x^4+3/7*d*x^2+c)*a^2*b^3+99/80*(10/33*f*x^4+4/9*e*x^2+d)*a^3*b^2-11/10*(5/
11*f*x^2+e)*a^4*b+f*a^5)*(b*x^2+a)^(1/2)/b^6
```

3.143. $\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.143.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{(315\,b^5fx^{10} + 35\,(11\,b^5e - 10\,ab^4f)x^8 + 5\,(99\,b^5d - 88\,ab^4e + 80\,a^2b^3f)x^6 + 1848\,a^2b^3c - 1584\,a^3b^2d + 14}{}$$

input
```
integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88
*a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*
b*e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^
2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x
^2)*sqrt(b*x^2 + a)/b^6
```

3.143.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. 2(214) = 428.

Time = 0.46 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} \\ \frac{\frac{cx^6}{6}+\frac{dx^8}{8}+\frac{ex^{10}}{10}+\frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**5*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b
*x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*
d*sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4)
- 32*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(
15*b**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(
a + b*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*
c*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2)
- 8*a*e*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99
*b**2) + c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e
*x**8*sqrt(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)),
 ((c*x**6/6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^{10}}{11\,b} + \frac{\sqrt{bx^2+a}ex^8}{9\,b} - \frac{10\,\sqrt{bx^2+a}afx^8}{99\,b^2} + \frac{\sqrt{bx^2+a}dx^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}aex^6}{63\,b^2} + \frac{80\,\sqrt{bx^2+a}a^2fx^6}{693\,b^3} + \frac{\sqrt{bx^2+a}cx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}adx^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2ex^4}{105\,b^3} - \frac{32\,\sqrt{bx^2+a}a^3fx^4}{231\,b^4} - \frac{4\,\sqrt{bx^2+a}acx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2dx^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3ex^2}{315\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4fx^2}{693\,b^5} + \frac{8\,\sqrt{bx^2+a}a^2c}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3d}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4e}{315\,b^5} - \frac{256\,\sqrt{bx^2+a}a^5f}{693\,b^6}$$

input
```
integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b
*x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)
*a*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*
c*x^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*
x^4/b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*
e*x^2/b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a
^2*c/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e
/b^5 - 256/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

3.143.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.21

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(a^2b^3c-a^3b^2d+a^4be-a^5f)\sqrt{bx^2+a}}{b^6} + \frac{693\,(bx^2+a)^{\frac{5}{2}}b^3c-2310\,(bx^2+a)^{\frac{3}{2}}ab^3c+495\,(bx^2+a)^{\frac{7}{2}}b^2d-2079\,(bx^2+a)^{\frac{5}{2}}ab^2d+3465\,(bx^2+a)^{\frac{3}{2}}}{}$$

input
```
integrate(x^5*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*sqrt(b*x^2 + a)/b^6 + 1/3465*(69
3*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 +
a)^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a
^2*b^2*d + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158
*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e + 315*(b*x^2 +
 a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f -
 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f)/b^6
```

3.143.9 Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{x^5(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6}-\frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6}+\frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6}+\frac{f\,x^{10}}{11\,b}+\frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6}-\frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

input
```
int((x^5*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b
^6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*
b^6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(34
65*b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (
4*a*x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.144 $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.144.1 Optimal result

Integrand size = 32, antiderivative size = 167

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5} + \frac{(b^3c-2ab^2d+3a^2be-4a^3f)\left(a+bx^2\right)^{3/2}}{3b^5} + \frac{(b^2d-3abe+6a^2f)\left(a+bx^2\right)^{5/2}}{5b^5} + \frac{(be-4af)\left(a+bx^2\right)^{7/2}}{7b^5} + \frac{f\left(a+bx^2\right)^{9/2}}{9b^5}$$

output
```
1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-
3*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/
9*f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b
^5
```

3.144.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{\sqrt{a+bx^2}(128a^4f-16a^3b(9e+4fx^2)+24a^2b^2(7d+3ex^2+2fx^4)-2ab^3(105c+42dx^2+27ex^4+20fx}{315b^5}$$

input
```
Integrate[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
 3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b
^4*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

3.144.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2331}$$
$$\frac{1}{2}\int \frac{x^2\left(fx^6+ex^4+dx^2+c\right)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{2}\int \left(\frac{f\left(bx^2+a\right)^{7/2}}{b^4}+\frac{(be-4af)\left(bx^2+a\right)^{5/2}}{b^4}+\frac{\left(6fa^2-3bea+b^2d\right)\left(bx^2+a\right)^{3/2}}{b^4}+\frac{\left(-4fa^3+3bea^2-2b^2da\right.}{b^4}\right.$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{2\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{2a\sqrt{a+bx^2}\left(a^3(-f)+\right.}{b^5}\right.$$

input
```
Int[(x^3*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5 + (2*(b^3*
c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + (2*(b^2*
d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + (2*(b*e - 4*a*f)*(a +
b*x^2)^(7/2))/(7*b^5) + (2*f*(a + b*x^2)^(9/2))/(9*b^5))/2
```

3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.144.4 Maple [A] (verified)

Time = 3.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{128\left(\frac{105\left(\frac{1}{3} f x^6+\frac{3}{7} e x^4+\frac{3}{5} d x^2+c\right) x^2 b^4}{128}-\frac{105\left(\frac{4}{21} f x^6+\frac{9}{35} e x^4+\frac{2}{5} d x^2+c\right) a b^3}{64}+\frac{21\left(\frac{2}{7} f x^4+\frac{3}{7} e x^2+d\right) a^2 b^2}{16}-\frac{9\left(\frac{4 f x^2}{9}+e\right) a^3 b}{8}+a^4 f \right.}{315 b^5}$
gosper	$\frac{\sqrt{b x^2+a}\,(35 f x^8 b^4-40 a b^3 f x^6+45 b^4 e x^6+48 a^2 b^2 f x^4-54 a b^3 e x^4+63 b^4 d x^4-64 a^3 b f x^2+72 a^2 b^2 e x^2-84 a b^3 d x^2+105 b^4 c x}{315 b^5}$
trager	$\frac{\sqrt{b x^2+a}\,(35 f x^8 b^4-40 a b^3 f x^6+45 b^4 e x^6+48 a^2 b^2 f x^4-54 a b^3 e x^4+63 b^4 d x^4-64 a^3 b f x^2+72 a^2 b^2 e x^2-84 a b^3 d x^2+105 b^4 c x}{315 b^5}$
risch	$\frac{\sqrt{b x^2+a}\,(35 f x^8 b^4-40 a b^3 f x^6+45 b^4 e x^6+48 a^2 b^2 f x^4-54 a b^3 e x^4+63 b^4 d x^4-64 a^3 b f x^2+72 a^2 b^2 e x^2-84 a b^3 d x^2+105 b^4 c x}{315 b^5}$
default	$f\left(\frac{x^8\sqrt{b x^2+a}}{9b}-\frac{8a\left(\frac{x^6\sqrt{b x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b x^2+a}}{3b}-\frac{2a\sqrt{b x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e\left(\frac{x^6\sqrt{b x^2+a}}{7b}-\frac{6a\left(\frac{x^4}{}\right.}{}\right.$

input
```
int(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
128/315*(105/128*(1/3*f*x^6+3/7*e*x^4+3/5*d*x^2+c)*x^2*b^4-105/64*(4/21*f*
x^6+9/35*e*x^4+2/5*d*x^2+c)*a*b^3+21/16*(2/7*f*x^4+3/7*e*x^2+d)*a^2*b^2-9/
8*(4/9*f*x^2+e)*a^3*b+a^4*f)*(b*x^2+a)^(1/2)/b^5
```

3.144.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(35\,b^4fx^8+5\,(9\,b^4e-8\,ab^3f)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\,(21\,b^4d-18\,ab^3e+16\,a}{315\,b^5}$$

input
```
integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*
b^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)
*x^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^
2 + a)/b^5
```

3.144.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(163) = 326$.

Time = 0.38 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4ad}{} \\ \frac{\frac{cx^4}{4}+\frac{dx^6}{6}+\frac{ex^8}{8}+\frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**3*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x
**2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqr
t(a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a*
*2*f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) -
4*a*d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b*
*2) - 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b
) + d*x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8
*sqrt(a + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x
**10/10)/sqrt(a), True))
```

3.144.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^8}{9\,b} + \frac{\sqrt{bx^2+a}ex^6}{7\,b} - \frac{8\,\sqrt{bx^2+a}afx^6}{63\,b^2} + \frac{\sqrt{bx^2+a}dx^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}aex^4}{35\,b^2} + \frac{16\,\sqrt{bx^2+a}a^2fx^4}{105\,b^3} + \frac{\sqrt{bx^2+a}cx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}adx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2ex^2}{35\,b^3} - \frac{64\,\sqrt{bx^2+a}a^3fx^2}{315\,b^4} - \frac{2\,\sqrt{bx^2+a}ac}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2d}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3e}{35\,b^4} + \frac{128\,\sqrt{bx^2+a}a^4f}{315\,b^5}$$

input
```
integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^
2 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*
e*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x
^2/b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b
^3 - 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 +
8/15*sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315
*sqrt(b*x^2 + a)*a^4*f/b^5
```

3.144.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{(ab^3c-a^2b^2d+a^3be-a^4f)\sqrt{bx^2+a}}{b^5} + \frac{105\,(bx^2+a)^{\frac{3}{2}}b^3c+63\,(bx^2+a)^{\frac{5}{2}}b^2d-210\,(bx^2+a)^{\frac{3}{2}}ab^2d+45\,(bx^2+a)^{\frac{7}{2}}be-189\,(bx^2+a)^{\frac{5}{2}}abe+31}{315\,b^5}$$

input
```
integrate(x^3*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*
(b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/
2)*a*b^2*d + 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*
(b*x^2 + a)^(3/2)*a^2*b*e + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)
*a*f + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f)/b^5
```

3.144.9 Mupad [B] (verification not implemented)

Time = 5.78 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5} + \frac{x^4\,\left(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4\right)}{315\,b^5} + \frac{f\,x^8}{9\,b} + \frac{x^6\,\left(45\,b^4\,e-40\,a\,b^3\,f\right)}{315\,b^5} + \frac{x^2\,\left(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4\right)}{315\,b^5}\right)$$

input
```
int((x^3*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)
/(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x
^8)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72
*a^2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.144. $\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.145 $\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.145.1 Optimal result

Integrand size = 30, antiderivative size = 121

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4} + \frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4} + \frac{(be-3af)(a+bx^2)^{5/2}}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

output
```
1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)
^(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2
+a)^(1/2)/b^4
```

3.145.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

input
```
Integrate[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14
*e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^
4)
```

3.145.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2389}$$

$$\frac{1}{2}\int \left(\frac{f(bx^2+a)^{5/2}}{b^3}+\frac{(be-3af)(bx^2+a)^{3/2}}{b^3}+\frac{(3fa^2-2bea+b^2d)\sqrt{bx^2+a}}{b^3}+\frac{-fa^3+bea^2-b^2da+b^3c}{b^3\sqrt{bx^2+a}}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2(a+bx^2)^{3/2}(3a^2f-2abe+b^2d)}{3b^4}+\frac{2\sqrt{a+bx^2}(a^3(-f)+a^2be-ab^2d+b^3c)}{b^4}+\frac{2(a+bx^2)^{5/2}(be-3af)}{5b^4}+\right.$$

input
```
Int[(x*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
((2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + (2*(b^2*d -
 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x
^2)^(5/2))/(5*b^4) + (2*f*(a + b*x^2)^(7/2))/(7*b^4))/2
```

3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.145.4 Maple [A] (verified)

Time = 3.47 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{16\left(\frac{\left(-5 f x^6-7 e x^4-\frac{35}{3} d x^2-35 c\right) b^3}{16}+\frac{35\left(\frac{9}{35} f x^4+\frac{2}{5} e x^2+d\right) a b^2}{24}-\frac{7\left(\frac{3 f x^2}{7}+e\right) a^2 b}{6}+f a^3\right) \sqrt{b x^2+a}}{35 b^4}$
gosper	$-\frac{\sqrt{b x^2+a}\,\left(-15 f x^6 b^3+18 a b^2 f x^4-21 b^3 e x^4-24 a^2 b f x^2+28 a b^2 e x^2-35 b^3 d x^2+48 f a^3-56 a^2 b e+70 a b^2 d-105 b^3 c\right)}{105 b^4}$
trager	$-\frac{\sqrt{b x^2+a}\,\left(-15 f x^6 b^3+18 a b^2 f x^4-21 b^3 e x^4-24 a^2 b f x^2+28 a b^2 e x^2-35 b^3 d x^2+48 f a^3-56 a^2 b e+70 a b^2 d-105 b^3 c\right)}{105 b^4}$
risch	$-\frac{\sqrt{b x^2+a}\,\left(-15 f x^6 b^3+18 a b^2 f x^4-21 b^3 e x^4-24 a^2 b f x^2+28 a b^2 e x^2-35 b^3 d x^2+48 f a^3-56 a^2 b e+70 a b^2 d-105 b^3 c\right)}{105 b^4}$
default	$f\left(\frac{x^6\sqrt{b x^2+a}}{7b}-\frac{6a\left(\frac{x^4\sqrt{b x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b x^2+a}}{3b}-\frac{2a\sqrt{b x^2+a}}{3b^2}\right)}{5b}\right)}{7b}\right)+e\left(\frac{x^4\sqrt{b x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b x^2+a}}{3b}-\frac{2a\sqrt{b x^2-}}{3b^2}\right.}{5b}\right.$

input
```
int(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16/35*(1/16*(-5*f*x^6-7*e*x^4-35/3*d*x^2-35*c)*b^3+35/24*(9/35*f*x^4+2/5*
e*x^2+d)*a*b^2-7/6*(3/7*f*x^2+e)*a^2*b+f*a^3)*(b*x^2+a)^(1/2)/b^4
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{(15\,b^3fx^6+3\,(7\,b^3e-6\,ab^2f)x^4+105\,b^3c-70\,ab^2d+56\,a^2be-48\,a^3f+(35\,b^3d-28\,ab^2e+24\,a^2bf)x^2)}{105\,b^4}$$

input
```
integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
 + 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(
b*x^2 + a)/b^4
```

3.145.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(112) = 224$.

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4}+\frac{8a^2e\sqrt{a+bx^2}}{15b^3}+\frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3}-\frac{2ad\sqrt{a+bx^2}}{3b^2}-\frac{4aex^2\sqrt{a+bx^2}}{15b^2}-\frac{6afx^4\sqrt{a+bx^2}}{35b^2}+\frac{c\sqrt{a+bx^2}}{b}+\frac{dx^2\sqrt{a+bx^2}}{3b} \\ \frac{\frac{cx^2}{2}+\frac{dx^4}{4}+\frac{ex^6}{6}+\frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**
2)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b
*x**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(
a + b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*
b) + e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b,
0)), ((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^6}{7\,b} + \frac{\sqrt{bx^2+a}ex^4}{5\,b} - \frac{6\,\sqrt{bx^2+a}afx^4}{35\,b^2} + \frac{\sqrt{bx^2+a}dx^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}aex^2}{15\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2fx^2}{35\,b^3} + \frac{\sqrt{bx^2+a}c}{b} - \frac{2\,\sqrt{bx^2+a}ad}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2e}{15\,b^3} - \frac{16\,\sqrt{bx^2+a}a^3f}{35\,b^4}$$

input
```
integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^
2 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*
e*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3
*sqrt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x
^2 + a)*a^3*f/b^4
```

3.145.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{bx^2+a}}{b^4} + \frac{35\,(bx^2+a)^{\frac{3}{2}}b^2d+21\,(bx^2+a)^{\frac{5}{2}}be-70\,(bx^2+a)^{\frac{3}{2}}abe+15\,(bx^2+a)^{\frac{7}{2}}f-63\,(bx^2+a)^{\frac{5}{2}}af+105\,(bx^2}{105\,b^4}$$

input
```
integrate(x*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
 + a)^(3/2)*b^2*d + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e
+ 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2
)*a^2*f)/b^4
```

3.145.9 Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{x(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4} + \frac{f\,x^6}{7\,b} + \frac{x^2\,(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3)}{105\,b^4} + \frac{x^4\,(21\,b^3\,e-18\,a\,b^2\,f)}{105\,b^4}\right)$$

input
```
int((x*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b
^4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4)
 + (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.146 $\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$

3.146.1 Optimal result

Integrand size = 32, antiderivative size = 103

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx=\frac{(b^2d-abe+a^2f)\sqrt{a+bx^2}}{b^3}+\frac{(be-2af)\left(a+bx^2\right)^{3/2}}{3b^3}+\frac{f\left(a+bx^2\right)^{5/2}}{5b^3}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
1/3*(-2*a*f+b*e)*(b*x^2+a)^(3/2)/b^3+1/5*f*(b*x^2+a)^(5/2)/b^3-c*arctanh((
b*x^2+a)^(1/2)/a^(1/2))/a^(1/2)+(a^2*f-a*b*e+b^2*d)*(b*x^2+a)^(1/2)/b^3
```

3.146.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(8a^2f-2ab(5e+2fx^2)+b^2(15d+5ex^2+3fx^4))}{15b^3}-\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(8*a^2*f - 2*a*b*(5*e + 2*f*x^2) + b^2*(15*d + 5*e*x^2 +
3*f*x^4)))/(15*b^3) - (c*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a]
```

3.146.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{x^2\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2123}$$

$$\frac{1}{2}\int \left(\frac{f\left(bx^2+a\right)^{3/2}}{b^2}+\frac{(be-2af)\sqrt{bx^2+a}}{b^2}+\frac{fa^2-bea+b^2d}{b^2\sqrt{bx^2+a}}+\frac{c}{x^2\sqrt{bx^2+a}}\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\sqrt{a+bx^2}(a^2f-abe+b^2d)}{b^3}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\left(a+bx^2\right)^{3/2}(be-2af)}{3b^3}+\frac{2f\left(a+bx^2\right)^{5/2}}{5b^3}\right)$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x*Sqrt[a + b*x^2]),x]
```

output
```
((2*(b^2*d - a*b*e + a^2*f)*Sqrt[a + b*x^2])/b^3 + (2*(b*e - 2*a*f)*(a + b
*x^2)^(3/2))/(3*b^3) + (2*f*(a + b*x^2)^(5/2))/(5*b^3) - (2*c*ArcTanh[Sqrt
[a + b*x^2]/Sqrt[a]])/Sqrt[a])/2
```

3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.146.4 Maple [A] (verified)

Time = 3.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$-\frac{2\left(\frac{3b^3 c \operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2}+\sqrt{b\,x^2+a}\left(b\left(\frac{2f\,x^2}{5}+e\right)a^{\frac{3}{2}}-\frac{4a^{\frac{5}{2}}f}{5}-\frac{3\sqrt{a}\,b^2\left(\frac{1}{5}f\,x^4+\frac{1}{3}e\,x^2+d\right)}{2}\right)\right)}{3\sqrt{a}\,b^3}$
default	$f\left(\frac{x^4\sqrt{b\,x^2+a}}{5b}-\frac{4a\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)}{5b}\right)+e\left(\frac{x^2\sqrt{b\,x^2+a}}{3b}-\frac{2a\sqrt{b\,x^2+a}}{3b^2}\right)+\frac{d\sqrt{b\,x^2+a}}{b}-\frac{c\ln\left(\frac{2a+2\sqrt{a}}{}\right.}{\sqrt{a}}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3/2*b^3*c*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(b*x^2+a)^(1/2)*(b*(2/5*f
*x^2+e)*a^(3/2)-4/5*a^(5/2)*f-3/2*a^(1/2)*b^2*(1/5*f*x^4+1/3*e*x^2+d)))/a^
(1/2)/b^3
```

3.146.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{15\sqrt{a}b^3c\log\left(-\frac{bx^2-2\sqrt{bx^2+a}\sqrt{a}+2a}{x^2}\right)+2\left(3ab^2fx^4+15ab^2d-10a^2be+8a^3f+(5ab^2e-4a^2bf)x^2\right)\sqrt{b}}{30ab^3}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*sqrt(a)*b^3*c*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2
) + 2*(3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*f + (5*a*b^2*e - 4*
a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3), 1/15*(15*sqrt(-a)*b^3*c*arctan(sqr
t(-a)/sqrt(b*x^2 + a)) + (3*a*b^2*f*x^4 + 15*a*b^2*d - 10*a^2*b*e + 8*a^3*
f + (5*a*b^2*e - 4*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3)]
```

3.146.6 Sympy [A] (verification not implemented)

Time = 4.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.30

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\begin{cases} \frac{2c\,\mathrm{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{2f(a+bx^2)^{\frac{5}{2}}}{5b^3} + \frac{2(a+bx^2)^{\frac{3}{2}}(-2af+be)}{3b^3} + \frac{2\sqrt{a+bx^2}(a^2f-abe+b^2d)}{b^3} & \text{for } b\neq 0 \\ \frac{c\log(x^2)+dx^2+\frac{ex^4}{2}+\frac{fx^6}{3}}{\sqrt{a}} & \text{otherwise} \end{cases}}{2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((2*c*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a) + 2*f*(a + b*x**2)
**(5/2)/(5*b**3) + 2*(a + b*x**2)**(3/2)*(-2*a*f + b*e)/(3*b**3) + 2*sqrt(
a + b*x**2)*(a**2*f - a*b*e + b**2*d)/b**3, Ne(b, 0)), ((c*log(x**2) + d*x
**2 + e*x**4/2 + f*x**6/3)/sqrt(a), True))/2
```

3.146.7 Maxima [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.18

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^4}{5\,b} + \frac{\sqrt{bx^2+a}ex^2}{3\,b} - \frac{4\,\sqrt{bx^2+a}afx^2}{15\,b^2} - \frac{c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}d}{b} - \frac{2\,\sqrt{bx^2+a}ae}{3\,b^2} + \frac{8\,\sqrt{bx^2+a}a^2f}{15\,b^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(b*x^2 + a)*f*x^4/b + 1/3*sqrt(b*x^2 + a)*e*x^2/b - 4/15*sqrt(b*x^
2 + a)*a*f*x^2/b^2 - c*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2
+ a)*d/b - 2/3*sqrt(b*x^2 + a)*a*e/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*f/b^3
```

3.146.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.21

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \frac{c\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{15\sqrt{bx^2+a}ab^{14}d+5\left(bx^2+a\right)^{\frac{3}{2}}b^{13}e-15\sqrt{bx^2+a}ab^{13}e+3\left(bx^2+a\right)^{\frac{5}{2}}b^{12}f-10\left(bx^2+a\right)^{\frac{3}{2}}ab^{12}f+15}{15\,b^{15}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
c*arctan(sqrt(b*x^2 + a)/sqrt(-a))/sqrt(-a) + 1/15*(15*sqrt(b*x^2 + a)*b^1
4*d + 5*(b*x^2 + a)^(3/2)*b^13*e - 15*sqrt(b*x^2 + a)*a*b^13*e + 3*(b*x^2
+ a)^(5/2)*b^12*f - 10*(b*x^2 + a)^(3/2)*a*b^12*f + 15*sqrt(b*x^2 + a)*a^2
*b^12*f)/b^15
```

3.146.9 Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.96

$$\int \frac{c+dx^2+ex^4+fx^6}{x\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{8\,a^2\,f}{15\,b^3}+\frac{f\,x^4}{5\,b}-\frac{4\,a\,f\,x^2}{15\,b^2}\right)+\frac{d\,\sqrt{b\,x^2+a}}{b} - \frac{c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{e\,\sqrt{b\,x^2+a}\,\left(2\,a-b\,x^2\right)}{3\,b^2}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x*(a + b*x^2)^(1/2)),x)
```

output
```
(a + b*x^2)^(1/2)*((8*a^2*f)/(15*b^3) + (f*x^4)/(5*b) - (4*a*f*x^2)/(15*b^
2)) + (d*(a + b*x^2)^(1/2))/b - (c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/
2) - (e*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.147 $\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$

3.147.1 Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{(be-af)\sqrt{a+bx^2}}{b^2} - \frac{c\sqrt{a+bx^2}}{2ax^2} + \frac{f(a+bx^2)^{3/2}}{3b^2} + \frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

output
```
1/3*f*(b*x^2+a)^(3/2)/b^2+1/2*(-2*a*d+b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2)
)/a^(3/2)+(-a*f+b*e)*(b*x^2+a)^(1/2)/b^2-1/2*c*(b*x^2+a)^(1/2)/a/x^2
```

3.147.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.92

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-3b^2c+6abex^2-4a^2fx^2+2abfx^4)}{6ab^2x^2} + \frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-3*b^2*c + 6*a*b*e*x^2 - 4*a^2*f*x^2 + 2*a*b*f*x^4))/(6*
a*b^2*x^2) + ((b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))
```

3.147.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2331, 2124, 27, 1192, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{x^4\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-2afx^4-2aex^2+bc-2ad}{2x^2\sqrt{bx^2+a}}dx^2}{a}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-2afx^4-2aex^2+bc-2ad}{x^2\sqrt{bx^2+a}}dx^2}{2a}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-2afx^8-2a(be-2af)x^4+b^3c-2ab^2d+2a^2be-2a^3f}{a-x^4}d\sqrt{bx^2+a}}{ab^2}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{-2afx^8-2a(be-2af)x^4+b^3c-2ab^2d+2a^2be-2a^3f}{a-x^4}d\sqrt{bx^2+a}}{ab^2}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{1467}$$

$$\frac{1}{2}\left(\frac{\int \left(2afx^4+2a(be-af)+\frac{b^3c-2ab^2d}{a-x^4}\right)d\sqrt{bx^2+a}}{ab^2}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(bc-2ad)}{\sqrt{a}}-2a\sqrt{a+bx^2}(be-af)-\frac{2}{3}afx^6}{ab^2}-\frac{c\sqrt{a+bx^2}}{ax^2}\right)$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^3*Sqrt[a + b*x^2]),x]
```

output
```
(-((c*Sqrt[a + b*x^2])/(a*x^2)) - ((-2*a*f*x^6)/3 - 2*a*(b*e - a*f)*Sqrt[a
 + b*x^2] - (b^2*(b*c - 2*a*d)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/
(a*b^2))/2
```

3.147.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.147.4 Maple [A] (verified)

Time = 3.54 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{b^2x^2\left(ad-\frac{bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)+\frac{\sqrt{bx^2+a}\left(-2\left(\frac{fx^2}{3}+e\right)bx^2a^{\frac{3}{2}}+\sqrt{a}\,b^2c+\frac{4a^{\frac{5}{2}}fx^2}{3}\right)}{2}}{a^{\frac{3}{2}}b^2x^2}$
risch	$-\frac{c\sqrt{bx^2+a}}{2ax^2}+\frac{2af\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2+a}}{3b^2}\right)+\frac{2ae\sqrt{bx^2+a}}{b}-\frac{(2ad-bc)\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{\sqrt{a}}}{2a}$
default	$f\left(\frac{x^2\sqrt{bx^2+a}}{3b}-\frac{2a\sqrt{bx^2+a}}{3b^2}\right)+\frac{e\sqrt{bx^2+a}}{b}-\frac{d\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{\sqrt{a}}+c\left(-\frac{\sqrt{bx^2+a}}{2ax^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b^2*x^2*(a*d-1/2*b*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))+1/2*(b*x^2+a)^(1/
2)*(-2*(1/3*f*x^2+e)*b*x^2*a^(3/2)+a^(1/2)*b^2*c+4/3*a^(5/2)*f*x^2))/a^(3/
2)/b^2/x^2
```

3.147.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(b^3c-2\,ab^2d)\sqrt{a}x^2\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(2\,a^2bfx^4-3\,ab^2c+2\,(3\,a^2be-2\,a^3f)x^2)\sqrt{bx^2+\cdots}}{12\,a^2b^2x^2}\right.$$
$$\left.-\frac{3\,(b^3c-2\,ab^2d)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)-(2\,a^2bfx^4-3\,ab^2c+2\,(3\,a^2be-2\,a^3f)x^2)\sqrt{bx^2+a}}{6\,a^2b^2x^2}\right]$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^2*log(-(b*x^2 - 2*sqrt(b*x^2 + a)*
sqrt(a) + 2*a)/x^2) - 2*(2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(3*a^2*b*e - 2*a^3*
f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2), -1/6*(3*(b^3*c - 2*a*b^2*d)*sqrt(-
a)*x^2*arctan(sqrt(-a)/sqrt(b*x^2 + a)) - (2*a^2*b*f*x^4 - 3*a*b^2*c + 2*(
3*a^2*b*e - 2*a^3*f)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^2)]
```

3.147.6 Sympy [A] (verification not implemented)

Time = 12.83 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = e\left(\begin{cases}\frac{\sqrt{a+bx^2}}{b} & \text{for}\ b\neq 0\\ \frac{x^2}{2\sqrt{a}} & \text{otherwise}\end{cases}\right) + f\left(\begin{cases}-\frac{2a\sqrt{a+bx^2}}{3b^2}+\frac{x^2\sqrt{a+bx^2}}{3b} & \text{for}\ b\neq 0\\ \frac{x^4}{4\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{d\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{bc\,\mathrm{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**3/(b*x**2+a)**(1/2),x)
```

output
```
e*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) + f*
Piecewise((-2*a*sqrt(a + b*x**2)/(3*b**2) + x**2*sqrt(a + b*x**2)/(3*b), N
e(b, 0)), (x**4/(4*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(2*a*
x) - d*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*c*asinh(sqrt(a)/(sqrt(b)*x))
/(2*a**(3/2))
```

3.147.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.04

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^2}{3\,b} + \frac{bc\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}e}{b} - \frac{2\sqrt{bx^2+a}af}{3\,b^2} - \frac{\sqrt{bx^2+a}c}{2\,ax^2}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(b*x^2 + a)*f*x^2/b + 1/2*b*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2
) - d*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*x^2 + a)*e/b - 2/3*sq
rt(b*x^2 + a)*a*f/b^2 - 1/2*sqrt(b*x^2 + a)*c/(a*x^2)
```

3.147.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.13

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = -\frac{\frac{3\,(b^2c-2\,abd)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{3\sqrt{bx^2+a}bc}{ax^2} - \frac{2\left(3\sqrt{bx^2+a}b^3e+(bx^2+a)^{\frac{3}{2}}b^2f-3\sqrt{bx^2+a}ab^2f\right)}{b^3}}{6\,b}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^3/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/6*(3*(b^2*c - 2*a*b*d)*arctan(sqrt(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a) +
3*sqrt(b*x^2 + a)*b*c/(a*x^2) - 2*(3*sqrt(b*x^2 + a)*b^3*e + (b*x^2 + a)^(
3/2)*b^2*f - 3*sqrt(b*x^2 + a)*a*b^2*f)/b^3)/b
```

3.147.9 Mupad [B] (verification not implemented)

Time = 6.42 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.99

$$\int \frac{c+dx^2+ex^4+fx^6}{x^3\sqrt{a+bx^2}}\,dx = \frac{e\sqrt{b\,x^2+a}}{b} - \frac{d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{c\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{f\sqrt{b\,x^2+a}\,(2\,a-b\,x^2)}{3\,b^2}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^3*(a + b*x^2)^(1/2)),x)
```

output
```
(e*(a + b*x^2)^(1/2))/b - (d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (
c*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2
*a^(3/2)) - (f*(a + b*x^2)^(1/2)*(2*a - b*x^2))/(3*b^2)
```

3.148 $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

3.148.1 Optimal result

Integrand size = 32, antiderivative size = 114

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx=\frac{f\sqrt{a+bx^2}}{b}-\frac{c\sqrt{a+bx^2}}{4ax^4}+\frac{(3bc-4ad)\sqrt{a+bx^2}}{8a^2x^2}-\frac{(3b^2c-4abd+8a^2e)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

output
```
-1/8*(8*a^2*e-4*a*b*d+3*b^2*c)*arctanh((b*x^2+a)^(1/2)/a^(1/2))/a^(5/2)+f*
(b*x^2+a)^(1/2)/b-1/4*c*(b*x^2+a)^(1/2)/a/x^4+1/8*(-4*a*d+3*b*c)*(b*x^2+a)
^(1/2)/a^2/x^2
```

3.148.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-2abc+3b^2cx^2-4abdx^2+8a^2fx^4)}{8a^2bx^4}+\frac{(-3b^2c+4abd-8a^2e)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{8a^{5/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*b*c + 3*b^2*c*x^2 - 4*a*b*d*x^2 + 8*a^2*f*x^4))/(8*
a^2*b*x^4) + ((-3*b^2*c + 4*a*b*d - 8*a^2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[
a]])/(8*a^(5/2))
```

3.148.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2331, 2124, 27, 1192, 1471, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2331}$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{x^6\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4afx^4-4aex^2+3bc-4ad}{2x^4\sqrt{bx^2+a}}dx^2}{2a}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4afx^4-4aex^2+3bc-4ad}{x^4\sqrt{bx^2+a}}dx^2}{4a}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{1192}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-4afx^8-4a(be-2af)x^4+3b^3c-4ab^2d+4a^2be-4a^3f}{(a-x^4)^2}d\sqrt{bx^2+a}}{2ab}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{1471}$$

$$\frac{1}{2}\left(-\frac{\frac{b^2\sqrt{a+bx^2}(3bc-4ad)}{2a(a-x^4)}-\frac{\int -\frac{8a^2fx^4+3b^3c-4ab^2d+8a^2be-8a^3f}{a-x^4}d\sqrt{bx^2+a}}{2a}}{2ab}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(-\frac{\frac{\int \frac{8a^2fx^4+3b^3c-4ab^2d+8a^2be-8a^3f}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{b^2\sqrt{a+bx^2}(3bc-4ad)}{2a(a-x^4)}}{2ab}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 299$$

$$\frac{1}{2}\left(-\frac{\frac{b(8a^2e-4abd+3b^2c)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}-8a^2f\sqrt{a+bx^2}}{2a}+\frac{b^2\sqrt{a+bx^2}(3bc-4ad)}{2a(a-x^4)}}{2ab}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}\left(-\frac{\frac{\frac{b\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)(8a^2e-4abd+3b^2c)}{\sqrt{a}}-8a^2f\sqrt{a+bx^2}}{2a}+\frac{b^2\sqrt{a+bx^2}(3bc-4ad)}{2a(a-x^4)}}{2ab}-\frac{c\sqrt{a+bx^2}}{2ax^4}\right)$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^5*Sqrt[a + b*x^2]),x]
```

output
```
(-1/2*(c*Sqrt[a + b*x^2])/(a*x^4) - ((b^2*(3*b*c - 4*a*d)*Sqrt[a + b*x^2])
/(2*a*(a - x^4)) + (-8*a^2*f*Sqrt[a + b*x^2] + (b*(3*b^2*c - 4*a*b*d + 8*a
^2*e)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/Sqrt[a])/(2*a))/(2*a*b))/2
```

3.148.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.148. $\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$

3.148.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$\frac{-(a^2e-\frac{1}{2}abd+\frac{3}{8}b^2c)bx^4a^2\,\text{arctanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a}}\right)+\sqrt{bx^2+a}\left(a^2fx^4-\frac{b(2dx^2+c)a}{4}+\frac{3b^2cx^2}{8}\right)a^{\frac{5}{2}}}{b\,a^{\frac{9}{2}}x^4}$
risch	$-\frac{\sqrt{bx^2+a}\,(4adx^2-3cbx^2+2ac)}{8a^2x^4}+\frac{\frac{8a^2f\sqrt{bx^2+a}}{b}-\frac{(8a^2e-4abd+3b^2c)\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{\sqrt{a}}}{8a^2}$
default	$\frac{f\sqrt{bx^2+a}}{b}-\frac{e\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{\sqrt{a}}+d\left(-\frac{\sqrt{bx^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\sqrt{bx^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)+c\left(-\frac{\sqrt{bx^2+a}}{4a\,x^4}-\frac{3b\left(-\sqrt{}\right.}{}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(a^2*e-1/2*a*b*d+3/8*b^2*c)*b*x^4*a^2*arctanh((b*x^2+a)^(1/2)/a^(1/2))+(
b*x^2+a)^(1/2)*(a^2*f*x^4-1/4*b*(2*d*x^2+c)*a+3/8*b^2*c*x^2)*a^(5/2))/b/a^
(9/2)/x^4
```

3.148.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.94

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx$$
$$=\left[\frac{(3\,b^3c-4\,ab^2d+8\,a^2be)\sqrt{a}x^4\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(8\,a^3fx^4-2\,a^2bc+(3\,ab^2c-4\,a^2bd)x^2)\sqrt{bx^2}}{16\,a^3bx^4}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*sqrt(a)*x^4*log(-(b*x^2 - 2*sqrt(
b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(8*a^3*f*x^4 - 2*a^2*b*c + (3*a*b^2*c -
 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4), 1/8*((3*b^3*c - 4*a*b^2*d +
 8*a^2*b*e)*sqrt(-a)*x^4*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (8*a^3*f*x^4 -
 2*a^2*b*c + (3*a*b^2*c - 4*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^4)]
```

3.148.6 Sympy [A] (verification not implemented)

Time = 33.69 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.70

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = f\left(\begin{cases} \frac{\sqrt{a+bx^2}}{b} & \text{for } b \neq 0 \\ \frac{x^2}{2\sqrt{a}} & \text{otherwise} \end{cases}\right) - \frac{c}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{2ax} + \frac{3b^{\frac{3}{2}}c}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{bd\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**5/(b*x**2+a)**(1/2),x)
```

output
```
f*Piecewise((sqrt(a + b*x**2)/b, Ne(b, 0)), (x**2/(2*sqrt(a)), True)) - c/
(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(8*a*x**3*sqrt(a/(b*x**2
) + 1)) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/(2*a*x) + 3*b**(3/2)*c/(8*a**2*x*
sqrt(a/(b*x**2) + 1)) - e*asinh(sqrt(a)/(sqrt(b)*x))/sqrt(a) + b*d*asinh(s
qrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*c*asinh(sqrt(a)/(sqrt(b)*x))/(8*
a**(5/2))
```

3.148.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.12

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = -\frac{3\,b^2c\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{bd\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{e\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} + \frac{\sqrt{bx^2+a}f}{b} + \frac{3\sqrt{bx^2+a}bc}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}d}{2\,ax^2} - \frac{\sqrt{bx^2+a}c}{4\,ax^4}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-3/8*b^2*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(5/2) + 1/2*b*d*arcsinh(a/(sqrt
(a*b)*abs(x)))/a^(3/2) - e*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) + sqrt(b*
x^2 + a)*f/b + 3/8*sqrt(b*x^2 + a)*b*c/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*d/(
a*x^2) - 1/4*sqrt(b*x^2 + a)*c/(a*x^4)
```

3.148.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.23

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{8\sqrt{bx^2+a}f + \frac{(3b^3c-4ab^2d+8a^2be)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{3(bx^2+a)^{\frac{3}{2}}b^3c-5\sqrt{bx^2+a}ab^3c-4(bx^2+a)^{\frac{3}{2}}ab^2d+4\sqrt{bx^2+a}a^2b^2d}{a^2b^2x^4}}{8b}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^5/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*(8*sqrt(b*x^2 + a)*f + (3*b^3*c - 4*a*b^2*d + 8*a^2*b*e)*arctan(sqrt(b
*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^2) + (3*(b*x^2 + a)^(3/2)*b^3*c - 5*sqrt(b
*x^2 + a)*a*b^3*c - 4*(b*x^2 + a)^(3/2)*a*b^2*d + 4*sqrt(b*x^2 + a)*a^2*b^
2*d)/(a^2*b^2*x^4))/b
```

3.148.9 Mupad [B] (verification not implemented)

Time = 6.99 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17

$$\int \frac{c+dx^2+ex^4+fx^6}{x^5\sqrt{a+bx^2}}\,dx = \frac{f\sqrt{b\,x^2+a}}{b} - \frac{e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{5\,c\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,c\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4} - \frac{d\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{3\,b^2\,c\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^5*(a + b*x^2)^(1/2)),x)
```

output
```
(f*(a + b*x^2)^(1/2))/b - (e*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (
5*c*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*c*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (
d*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2
*a^(3/2)) - (3*b^2*c*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.149 $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

3.149.1 Optimal result

Integrand size = 32, antiderivative size = 146

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{6ax^6}+\frac{(5bc-6ad)\sqrt{a+bx^2}}{24a^2x^4} - \frac{(5b^2c-6abd+8a^2e)\sqrt{a+bx^2}}{16a^3x^2} + \frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

output
```
1/16*(-16*a^3*f+8*a^2*b*e-6*a*b^2*d+5*b^3*c)*arctanh((b*x^2+a)^(1/2)/a^(1/
2))/a^(7/2)-1/6*c*(b*x^2+a)^(1/2)/a/x^6+1/24*(-6*a*d+5*b*c)*(b*x^2+a)^(1/2
)/a^2/x^4-1/16*(8*a^2*e-6*a*b*d+5*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^2
```

3.149.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-8a^2c+10abcx^2-12a^2dx^2-15b^2cx^4+18abdx^4-24a^2ex^4)}{48a^3x^6} + \frac{(5b^3c-6ab^2d+8a^2be-16a^3f)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{16a^{7/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-8*a^2*c + 10*a*b*c*x^2 - 12*a^2*d*x^2 - 15*b^2*c*x^4 +
18*a*b*d*x^4 - 24*a^2*e*x^4))/(48*a^3*x^6) + ((5*b^3*c - 6*a*b^2*d + 8*a^2
*b*e - 16*a^3*f)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(16*a^(7/2))
```

3.149.3 Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2331, 2124, 27, 1192, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{ 2331}$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{x^8\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \text{ 2124}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-6afx^4-6aex^2+5bc-6ad}{2x^6\sqrt{bx^2+a}}dx^2}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2}\left(-\frac{\int \frac{-6afx^4-6aex^2+5bc-6ad}{x^6\sqrt{bx^2+a}}dx^2}{6a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 1192}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-6afx^8-6a(be-2af)x^4+5b^3c-6ab^2d+6a^2be-6a^3f}{(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\left(\frac{\int \frac{-6afx^8-6a(be-2af)x^4+5b^3c-6ab^2d+6a^2be-6a^3f}{(a-x^4)^3}d\sqrt{bx^2+a}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{1471}$$

$$\frac{1}{2}\left(-\frac{\frac{\int -\frac{3\left(8a^2fx^4+5b^3c-6ab^2d+8a^2be-8a^3f\right)}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}-\frac{b^2\sqrt{a+bx^2}(5bc-6ad)}{4a(a-x^4)^2}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(-\frac{-\frac{3\int \frac{8a^2fx^4+5b^3c-6ab^2d+8a^2be-8a^3f}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}-\frac{b^2\sqrt{a+bx^2}(5bc-6ad)}{4a(a-x^4)^2}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{298}$$

$$\frac{1}{2}\left(-\frac{-\frac{3\left(\frac{\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)\int \frac{1}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{b\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{2a\left(a-x^4\right)}\right)}{4a}-\frac{b^2\sqrt{a+bx^2}(5bc-6ad)}{4a(a-x^4)^2}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{-\frac{3\left(\frac{b\sqrt{a+bx^2}\left(8a^2e-6abd+5b^2c\right)}{2a\left(a-x^4\right)}+\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\left(-16a^3f+8a^2be-6ab^2d+5b^3c\right)}{2a^{3/2}}\right)}{4a}-\frac{b^2\sqrt{a+bx^2}(5bc-6ad)}{4a(a-x^4)^2}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^6}\right)$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^7*Sqrt[a + b*x^2]),x]
```

output
```
(-1/3*(c*Sqrt[a + b*x^2])/(a*x^6) - (-1/4*(b^2*(5*b*c - 6*a*d)*Sqrt[a + b*
x^2])/(a*(a - x^4)^2) - (3*((b*(5*b^2*c - 6*a*b*d + 8*a^2*e)*Sqrt[a + b*x^
2])/(2*a*(a - x^4)) + ((5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*ArcTan
h[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2))))/(4*a))/(3*a))/2
```

3.149. $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

3.149.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

3.149. $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.149.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{x^{6}\left(f\,a^{3}-\frac{1}{2}a^{2}be+\frac{3}{8}a\,b^{2}d-\frac{5}{16}b^{3}c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{\sqrt{a}}\right)+\frac{5\left(\frac{4\left(2e\,x^{4}+d\,x^{2}+\frac{2}{3}c\right)a^{\frac{5}{2}}}{5}+b\left(2\left(-\frac{3d\,x^{2}}{5}-\frac{c}{3}\right)a^{\frac{3}{2}}+bc\,x^{2}\sqrt{a}\right)x^{2}\right)\sqrt{b\,x^{2}\cdots}}{16}}{a^{\frac{7}{2}}x^{6}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(24a^{2}e\,x^{4}-18abd\,x^{4}+15b^{2}c\,x^{4}+12a^{2}d\,x^{2}-10abc\,x^{2}+8a^{2}c\right)}{48a^{3}x^{6}}-\frac{\left(16f\,a^{3}-8a^{2}be+6a\,b^{2}d-5b^{3}c\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}\cdots}}{x}\right.}{16a^{\frac{7}{2}}}$
default	$-\frac{f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{\sqrt{a}}+c\left(-\frac{\sqrt{b\,x^{2}+a}}{6a\,x^{6}}-\frac{5b\left(-\frac{\sqrt{b\,x^{2}+a}}{4a\,x^{4}}-\frac{3b\left(-\frac{\sqrt{b\,x^{2}+a}}{2a\,x^{2}}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{b\,x^{2}+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right)+e\Big(\cdots$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(x^6*(f*a^3-1/2*a^2*b*e+3/8*a*b^2*d-5/16*b^3*c)*arctanh((b*x^2+a)^(1/2)/a
^(1/2))+5/16*(4/5*(2*e*x^4+d*x^2+2/3*c)*a^(5/2)+b*(2*(-3/5*d*x^2-1/3*c)*a^
(3/2)+b*c*x^2*a^(1/2))*x^2)*(b*x^2+a)^(1/2))/a^(7/2)/x^6
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.79

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f)\sqrt{a}x^6\log\left(-\frac{bx^2-2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)+2\,(3\,(5\,ab^2c-6\,a^2bd+8\,a^3e)x^4 -}{96\,a^4x^6}\right.$$
$$-\frac{3\,(5\,b^3c-6\,ab^2d+8\,a^2be-16\,a^3f)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{-a}}{\sqrt{bx^2+a}}\right)+(3\,(5\,ab^2c-6\,a^2bd+8\,a^3e)x^4+8\,a^3c-}{48\,a^4x^6}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(a)*x^6*log(-(b
*x^2 - 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) + 2*(3*(5*a*b^2*c - 6*a^2*b*d
 + 8*a^3*e)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/
(a^4*x^6), -1/48*(3*(5*b^3*c - 6*a*b^2*d + 8*a^2*b*e - 16*a^3*f)*sqrt(-a)*
x^6*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(5*a*b^2*c - 6*a^2*b*d + 8*a^3*e
)*x^4 + 8*a^3*c - 2*(5*a^2*b*c - 6*a^3*d)*x^2)*sqrt(b*x^2 + a))/(a^4*x^6)]
```

3.149.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(141) = 282.

Time = 40.70 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{c}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{d}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{24ax^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}d}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{5b^{\frac{3}{2}}c}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{3}{2}}d}{8a^2x\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{5}{2}}c}{16a^3x\sqrt{\frac{a}{bx^2}+1}} - \frac{f\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{\sqrt{a}} + \frac{be\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} + \frac{5b^3c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**7/(b*x**2+a)**(1/2),x)
```

output
```
-c/(6*sqrt(b)*x**7*sqrt(a/(b*x**2) + 1)) - d/(4*sqrt(b)*x**5*sqrt(a/(b*x**
2) + 1)) + sqrt(b)*c/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(8*a*x**
3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/(2*a*x) - 5*b**(3
/2)*c/(48*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*d/(8*a**2*x*sqrt(a/
(b*x**2) + 1)) - 5*b**(5/2)*c/(16*a**3*x*sqrt(a/(b*x**2) + 1)) - f*asinh(s
qrt(a)/(sqrt(b)*x))/sqrt(a) + b*e*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2))
- 3*b**2*d*asinh(sqrt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*c*asinh(sqrt(a
)/(sqrt(b)*x))/(16*a**(7/2))
```

3.149.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.32

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = \frac{5\,b^3c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{3\,b^2d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{be\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} - \frac{f\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{\sqrt{a}} - \frac{5\,\sqrt{bx^2+a}b^2c}{16\,a^3x^2} + \frac{3\,\sqrt{bx^2+a}bd}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}e}{2\,ax^2} + \frac{5\,\sqrt{bx^2+a}bc}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}d}{4\,ax^4} - \frac{\sqrt{bx^2+a}c}{6\,ax^6}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
5/16*b^3*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*d*arcsinh(a/(sq
rt(a*b)*abs(x)))/a^(5/2) + 1/2*b*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) -
 f*arcsinh(a/(sqrt(a*b)*abs(x)))/sqrt(a) - 5/16*sqrt(b*x^2 + a)*b^2*c/(a^3
*x^2) + 3/8*sqrt(b*x^2 + a)*b*d/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*e/(a*x^2)
+ 5/24*sqrt(b*x^2 + a)*b*c/(a^2*x^4) - 1/4*sqrt(b*x^2 + a)*d/(a*x^4) - 1/6
*sqrt(b*x^2 + a)*c/(a*x^6)
```

3.149.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.56

$$\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx = -\frac{\frac{3\left(5\,b^4c-6\,ab^3d+8\,a^2b^2e-16\,a^3bf\right)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\left(bx^2+a\right)^{\frac{5}{2}}b^4c-40\left(bx^2+a\right)^{\frac{3}{2}}ab^4c+33\sqrt{bx^2+a}a^2b^4c-18\left(bx^2+a\right)^{\frac{5}{2}}ab^3d+48\left(bx\right.}{}}{48\,b}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^7/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/48*(3*(5*b^4*c - 6*a*b^3*d + 8*a^2*b^2*e - 16*a^3*b*f)*arctan(sqrt(b*x^
2 + a)/sqrt(-a))/(sqrt(-a)*a^3) + (15*(b*x^2 + a)^(5/2)*b^4*c - 40*(b*x^2
+ a)^(3/2)*a*b^4*c + 33*sqrt(b*x^2 + a)*a^2*b^4*c - 18*(b*x^2 + a)^(5/2)*a
*b^3*d + 48*(b*x^2 + a)^(3/2)*a^2*b^3*d - 30*sqrt(b*x^2 + a)*a^3*b^3*d + 2
4*(b*x^2 + a)^(5/2)*a^2*b^2*e - 48*(b*x^2 + a)^(3/2)*a^3*b^2*e + 24*sqrt(b
*x^2 + a)*a^4*b^2*e)/(a^3*b^3*x^6))/b
```

3.149.9 Mupad [B] (verification not implemented)

Time = 7.62 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.36

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx &= \frac{5\,c\,\left(b\,x^2+a\right)^{3/2}}{6\,a^2\,x^6}-\frac{11\,c\,\sqrt{b\,x^2+a}}{16\,a\,x^6}-\frac{f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{\sqrt{a}}\\ &\quad-\frac{5\,c\,\left(b\,x^2+a\right)^{5/2}}{16\,a^3\,x^6}-\frac{5\,d\,\sqrt{b\,x^2+a}}{8\,a\,x^4}+\frac{3\,d\,\left(b\,x^2+a\right)^{3/2}}{8\,a^2\,x^4}\\ &\quad-\frac{e\,\sqrt{b\,x^2+a}}{2\,a\,x^2}+\frac{b\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}}\\ &\quad-\frac{3\,b^2\,d\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}}-\frac{b^3\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,a^{7/2}}\end{aligned}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^7*(a + b*x^2)^(1/2)),x)
```

output
```
(5*c*(a + b*x^2)^(3/2))/(6*a^2*x^6) - (11*c*(a + b*x^2)^(1/2))/(16*a*x^6)
- (f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/a^(1/2) - (5*c*(a + b*x^2)^(5/2))/(
16*a^3*x^6) - (5*d*(a + b*x^2)^(1/2))/(8*a*x^4) + (3*d*(a + b*x^2)^(3/2))/
(8*a^2*x^4) - (e*(a + b*x^2)^(1/2))/(2*a*x^2) + (b*e*atanh((a + b*x^2)^(1/
2)/a^(1/2)))/(2*a^(3/2)) - (b^3*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)
/(16*a^(7/2)) - (3*b^2*d*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.149. $\int \frac{c+dx^2+ex^4+fx^6}{x^7\sqrt{a+bx^2}}\,dx$

3.150 $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

3.150.1 Optimal result

Integrand size = 32, antiderivative size = 195

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{8ax^8}+\frac{(7bc-8ad)\sqrt{a+bx^2}}{48a^2x^6} - \frac{(35b^2c-40abd+48a^2e)\sqrt{a+bx^2}}{192a^3x^4} + \frac{(35b^3c-40ab^2d+48a^2be-64a^3f)\sqrt{a+bx^2}}{128a^4x^2} - \frac{b(35b^3c-40ab^2d+48a^2be-64a^3f)\,\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{128a^{9/2}}$$

output
```
-1/128*b*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*arctanh((b*x^2+a)^(1/2
)/a^(1/2))/a^(9/2)-1/8*c*(b*x^2+a)^(1/2)/a/x^8+1/48*(-8*a*d+7*b*c)*(b*x^2+
a)^(1/2)/a^2/x^6-1/192*(48*a^2*e-40*a*b*d+35*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^
4+1/128*(-64*a^3*f+48*a^2*b*e-40*a*b^2*d+35*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^2
```

3.150.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$$
$$= \frac{\frac{\sqrt{a}\sqrt{a+bx^2}(105b^3cx^6-10ab^2x^4(7c+12dx^2)+8a^2bx^2(7c+10dx^2+18ex^4)-16a^3(3c+4dx^2+6ex^4+12fx^6))}{x^8} - 3b(35b^3c-40ab^2d+48a}{384a^{9/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

output
```
((Sqrt[a]*Sqrt[a + b*x^2]*(105*b^3*c*x^6 - 10*a*b^2*x^4*(7*c + 12*d*x^2) +
 8*a^2*b*x^2*(7*c + 10*d*x^2 + 18*e*x^4) - 16*a^3*(3*c + 4*d*x^2 + 6*e*x^4
 + 12*f*x^6)))/x^8 - 3*b*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)*A
rcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(384*a^(9/2))
```

3.150.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.281, Rules used = {2331, 2124, 27, 1192, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \; 2331$$

$$\frac{1}{2}\int \frac{fx^6+ex^4+dx^2+c}{x^{10}\sqrt{bx^2+a}}dx^2$$

$$\Big\downarrow \; 2124$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8afx^4-8aex^2+7bc-8ad}{2x^8\sqrt{bx^2+a}}dx^2}{4a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(-\frac{\int \frac{-8afx^4-8aex^2+7bc-8ad}{x^8\sqrt{bx^2+a}}dx^2}{8a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 1192$$

$$\frac{1}{2}\left(-\frac{b\int \frac{-8afx^8-8a(be-2af)x^4+7b^3c-8ab^2d+8a^2be-8a^3f}{(a-x^4)^4}d\sqrt{bx^2+a}}{4a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 1471$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{b^2\sqrt{a+bx^2}(7bc-8ad)}{6a(a-x^4)^3}-\frac{\int -\frac{48a^2fx^4+35b^3c-40ab^2d+48a^2be-48a^3f}{\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a}\right)}{4a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\int \frac{48a^2fx^4+35b^3c-40ab^2d+48a^2be-48a^3f}{\left(a-x^4\right)^3}d\sqrt{bx^2+a}}{6a}+\frac{b^2\sqrt{a+bx^2}(7bc-8ad)}{6a(a-x^4)^3}\right)}{4a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 298$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)\int \frac{1}{\left(a-x^4\right)^2}d\sqrt{bx^2+a}}{4a}+\frac{b\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{4a\left(a-x^4\right)^2}}{6a}+\frac{b^2\sqrt{a+bx^2}(7bc-8ad)}{6a(a-x^4)^3}\right)}{4a}-\frac{c\sqrt{a+bx^2}}{4ax^8}\right)$$

$$\Big\downarrow \; 215$$

3.150. $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{3\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)\left(\frac{\int \frac{1}{a-x^4}d\sqrt{bx^2+a}}{2a}+\frac{\sqrt{a+bx^2}}{2a\left(a-x^4\right)}\right)}{4a}+\frac{b\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{4a\left(a-x^4\right)^2}}{6a}+\frac{b^2\sqrt{a+bx^2}(7bc-8ad)}{6a(a-x^4)^3}\right)}{4a}-\frac{c\sqrt{\cdots}}{4\cdots}\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{b\left(\frac{\frac{b\sqrt{a+bx^2}\left(48a^2e-40abd+35b^2c\right)}{4a\left(a-x^4\right)^2}+\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}}+\frac{\sqrt{a+bx^2}}{2a\left(a-x^4\right)}\right)\left(-64a^3f+48a^2be-40ab^2d+35b^3c\right)}{4a}}{6a}+\frac{b^2\sqrt{a+bx^2}(7bc-8ad)}{6a(a-x^4)^3}\right)}{4a}-\cdots\right.$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^9*Sqrt[a + b*x^2]),x]
```

output
```
(-1/4*(c*Sqrt[a + b*x^2])/(a*x^8) - (b*((b^2*(7*b*c - 8*a*d)*Sqrt[a + b*x^
2])/(6*a*(a - x^4)^3) + ((b*(35*b^2*c - 40*a*b*d + 48*a^2*e)*Sqrt[a + b*x^
2])/(4*a*(a - x^4)^2) + (3*(35*b^3*c - 40*a*b^2*d + 48*a^2*b*e - 64*a^3*f)
*(Sqrt[a + b*x^2]/(2*a*(a - x^4)) + ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(2*a^
(3/2))))/(4*a))/(6*a)))/(4*a))/2
```

3.150. $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

3.150.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1192
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[2/e^(n + 2*p + 1)   Subst[Int[x^(
2*m + 1)*(e*f - d*g + g*x^2)^n*(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4)^p, x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &&
IGtQ[p, 0] && ILtQ[n, 0] && IntegerQ[m + 1/2]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

3.150. $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

rule 2124
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[R*(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((m + 1)*(b*c -
 a*d))), x] + Simp[1/((m + 1)*(b*c - a*d))   Int[(a + b*x)^(m + 1)*(c + d*x
)^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fr
eeQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && LtQ[m, -1] && (IntegerQ[m] ||  !
ILtQ[n, -1])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.150.4 Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{b x^8 \left(f a^3 - \frac{3}{4} a^2 b e + \frac{5}{8} a b^2 d - \frac{35}{64} b^3 c\right) \operatorname{arctanh}\left(\frac{\sqrt{b x^2+a}}{\sqrt{a}}\right) + \frac{35\left(\left(-\frac{64}{35} f x^6 - \frac{32}{35} e x^4 - \frac{64}{105} d x^2 - \frac{16}{35} c\right) a^{\frac{7}{2}} + b x^2 \left(\left(\frac{48}{35} e x^4 + \frac{16}{21} d x^2 + \frac{8}{15} c \ldots\right.\right.\right.}{64}}{2 a^{\frac{9}{2}} x^8}$
risch	$-\frac{\sqrt{b x^2+a}\,(192 a^3 f x^6 - 144 a^2 b e x^6 + 120 a b^2 d x^6 - 105 b^3 c x^6 + 96 a^3 e x^4 - 80 a^2 b d x^4 + 70 a b^2 c x^4 + 64 a^3 d x^2 - 56 a^2 b c x^2 + 48 c \ldots}{384 a^4 x^8}$
default	$d\left(-\frac{\sqrt{b x^2+a}}{6 a x^6} - \frac{5b\left(-\frac{\sqrt{b x^2+a}}{4 a x^4} - \frac{3b\left(-\frac{\sqrt{b x^2+a}}{2 a x^2} + \frac{b \ln\left(\frac{2a+2\sqrt{a}\sqrt{b x^2+a}}{x}\right)}{2 a^{\frac{3}{2}}}\right)}{4a}\right)}{6a}\right) + c\left(-\frac{\sqrt{b x^2+a}}{8 a x^8} - \frac{7b\left(-\frac{\sqrt{b x^2+a}}{6 a x^6} \ldots\right.}{\ldots}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b*x^8*(f*a^3-3/4*a^2*b*e+5/8*a*b^2*d-35/64*b^3*c)*arctanh((b*x^2+a)^(
1/2)/a^(1/2))+35/64*((-64/35*f*x^6-32/35*e*x^4-64/105*d*x^2-16/35*c)*a^(7/
2)+b*x^2*((48/35*e*x^4+16/21*d*x^2+8/15*c)*a^(5/2)+((-8/7*d*x^2-2/3*c)*a^(
3/2)+b*c*x^2*a^(1/2))*b*x^2))*(b*x^2+a)^(1/2))/a^(9/2)/x^8
```

3.150. $\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$

3.150.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.75

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(35\,b^4c-40\,ab^3d+48\,a^2b^2e-64\,a^3bf)\sqrt{a}x^8\log\left(-\frac{bx^2+2\,\sqrt{bx^2+a}\sqrt{a}+2\,a}{x^2}\right)-2\,(3\,(35\,ab^3c-40\,a^2b^2d+4}{768\,a^5x}\right.$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sqrt(a)*x^8
*log(-(b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*(3*(35*a*b^3*c -
40*a^2*b^2*d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 4
0*a^3*b*d + 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/
(a^5*x^8), 1/384*(3*(35*b^4*c - 40*a*b^3*d + 48*a^2*b^2*e - 64*a^3*b*f)*sq
rt(-a)*x^8*arctan(sqrt(-a)/sqrt(b*x^2 + a)) + (3*(35*a*b^3*c - 40*a^2*b^2*
d + 48*a^3*b*e - 64*a^4*f)*x^6 - 48*a^4*c - 2*(35*a^2*b^2*c - 40*a^3*b*d +
 48*a^4*e)*x^4 + 8*(7*a^3*b*c - 8*a^4*d)*x^2)*sqrt(b*x^2 + a))/(a^5*x^8)]
```

3.150.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(196) = 392$.

Time = 103.42 (sec) , antiderivative size = 444, normalized size of antiderivative = 2.28

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{c}{8\sqrt{b}x^9\sqrt{\frac{a}{bx^2}+1}} - \frac{d}{6\sqrt{b}x^7\sqrt{\frac{a}{bx^2}+1}} - \frac{e}{4\sqrt{b}x^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}c}{48ax^7\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}d}{24ax^5\sqrt{\frac{a}{bx^2}+1}} + \frac{\sqrt{b}e}{8ax^3\sqrt{\frac{a}{bx^2}+1}} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^2}+1}}{2ax} - \frac{7b^{\frac{3}{2}}c}{192a^2x^5\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{3}{2}}d}{48a^2x^3\sqrt{\frac{a}{bx^2}+1}} + \frac{3b^{\frac{3}{2}}e}{8a^2x\sqrt{\frac{a}{bx^2}+1}} + \frac{35b^{\frac{5}{2}}c}{384a^3x^3\sqrt{\frac{a}{bx^2}+1}} - \frac{5b^{\frac{5}{2}}d}{16a^3x\sqrt{\frac{a}{bx^2}+1}} + \frac{35b^{\frac{7}{2}}c}{128a^4x\sqrt{\frac{a}{bx^2}+1}} + \frac{bf\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{2a^{\frac{3}{2}}} - \frac{3b^2e\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{8a^{\frac{5}{2}}} + \frac{5b^3d\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{16a^{\frac{7}{2}}} - \frac{35b^4c\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}x}\right)}{128a^{\frac{9}{2}}}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**9/(b*x**2+a)**(1/2),x)
```

output
```
-c/(8*sqrt(b)*x**9*sqrt(a/(b*x**2) + 1)) - d/(6*sqrt(b)*x**7*sqrt(a/(b*x**
2) + 1)) - e/(4*sqrt(b)*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(b)*c/(48*a*x**7*
sqrt(a/(b*x**2) + 1)) + sqrt(b)*d/(24*a*x**5*sqrt(a/(b*x**2) + 1)) + sqrt(
b)*e/(8*a*x**3*sqrt(a/(b*x**2) + 1)) - sqrt(b)*f*sqrt(a/(b*x**2) + 1)/(2*a
*x) - 7*b**(3/2)*c/(192*a**2*x**5*sqrt(a/(b*x**2) + 1)) - 5*b**(3/2)*d/(48
*a**2*x**3*sqrt(a/(b*x**2) + 1)) + 3*b**(3/2)*e/(8*a**2*x*sqrt(a/(b*x**2)
+ 1)) + 35*b**(5/2)*c/(384*a**3*x**3*sqrt(a/(b*x**2) + 1)) - 5*b**(5/2)*d/
(16*a**3*x*sqrt(a/(b*x**2) + 1)) + 35*b**(7/2)*c/(128*a**4*x*sqrt(a/(b*x**
2) + 1)) + b*f*asinh(sqrt(a)/(sqrt(b)*x))/(2*a**(3/2)) - 3*b**2*e*asinh(sq
rt(a)/(sqrt(b)*x))/(8*a**(5/2)) + 5*b**3*d*asinh(sqrt(a)/(sqrt(b)*x))/(16*
a**(7/2)) - 35*b**4*c*asinh(sqrt(a)/(sqrt(b)*x))/(128*a**(9/2))
```

3.150.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.41

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = -\frac{35\,b^4c\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{128\,a^{\frac{9}{2}}} + \frac{5\,b^3d\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{16\,a^{\frac{7}{2}}} - \frac{3\,b^2e\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{8\,a^{\frac{5}{2}}} + \frac{bf\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}|x|}\right)}{2\,a^{\frac{3}{2}}} + \frac{35\,\sqrt{bx^2+a}b^3c}{128\,a^4x^2} - \frac{5\,\sqrt{bx^2+a}b^2d}{16\,a^3x^2} + \frac{3\,\sqrt{bx^2+a}be}{8\,a^2x^2} - \frac{\sqrt{bx^2+a}f}{2\,ax^2} - \frac{35\,\sqrt{bx^2+a}b^2c}{192\,a^3x^4} + \frac{5\,\sqrt{bx^2+a}bd}{24\,a^2x^4} - \frac{\sqrt{bx^2+a}e}{4\,ax^4} + \frac{7\,\sqrt{bx^2+a}bc}{48\,a^2x^6} - \frac{\sqrt{bx^2+a}d}{6\,ax^6} - \frac{\sqrt{bx^2+a}c}{8\,ax^8}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
-35/128*b^4*c*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(9/2) + 5/16*b^3*d*arcsinh(a
/(sqrt(a*b)*abs(x)))/a^(7/2) - 3/8*b^2*e*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(
5/2) + 1/2*b*f*arcsinh(a/(sqrt(a*b)*abs(x)))/a^(3/2) + 35/128*sqrt(b*x^2 +
 a)*b^3*c/(a^4*x^2) - 5/16*sqrt(b*x^2 + a)*b^2*d/(a^3*x^2) + 3/8*sqrt(b*x^
2 + a)*b*e/(a^2*x^2) - 1/2*sqrt(b*x^2 + a)*f/(a*x^2) - 35/192*sqrt(b*x^2 +
 a)*b^2*c/(a^3*x^4) + 5/24*sqrt(b*x^2 + a)*b*d/(a^2*x^4) - 1/4*sqrt(b*x^2
+ a)*e/(a*x^4) + 7/48*sqrt(b*x^2 + a)*b*c/(a^2*x^6) - 1/6*sqrt(b*x^2 + a)*
d/(a*x^6) - 1/8*sqrt(b*x^2 + a)*c/(a*x^8)
```

3.150.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(171) = 342$.

Time = 0.29 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.83

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^9\sqrt{a + bx^2}}\,dx$$
$$= \frac{\frac{3\,(35\,b^5c-40\,ab^4d+48\,a^2b^3e-64\,a^3b^2f)\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\,(bx^2+a)^{\frac{7}{2}}b^5c-385\,(bx^2+a)^{\frac{5}{2}}ab^5c+511\,(bx^2+a)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx^2+a}a^3b^5c-}{}}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^9/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(3*(35*b^5*c - 40*a*b^4*d + 48*a^2*b^3*e - 64*a^3*b^2*f)*arctan(sqrt
(b*x^2 + a)/sqrt(-a))/(sqrt(-a)*a^4) + (105*(b*x^2 + a)^(7/2)*b^5*c - 385*
(b*x^2 + a)^(5/2)*a*b^5*c + 511*(b*x^2 + a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x
^2 + a)*a^3*b^5*c - 120*(b*x^2 + a)^(7/2)*a*b^4*d + 440*(b*x^2 + a)^(5/2)*
a^2*b^4*d - 584*(b*x^2 + a)^(3/2)*a^3*b^4*d + 264*sqrt(b*x^2 + a)*a^4*b^4*
d + 144*(b*x^2 + a)^(7/2)*a^2*b^3*e - 528*(b*x^2 + a)^(5/2)*a^3*b^3*e + 62
4*(b*x^2 + a)^(3/2)*a^4*b^3*e - 240*sqrt(b*x^2 + a)*a^5*b^3*e - 192*(b*x^2
 + a)^(7/2)*a^3*b^2*f + 576*(b*x^2 + a)^(5/2)*a^4*b^2*f - 576*(b*x^2 + a)^
(3/2)*a^5*b^2*f + 192*sqrt(b*x^2 + a)*a^6*b^2*f)/(a^4*b^4*x^8))/b
```

3.150.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.42

$$\int \frac{c+dx^2+ex^4+fx^6}{x^9\sqrt{a+bx^2}}\,dx = \frac{511\,c\,(b\,x^2+a)^{3/2}}{384\,a^2\,x^8} - \frac{93\,c\,\sqrt{b\,x^2+a}}{128\,a\,x^8} - \frac{385\,c\,(b\,x^2+a)^{5/2}}{384\,a^3\,x^8} + \frac{35\,c\,(b\,x^2+a)^{7/2}}{128\,a^4\,x^8} - \frac{11\,d\,\sqrt{b\,x^2+a}}{16\,a\,x^6} + \frac{5\,d\,(b\,x^2+a)^{3/2}}{6\,a^2\,x^6} - \frac{5\,d\,(b\,x^2+a)^{5/2}}{16\,a^3\,x^6} - \frac{5\,e\,\sqrt{b\,x^2+a}}{8\,a\,x^4} + \frac{3\,e\,(b\,x^2+a)^{3/2}}{8\,a^2\,x^4} - \frac{f\,\sqrt{b\,x^2+a}}{2\,a\,x^2} + \frac{b\,f\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{2\,a^{3/2}} - \frac{3\,b^2\,e\,\mathrm{atanh}\left(\frac{\sqrt{b\,x^2+a}}{\sqrt{a}}\right)}{8\,a^{5/2}} + \frac{b^4\,c\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,35\mathrm{i}}{128\,a^{9/2}} - \frac{b^3\,d\,\mathrm{atan}\left(\frac{\sqrt{b\,x^2+a}\,1\mathrm{i}}{\sqrt{a}}\right)\,5\mathrm{i}}{16\,a^{7/2}}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^9*(a + b*x^2)^(1/2)),x)
```

output
```
(511*c*(a + b*x^2)^(3/2))/(384*a^2*x^8) - (93*c*(a + b*x^2)^(1/2))/(128*a*
x^8) - (385*c*(a + b*x^2)^(5/2))/(384*a^3*x^8) + (35*c*(a + b*x^2)^(7/2))/
(128*a^4*x^8) - (11*d*(a + b*x^2)^(1/2))/(16*a*x^6) + (5*d*(a + b*x^2)^(3/
2))/(6*a^2*x^6) - (5*d*(a + b*x^2)^(5/2))/(16*a^3*x^6) - (5*e*(a + b*x^2)^
(1/2))/(8*a*x^4) + (3*e*(a + b*x^2)^(3/2))/(8*a^2*x^4) - (f*(a + b*x^2)^(1
/2))/(2*a*x^2) + (b*f*atanh((a + b*x^2)^(1/2)/a^(1/2)))/(2*a^(3/2)) + (b^4
*c*atan(((a + b*x^2)^(1/2)*1i)/a^(1/2))*35i)/(128*a^(9/2)) - (b^3*d*atan((
(a + b*x^2)^(1/2)*1i)/a^(1/2))*5i)/(16*a^(7/2)) - (3*b^2*e*atanh((a + b*x^
2)^(1/2)/a^(1/2)))/(8*a^(5/2))
```

3.151 $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.151.1 Optimal result

Integrand size = 32, antiderivative size = 245

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = -\frac{a(96b^3c-80ab^2d+70a^2be-63a^3f)\,x\sqrt{a+bx^2}}{256b^5} + \frac{(96b^3c-80ab^2d+70a^2be-63a^3f)\,x^3\sqrt{a+bx^2}}{384b^4} + \frac{(80b^2d-70abe+63a^2f)\,x^5\sqrt{a+bx^2}}{480b^3} + \frac{(10be-9af)x^7\sqrt{a+bx^2}}{80b^2} + \frac{fx^9\sqrt{a+bx^2}}{10b} + \frac{a^2(96b^3c-80ab^2d+70a^2be-63a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256b^{11/2}}$$

output
```
1/256*a^2*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*arctanh(x*b^(1/2)/(b*
x^2+a)^(1/2))/b^(11/2)-1/256*a*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*
x*(b*x^2+a)^(1/2)/b^5+1/384*(-63*a^3*f+70*a^2*b*e-80*a*b^2*d+96*b^3*c)*x^3
*(b*x^2+a)^(1/2)/b^4+1/480*(63*a^2*f-70*a*b*e+80*b^2*d)*x^5*(b*x^2+a)^(1/2
)/b^3+1/80*(-9*a*f+10*b*e)*x^7*(b*x^2+a)^(1/2)/b^2+1/10*f*x^9*(b*x^2+a)^(1
/2)/b
```

3.151.2 Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{x\sqrt{a+bx^2}(-1440ab^3c+1200a^2b^2d-1050a^3be+945a^4f+960b^4cx^2-800ab^3dx^2+700a^2b^2ex^2-630a^3}{3840b^5}$$
$$-\frac{a^2(-96b^3c+80ab^2d-70a^2be+63a^3f)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{128b^{11/2}}$$

input
```
Integrate[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(-1440*a*b^3*c + 1200*a^2*b^2*d - 1050*a^3*b*e + 945*a^
4*f + 960*b^4*c*x^2 - 800*a*b^3*d*x^2 + 700*a^2*b^2*e*x^2 - 630*a^3*b*f*x^
2 + 640*b^4*d*x^4 - 560*a*b^3*e*x^4 + 504*a^2*b^2*f*x^4 + 480*b^4*e*x^6 -
432*a*b^3*f*x^6 + 384*b^4*f*x^8))/(3840*b^5) - (a^2*(-96*b^3*c + 80*a*b^2*
d - 70*a^2*b*e + 63*a^3*f)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2]
)])/(128*b^(11/2))
```

3.151.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2340, 1590, 363, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \; 2340$$
$$\frac{\int \frac{x^4\left((10be-9af)x^4+10bdx^2+10bc\right)}{\sqrt{bx^2+a}}dx}{10b}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$
$$\Big\downarrow \; 1590$$
$$\frac{\frac{\int \frac{x^4\left(80cb^2+\left(63fa^2-70bea+80b^2d\right)x^2\right)}{\sqrt{bx^2+a}}dx}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{8b}}{10b}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{\frac{5\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)\int \frac{x^4}{\sqrt{bx^2+a}}dx}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{8b}}{10b}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\frac{5\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{8b}}{10b}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{\frac{5\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-9af)}{8b}}{10b}+\frac{fx^9\sqrt{a+bx^2}}{10b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{\frac{5\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{6b}+\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-}{8b}}{10b}$$

$$\frac{fx^9\sqrt{a+bx^2}}{10b}$$

$$\Big\downarrow \text{219}$$

3.151. $\int \frac{x^4\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$

$$\frac{\frac{\frac{x^5\sqrt{a+bx^2}\left(63a^2f-70abe+80b^2d\right)}{6b}+\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)\left(-63a^3f+70a^2be-80ab^2d+96b^3c\right)}{6b}}{8b}+\frac{x^7\sqrt{a+bx^2}(10be-}{8b}}{10b}$$

$$\frac{fx^9\sqrt{a+bx^2}}{10b}$$

input
```
Int[(x^4*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(f*x^9*Sqrt[a + b*x^2])/(10*b) + (((10*b*e - 9*a*f)*x^7*Sqrt[a + b*x^2])/(
8*b) + (((80*b^2*d - 70*a*b*e + 63*a^2*f)*x^5*Sqrt[a + b*x^2])/(6*b) + (5*
(96*b^3*c - 80*a*b^2*d + 70*a^2*b*e - 63*a^3*f)*((x^3*Sqrt[a + b*x^2])/(4*
b) - (3*a*((x*Sqrt[a + b*x^2])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x
^2]])/(2*b^(3/2))))/(4*b)))/(6*b))/(8*b))/(10*b)
```

3.151.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.151.4 Maple [A] (verified)

Time = 3.70 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{63\left(a^{2}\left(f\,a^{3}-\frac{10}{9}a^{2}be+\frac{80}{63}a\,b^{2}d-\frac{32}{21}b^{3}c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\left(-\frac{32\left(\frac{3}{10}f\,x^{6}+\frac{7}{18}e\,x^{4}+\frac{5}{9}d\,x^{2}+c\right)a\,b^{\frac{7}{2}}}{21}+\frac{64x^{2}\left(\frac{2}{5}f\,x^{6}+\frac{1}{2}e\,x^{4}+\right.}{63}\right.\right.}{256b^{\frac{11}{2}}}$
risch	$\frac{x\left(384f\,x^{8}b^{4}-432a\,b^{3}f\,x^{6}+480b^{4}e\,x^{6}+504a^{2}b^{2}f\,x^{4}-560a\,b^{3}e\,x^{4}+640b^{4}d\,x^{4}-630a^{3}bf\,x^{2}+700a^{2}b^{2}e\,x^{2}-800a\,b^{3}d\,x^{2}+960b^{\cdot}\right.}{3840b^{5}}$
default	$e\left(\frac{x^{7}\sqrt{b\,x^{2}+a}}{8b}-\frac{7a\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)}{8b}\right)+d\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\right.$

input

```
int(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-63/256/b^(11/2)*(a^2*(f*a^3-10/9*a^2*b*e+80/63*a*b^2*d-32/21*b^3*c)*arcta
nh((b*x^2+a)^(1/2)/x/b^(1/2))-(-32/21*(3/10*f*x^6+7/18*e*x^4+5/9*d*x^2+c)*
a*b^(7/2)+64/63*x^2*(2/5*f*x^6+1/2*e*x^4+2/3*d*x^2+c)*b^(9/2)+((8/15*f*x^4
+20/27*e*x^2+80/63*d)*b^(5/2)+((-2/3*f*x^2-10/9*e)*b^(3/2)+a*f*b^(1/2))*a)
*a^2)*(b*x^2+a)^(1/2)*x)
```

3.151. $\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.151.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.69

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{15\,(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(384\,b^5fx^9+48\right.$$
$$-\frac{15\,(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(384\,b^5fx^9+48\,(10\,b^5e-9\,ab^4f)x^7$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(b)
*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(384*b^5*f*x^9 + 48*(
10*b^5*e - 9*a*b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 +
 10*(96*b^5*c - 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b
^4*c - 80*a^2*b^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6,
-1/3840*(15*(96*a^2*b^3*c - 80*a^3*b^2*d + 70*a^4*b*e - 63*a^5*f)*sqrt(-b)
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (384*b^5*f*x^9 + 48*(10*b^5*e - 9*a*
b^4*f)*x^7 + 8*(80*b^5*d - 70*a*b^4*e + 63*a^2*b^3*f)*x^5 + 10*(96*b^5*c -
 80*a*b^4*d + 70*a^2*b^3*e - 63*a^3*b^2*f)*x^3 - 15*(96*a*b^4*c - 80*a^2*b
^3*d + 70*a^3*b^2*e - 63*a^4*b*f)*x)*sqrt(b*x^2 + a))/b^6]
```

3.151.6 Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.99

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \frac{3a^2\left(-\frac{5a\left(-\frac{7a\left(-\frac{9af}{10b}+e\right)}{8b}+d\right)}{6b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{8b^2}+\sqrt{a+bx^2}\left(-\frac{3ax\left(-\frac{5a\left(-\frac{7a\left(-\frac{9af}{10b}+e\right)}{8b}+d\right)}{6b}+c\right)}{8b^2}\right. \\ \frac{\frac{cx^5}{5}+\frac{dx^7}{7}+\frac{ex^9}{9}+\frac{fx^{11}}{11}}{\sqrt{a}} \end{cases}$$

input
```
integrate(x**4*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((3*a**2*(-5*a*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)*Pi
ecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*lo
g(x)/sqrt(b*x**2), True))/(8*b**2) + sqrt(a + b*x**2)*(-3*a*x*(-5*a*(-7*a*
(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)/(8*b**2) + f*x**9/(10*b) + x**7*
(-9*a*f/(10*b) + e)/(8*b) + x**5*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b
) + x**3*(-5*a*(-7*a*(-9*a*f/(10*b) + e)/(8*b) + d)/(6*b) + c)/(4*b)), Ne(
b, 0)), ((c*x**5/5 + d*x**7/7 + e*x**9/9 + f*x**11/11)/sqrt(a), True))
```

3.151.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.38

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^9}{10\,b} + \frac{\sqrt{bx^2+a}ex^7}{8\,b} - \frac{9\sqrt{bx^2+a}afx^7}{80\,b^2} + \frac{\sqrt{bx^2+a}dx^5}{6\,b} - \frac{7\sqrt{bx^2+a}aex^5}{48\,b^2} + \frac{21\sqrt{bx^2+a}a^2fx^5}{160\,b^3} + \frac{\sqrt{bx^2+a}cx^3}{4\,b} - \frac{5\sqrt{bx^2+a}adx^3}{24\,b^2} + \frac{35\sqrt{bx^2+a}a^2ex^3}{192\,b^3} - \frac{21\sqrt{bx^2+a}a^3fx^3}{128\,b^4} - \frac{3\sqrt{bx^2+a}acx}{8\,b^2} + \frac{5\sqrt{bx^2+a}a^2dx}{16\,b^3} - \frac{35\sqrt{bx^2+a}a^3ex}{128\,b^4} + \frac{63\sqrt{bx^2+a}a^4fx}{256\,b^5} + \frac{3\,a^2c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^3d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{35\,a^4e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}} - \frac{63\,a^5f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{256\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/10*sqrt(b*x^2 + a)*f*x^9/b + 1/8*sqrt(b*x^2 + a)*e*x^7/b - 9/80*sqrt(b*x
^2 + a)*a*f*x^7/b^2 + 1/6*sqrt(b*x^2 + a)*d*x^5/b - 7/48*sqrt(b*x^2 + a)*a
*e*x^5/b^2 + 21/160*sqrt(b*x^2 + a)*a^2*f*x^5/b^3 + 1/4*sqrt(b*x^2 + a)*c*
x^3/b - 5/24*sqrt(b*x^2 + a)*a*d*x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*e*x^
3/b^3 - 21/128*sqrt(b*x^2 + a)*a^3*f*x^3/b^4 - 3/8*sqrt(b*x^2 + a)*a*c*x/b
^2 + 5/16*sqrt(b*x^2 + a)*a^2*d*x/b^3 - 35/128*sqrt(b*x^2 + a)*a^3*e*x/b^4
 + 63/256*sqrt(b*x^2 + a)*a^4*f*x/b^5 + 3/8*a^2*c*arcsinh(b*x/sqrt(a*b))/b
^(5/2) - 5/16*a^3*d*arcsinh(b*x/sqrt(a*b))/b^(7/2) + 35/128*a^4*e*arcsinh(
b*x/sqrt(a*b))/b^(9/2) - 63/256*a^5*f*arcsinh(b*x/sqrt(a*b))/b^(11/2)
```

3.151.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.89

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{1}{3840}\left(2\left(4\left(6\left(\frac{8\,fx^2}{b}+\frac{10\,b^8e-9\,ab^7f}{b^9}\right)x^2+\frac{80\,b^8d-70\,ab^7e+63\,a^2b^6f}{b^9}\right)x^2+\frac{5\,(96\,b^8c-80\,ab^7d+7}{b^9}\right.\right.$$
$$-\frac{(96\,a^2b^3c-80\,a^3b^2d+70\,a^4be-63\,a^5f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{256\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(2*(4*(6*(8*f*x^2/b + (10*b^8*e - 9*a*b^7*f)/b^9)*x^2 + (80*b^8*d -
 70*a*b^7*e + 63*a^2*b^6*f)/b^9)*x^2 + 5*(96*b^8*c - 80*a*b^7*d + 70*a^2*b
^6*e - 63*a^3*b^5*f)/b^9)*x^2 - 15*(96*a*b^7*c - 80*a^2*b^6*d + 70*a^3*b^5
*e - 63*a^4*b^4*f)/b^9)*sqrt(b*x^2 + a)*x - 1/256*(96*a^2*b^3*c - 80*a^3*b
^2*d + 70*a^4*b*e - 63*a^5*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11
/2)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \int \frac{x^4\,(f\,x^6+e\,x^4+d\,x^2+c)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^4*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.152 $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.152.1 Optimal result

Integrand size = 32, antiderivative size = 194

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{(64b^3c-48ab^2d+40a^2be-35a^3f)\,x\sqrt{a+bx^2}}{128b^4} + \frac{(48b^2d-40abe+35a^2f)\,x^3\sqrt{a+bx^2}}{192b^3} + \frac{(8be-7af)x^5\sqrt{a+bx^2}}{48b^2} + \frac{fx^7\sqrt{a+bx^2}}{8b} - \frac{a(64b^3c-48ab^2d+40a^2be-35a^3f)\,\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{128b^{9/2}}$$

output
```
-1/128*a*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*arctanh(x*b^(1/2)/(b*x
^2+a)^(1/2))/b^(9/2)+1/128*(-35*a^3*f+40*a^2*b*e-48*a*b^2*d+64*b^3*c)*x*(b
*x^2+a)^(1/2)/b^4+1/192*(35*a^2*f-40*a*b*e+48*b^2*d)*x^3*(b*x^2+a)^(1/2)/b
^3+1/48*(-7*a*f+8*b*e)*x^5*(b*x^2+a)^(1/2)/b^2+1/8*f*x^7*(b*x^2+a)^(1/2)/b
```

3.152.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{b}x\sqrt{a+bx^2}(-105a^3f+10a^2b(12e+7fx^2)-8ab^2(18d+10ex^2+7fx^4)+16b^3(12c+6dx^2+4ex^4+3}{384b^{9/2}}$$

input
```
Integrate[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[b]*x*Sqrt[a + b*x^2]*(-105*a^3*f + 10*a^2*b*(12*e + 7*f*x^2) - 8*a*b
^2*(18*d + 10*e*x^2 + 7*f*x^4) + 16*b^3*(12*c + 6*d*x^2 + 4*e*x^4 + 3*f*x^
6)) + 6*a*(-64*b^3*c + 48*a*b^2*d - 40*a^2*b*e + 35*a^3*f)*ArcTanh[(Sqrt[b
]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(384*b^(9/2))
```

3.152.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {2340, 1590, 363, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2340}$$
$$\frac{\int \frac{x^2\left((8be-7af)x^4+8bdx^2+8bc\right)}{\sqrt{bx^2+a}}dx}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$
$$\Big\downarrow \text{1590}$$
$$\frac{\frac{\int \frac{x^2\left(48cb^2+\left(35fa^2-40bea+48b^2d\right)x^2\right)}{\sqrt{bx^2+a}}dx}{6b}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{6b}}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$
$$\Big\downarrow \text{363}$$
$$\frac{\frac{\frac{3\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)\int \frac{x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{6b}}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

$\Big\downarrow$ 262

$$\frac{\frac{\frac{3\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{6b}}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{3\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{6b}}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{x^3\sqrt{a+bx^2}\left(35a^2f-40abe+48b^2d\right)}{4b}+\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)\left(-35a^3f+40a^2be-48ab^2d+64b^3c\right)}{4b}}{6b}+\frac{x^5\sqrt{a+bx^2}(8be-7af)}{6b}}{8b}+\frac{fx^7\sqrt{a+bx^2}}{8b}$$

input
```
Int[(x^2*(c + d*x^2 + e*x^4 + f*x^6))/Sqrt[a + b*x^2],x]
```

output
```
(f*x^7*Sqrt[a + b*x^2])/(8*b) + (((8*b*e - 7*a*f)*x^5*Sqrt[a + b*x^2])/(6*
b) + (((48*b^2*d - 40*a*b*e + 35*a^2*f)*x^3*Sqrt[a + b*x^2])/(4*b) + (3*(6
4*b^3*c - 48*a*b^2*d + 40*a^2*b*e - 35*a^3*f)*((x*Sqrt[a + b*x^2])/(2*b) -
 (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))/(6*b))/(8*b
)
```

3.152. $\int\frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.152.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1590
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[c^p*(f*x)^(m + 4*p - 1)*((d + e*x^2)^
(q + 1)/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1))), x] + Simp[1/(e*(m + 4*p + 2*q
 + 1))   Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a +
b*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x],
x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p,
0] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

rule 2340
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[f*(c*x)^(m + q - 1
)*((a + b*x^2)^(p + 1)/(b*c^(q - 1)*(m + q + 2*p + 1))), x] + Simp[1/(b*(m
+ q + 2*p + 1))   Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)
*Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /;
GtQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ
[Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

3.152. $\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$

3.152.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{\frac{35a\left(f\,a^3-\frac{8}{7}a^2be+\frac{48}{35}a\,b^2d-\frac{64}{35}b^3c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)}{128}-\frac{35\left(\left(-\frac{16}{35}f\,x^6-\frac{64}{105}e\,x^4-\frac{32}{35}d\,x^2-\frac{64}{35}c\right)b^{\frac{7}{2}}+\left(\left(\frac{8}{15}f\,x^4+\frac{16}{21}e\,x^2+\frac{48}{35}d\right)b^{\frac{5}{2}}+\cdots\right.\right.}{128}}{b^{\frac{9}{2}}}$
risch	$-\frac{x\left(-48f\,x^6b^3+56a\,b^2f\,x^4-64b^3e\,x^4-70a^2bf\,x^2+80a\,b^2e\,x^2-96b^3d\,x^2+105f\,a^3-120a^2be+144a\,b^2d-192b^3c\right)\sqrt{b\,x^2+a}}{384b^4}+\cdots$
default	$f\left(\frac{x^7\sqrt{b\,x^2+a}}{8b}-\frac{7a\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\frac{5a\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)}{8b}\right)+e\left(\frac{x^5\sqrt{b\,x^2+a}}{6b}-\cdots\right.$

input
```
int(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
35/128*(a*(f*a^3-8/7*a^2*b*e+48/35*a*b^2*d-64/35*b^3*c)*arctanh((b*x^2+a)^
(1/2)/x/b^(1/2))-((-16/35*f*x^6-64/105*e*x^4-32/35*d*x^2-64/35*c)*b^(7/2)+
((8/15*f*x^4+16/21*e*x^2+48/35*d)*b^(5/2)+a*((-2/3*f*x^2-8/7*e)*b^(3/2)+a*
f*b^(1/2)))*a)*x*(b*x^2+a)^(1/2))/b^(9/2)
```

3.152.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.70

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx$$
$$=\left[-\frac{3\left(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f\right)\sqrt{b}\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\left(48\,b^4fx^7+8\left(8\,b\cdots\right.\right.}{7\cdots}\right.$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/768*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(b)*log
(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(48*b^4*f*x^7 + 8*(8*b^4*
e - 7*a*b^3*f)*x^5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*
b^4*c - 48*a*b^3*d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5, 1
/384*(3*(64*a*b^3*c - 48*a^2*b^2*d + 40*a^3*b*e - 35*a^4*f)*sqrt(-b)*arcta
n(sqrt(-b)*x/sqrt(b*x^2 + a)) + (48*b^4*f*x^7 + 8*(8*b^4*e - 7*a*b^3*f)*x^
5 + 2*(48*b^4*d - 40*a*b^3*e + 35*a^2*b^2*f)*x^3 + 3*(64*b^4*c - 48*a*b^3*
d + 40*a^2*b^2*e - 35*a^3*b*f)*x)*sqrt(b*x^2 + a))/b^5]
```

3.152.6 Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.03

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{a\left(-\frac{3a\left(-\frac{5a\left(-\frac{7af}{8b}+e\right)}{6b}+d\right)}{4b}+c\right)\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b} + \sqrt{a+bx^2}\left(\frac{fx^7}{8b}+\frac{x^5\left(-\frac{7af}{8b}+e\right)}{6b}+\frac{x^3\left(-\frac{5a(\cdots}{\cdots}\right.}{\cdots}\right. \\ \frac{\frac{cx^3}{3}+\frac{dx^5}{5}+\frac{ex^7}{7}+\frac{fx^9}{9}}{\sqrt{a}} & \end{cases}$$

input
```
integrate(x**2*(f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-a*(-3*a*(-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + c)*Piecewi
se((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/
sqrt(b*x**2), True))/(2*b) + sqrt(a + b*x**2)*(f*x**7/(8*b) + x**5*(-7*a*f
/(8*b) + e)/(6*b) + x**3*(-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + x*(-3
*a*(-5*a*(-7*a*f/(8*b) + e)/(6*b) + d)/(4*b) + c)/(2*b)), Ne(b, 0)), ((c*x
**3/3 + d*x**5/5 + e*x**7/7 + f*x**9/9)/sqrt(a), True))
```

3.152.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.31

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^7}{8\,b} + \frac{\sqrt{bx^2+a}ex^5}{6\,b} - \frac{7\,\sqrt{bx^2+a}afx^5}{48\,b^2} + \frac{\sqrt{bx^2+a}dx^3}{4\,b} - \frac{5\,\sqrt{bx^2+a}aex^3}{24\,b^2} + \frac{35\,\sqrt{bx^2+a}a^2fx^3}{192\,b^3} + \frac{\sqrt{bx^2+a}cx}{2\,b} - \frac{3\,\sqrt{bx^2+a}adx}{8\,b^2} + \frac{5\,\sqrt{bx^2+a}a^2ex}{16\,b^3} - \frac{35\,\sqrt{bx^2+a}a^3fx}{128\,b^4} - \frac{ac\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}} + \frac{3\,a^2d\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}} - \frac{5\,a^3e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}} + \frac{35\,a^4f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{128\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/8*sqrt(b*x^2 + a)*f*x^7/b + 1/6*sqrt(b*x^2 + a)*e*x^5/b - 7/48*sqrt(b*x^
2 + a)*a*f*x^5/b^2 + 1/4*sqrt(b*x^2 + a)*d*x^3/b - 5/24*sqrt(b*x^2 + a)*a*
e*x^3/b^2 + 35/192*sqrt(b*x^2 + a)*a^2*f*x^3/b^3 + 1/2*sqrt(b*x^2 + a)*c*x
/b - 3/8*sqrt(b*x^2 + a)*a*d*x/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*e*x/b^3 - 35
/128*sqrt(b*x^2 + a)*a^3*f*x/b^4 - 1/2*a*c*arcsinh(b*x/sqrt(a*b))/b^(3/2)
+ 3/8*a^2*d*arcsinh(b*x/sqrt(a*b))/b^(5/2) - 5/16*a^3*e*arcsinh(b*x/sqrt(a
*b))/b^(7/2) + 35/128*a^4*f*arcsinh(b*x/sqrt(a*b))/b^(9/2)
```

3.152.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.88

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx = \frac{1}{384}\left(2\left(4\left(\frac{6\,fx^2}{b} + \frac{8\,b^6e-7\,ab^5f}{b^7}\right)x^2 + \frac{48\,b^6d-40\,ab^5e+35\,a^2b^4f}{b^7}\right)x^2 + \frac{3\,(64\,b^6c-48\,ab^5d+40\,a^2b}{b^7} + \frac{(64\,ab^3c-48\,a^2b^2d+40\,a^3be-35\,a^4f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{128\,b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/384*(2*(4*(6*f*x^2/b + (8*b^6*e - 7*a*b^5*f)/b^7)*x^2 + (48*b^6*d - 40*a
*b^5*e + 35*a^2*b^4*f)/b^7)*x^2 + 3*(64*b^6*c - 48*a*b^5*d + 40*a^2*b^4*e
- 35*a^3*b^3*f)/b^7)*sqrt(b*x^2 + a)*x + 1/128*(64*a*b^3*c - 48*a^2*b^2*d
+ 40*a^3*b*e - 35*a^4*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}\,dx=\int \frac{x^2\,(f\,x^6+e\,x^4+d\,x^2+c)}{\sqrt{b\,x^2+a}}\,dx$$

input
```
int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2),x)
```

output
```
int((x^2*(c + d*x^2 + e*x^4 + f*x^6))/(a + b*x^2)^(1/2), x)
```

3.153 $\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} dx$

3.153.1 Optimal result

Integrand size = 29, antiderivative size = 145

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} dx = \frac{(8b^2d-6abe+5a^2f)\,x\sqrt{a+bx^2}}{16b^3} + \frac{(6be-5af)x^3\sqrt{a+bx^2}}{24b^2} + \frac{fx^5\sqrt{a+bx^2}}{6b} + \frac{(16b^3c-8ab^2d+6a^2be-5a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{7/2}}$$

output
```
1/16*(-5*a^3*f+6*a^2*b*e-8*a*b^2*d+16*b^3*c)*arctanh(x*b^(1/2)/(b*x^2+a)^(
1/2))/b^(7/2)+1/16*(5*a^2*f-6*a*b*e+8*b^2*d)*x*(b*x^2+a)^(1/2)/b^3+1/24*(-
5*a*f+6*b*e)*x^3*(b*x^2+a)^(1/2)/b^2+1/6*f*x^5*(b*x^2+a)^(1/2)/b
```

3.153.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}} dx = \frac{x\sqrt{a+bx^2}(24b^2d-18abe+15a^2f+12b^2ex^2-10abfx^2+8b^2fx^4)}{48b^3} + \frac{(16b^3c-8ab^2d+6a^2be-5a^3f)\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{7/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

output
```
(x*Sqrt[a + b*x^2]*(24*b^2*d - 18*a*b*e + 15*a^2*f + 12*b^2*e*x^2 - 10*a*b
*f*x^2 + 8*b^2*f*x^4))/(48*b^3) + ((16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a
^3*f)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/(8*b^(7/2))
```

3.153.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2346, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2346}$$

$$\frac{\int \frac{(6be-5af)x^4+6bdx^2+6bc}{\sqrt{bx^2+a}}dx}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{3\left(8cb^2+\left(5fa^2-6bea+8b^2d\right)x^2\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{4b}}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3\int \frac{8cb^2+\left(5fa^2-6bea+8b^2d\right)x^2}{\sqrt{bx^2+a}}dx}{4b}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{4b}}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{3\left(\frac{\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{4b}}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \text{224}$$

$$\frac{\frac{3\left(\frac{\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{2b}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{4b}}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{3\left(\frac{x\sqrt{a+bx^2}\left(5a^2f-6abe+8b^2d\right)}{2b}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)\left(-5a^3f+6a^2be-8ab^2d+16b^3c\right)}{2b^{3/2}}\right)}{4b}+\frac{x^3\sqrt{a+bx^2}(6be-5af)}{4b}}{6b}+\frac{fx^5\sqrt{a+bx^2}}{6b}$$

input

```
Int[(c + d*x^2 + e*x^4 + f*x^6)/Sqrt[a + b*x^2],x]
```

output

```
(f*x^5*Sqrt[a + b*x^2])/(6*b) + (((6*b*e - 5*a*f)*x^3*Sqrt[a + b*x^2])/(4*
b) + (3*(((8*b^2*d - 6*a*b*e + 5*a^2*f)*x*Sqrt[a + b*x^2])/(2*b) + ((16*b^
3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]
])/(2*b^(3/2))))/(4*b))/(6*b)
```

3.153.3.1 Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2346
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x^2)^(p + 1)/(b*(
q + 2*p + 1))), x] + Simp[1/(b*(q + 2*p + 1))   Int[(a + b*x^2)^p*ExpandToS
um[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x],
x], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

3.153.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{5\left(\left(f a^{3}-\frac{6}{5}a^{2}be+\frac{8}{5}a\,b^{2}d-\frac{16}{5}b^{3}c\right)\operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\left(\frac{4\left(\frac{2}{3}f\,x^{4}+e\,x^{2}+2d\right)b^{\frac{5}{2}}}{5}+\left(2\left(-\frac{f\,x^{2}}{3}-\frac{3e}{5}\right)b^{\frac{3}{2}}+af\sqrt{b}\right)a\right)x\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{7}{2}}}$
risch	$\frac{x\left(8b^{2}f\,x^{4}-10abf\,x^{2}+12b^{2}e\,x^{2}+15a^{2}f-18aeb+24b^{2}d\right)\sqrt{b\,x^{2}+a}}{48b^{3}}-\frac{\left(5f\,a^{3}-6a^{2}be+8a\,b^{2}d-16b^{3}c\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{16b^{\frac{7}{2}}}$
default	$\frac{c\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+f\left(\frac{x^{5}\sqrt{b\,x^{2}+a}}{6b}-\frac{5a\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^{2}+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)}{6b}\right)+e\left(\frac{x^{3}\sqrt{b\,x^{2}+a}}{4b} \cdots\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/16*((f*a^3-6/5*a^2*b*e+8/5*a*b^2*d-16/5*b^3*c)*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))-(4/5*(2/3*f*x^4+e*x^2+2*d)*b^(5/2)+(2*(-1/3*f*x^2-3/5*e)*b^(3/2
)+a*f*b^(1/2))*a)*x*(b*x^2+a)^(1/2))/b^(7/2)
```

3.153.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.72

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\sqrt{b}\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(8\,b^3fx^5+2\,(6\,b^3e-5\,}{96\,b^4}\right.$$
$$-\frac{3\,(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(8\,b^3fx^5+2\,(6\,b^3e-5\,ab^2f)x^3+3\,(8\,b^3d-}{48\,b^4}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/96*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(b)*log(-2*b*x^
2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*(8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b
^2*f)*x^3 + 3*(8*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4, -
1/48*(3*(16*b^3*c - 8*a*b^2*d + 6*a^2*b*e - 5*a^3*f)*sqrt(-b)*arctan(sqrt(
-b)*x/sqrt(b*x^2 + a)) - (8*b^3*f*x^5 + 2*(6*b^3*e - 5*a*b^2*f)*x^3 + 3*(8
*b^3*d - 6*a*b^2*e + 5*a^2*b*f)*x)*sqrt(b*x^2 + a))/b^4]
```

3.153.6 Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.07

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx$$
$$= \begin{cases} \sqrt{a+bx^2}\left(\frac{fx^5}{6b}+\frac{x^3\left(-\frac{5af}{6b}+e\right)}{4b}+\frac{x\left(-\frac{3a\left(-\frac{5af}{6b}+e\right)}{4b}+d\right)}{2b}\right)+\left(-\frac{a\left(-\frac{3a\left(-\frac{5af}{6b}+e\right)}{4b}+d\right)}{2b}+c\right)\left(\left\{\begin{matrix}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} \\ \frac{x\log(x)}{\sqrt{bx^2}}\end{matrix}\right.\right. \\ \frac{cx+\frac{dx^3}{3}+\frac{ex^5}{5}+\frac{fx^7}{7}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((sqrt(a + b*x**2)*(f*x**5/(6*b) + x**3*(-5*a*f/(6*b) + e)/(4*b)
+ x*(-3*a*(-5*a*f/(6*b) + e)/(4*b) + d)/(2*b)) + (-a*(-3*a*(-5*a*f/(6*b) +
 e)/(4*b) + d)/(2*b) + c)*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*
x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2), True)), Ne(b, 0)), ((c*x +
d*x**3/3 + e*x**5/5 + f*x**7/7)/sqrt(a), True))
```

3.153.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.20

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^5}{6\,b}+\frac{\sqrt{bx^2+a}ex^3}{4\,b}-\frac{5\,\sqrt{bx^2+a}afx^3}{24\,b^2}+\frac{\sqrt{bx^2+a}dx}{2\,b} - \frac{3\,\sqrt{bx^2+a}aex}{8\,b^2}+\frac{5\,\sqrt{bx^2+a}a^2fx}{16\,b^3}+\frac{c\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{ad\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{5\,a^3f\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(b*x^2 + a)*f*x^5/b + 1/4*sqrt(b*x^2 + a)*e*x^3/b - 5/24*sqrt(b*x^
2 + a)*a*f*x^3/b^2 + 1/2*sqrt(b*x^2 + a)*d*x/b - 3/8*sqrt(b*x^2 + a)*a*e*x
/b^2 + 5/16*sqrt(b*x^2 + a)*a^2*f*x/b^3 + c*arcsinh(b*x/sqrt(a*b))/sqrt(b)
 - 1/2*a*d*arcsinh(b*x/sqrt(a*b))/b^(3/2) + 3/8*a^2*e*arcsinh(b*x/sqrt(a*b
))/b^(5/2) - 5/16*a^3*f*arcsinh(b*x/sqrt(a*b))/b^(7/2)
```

3.153.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{\sqrt{a+bx^2}}\,dx = \frac{1}{48}\left(2\left(\frac{4\,fx^2}{b}+\frac{6\,b^4e-5\,ab^3f}{b^5}\right)x^2+\frac{3\,(8\,b^4d-6\,ab^3e+5\,a^2b^2f)}{b^5}\right)\sqrt{bx^2+a}x - \frac{(16\,b^3c-8\,ab^2d+6\,a^2be-5\,a^3f)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{7}{2}}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/48*(2*(4*f*x^2/b + (6*b^4*e - 5*a*b^3*f)/b^5)*x^2 + 3*(8*b^4*d - 6*a*b^3
*e + 5*a^2*b^2*f)/b^5)*sqrt(b*x^2 + a)*x - 1/16*(16*b^3*c - 8*a*b^2*d + 6*
a^2*b*e - 5*a^3*f)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(7/2)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c + dx^2 + ex^4 + fx^6}{\sqrt{a + bx^2}} \, dx = \int \frac{f\,x^6 + e\,x^4 + d\,x^2 + c}{\sqrt{b\,x^2 + a}} \, dx$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2),x)
```

output
```
int((c + d*x^2 + e*x^4 + f*x^6)/(a + b*x^2)^(1/2), x)
```

3.154 $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

3.154.1 Optimal result

Integrand size = 32, antiderivative size = 117

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=-\frac{c\sqrt{a+bx^2}}{ax}+\frac{(4be-3af)x\sqrt{a+bx^2}}{8b^2}+\frac{fx^3\sqrt{a+bx^2}}{4b}+\frac{(8b^2d-4abe+3a^2f)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{5/2}}$$

output
```
1/8*(3*a^2*f-4*a*b*e+8*b^2*d)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(5/2)-c
*(b*x^2+a)^(1/2)/a/x+1/8*(-3*a*f+4*b*e)*x*(b*x^2+a)^(1/2)/b^2+1/4*f*x^3*(b
*x^2+a)^(1/2)/b
```

3.154.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=\frac{\sqrt{a+bx^2}(-8b^2c+4abex^2-3a^2fx^2+2abfx^4)}{8ab^2x}+\frac{(-8b^2d+4abe-3a^2f)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{8b^{5/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-8*b^2*c + 4*a*b*e*x^2 - 3*a^2*f*x^2 + 2*a*b*f*x^4))/(8*
a*b^2*x) + ((-8*b^2*d + 4*a*b*e - 3*a^2*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x
^2]])/(8*b^(5/2))
```

3.154.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.13, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2338, 9, 25, 1473, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int -\frac{afx^5+aex^3+adx}{x\sqrt{bx^2+a}}dx}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int -\frac{afx^4+aex^2+ad}{\sqrt{bx^2+a}}dx}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{afx^4+aex^2+ad}{\sqrt{bx^2+a}}dx}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{1473}$$

$$\frac{\frac{\int \frac{a\left((4be-3af)x^2+4bd\right)}{\sqrt{bx^2+a}}dx}{4b}+\frac{afx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\int \frac{(4be-3af)x^2+4bd}{\sqrt{bx^2+a}}dx}{4b}+\frac{afx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{a\left(\frac{\left(3a^2f-4abe+8b^2d\right)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{x\sqrt{a+bx^2}(4be-3af)}{2b}\right)}{4b}+\frac{afx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow 224$$

$$\frac{\frac{a\left(\frac{(3a^2f-4abe+8b^2d)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{x\sqrt{a+bx^2}(4be-3af)}{2b}\right)}{4b}+\frac{afx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

$$\Big\downarrow 219$$

$$\frac{\frac{a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(3a^2f-4abe+8b^2d)}{2b^{3/2}}+\frac{x\sqrt{a+bx^2}(4be-3af)}{2b}\right)}{4b}+\frac{afx^3\sqrt{a+bx^2}}{4b}}{a}-\frac{c\sqrt{a+bx^2}}{ax}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^2*Sqrt[a + b*x^2]),x]
```

output
```
-((c*Sqrt[a + b*x^2])/(a*x)) + ((a*f*x^3*Sqrt[a + b*x^2])/(4*b) + (a*(((4*
b*e - 3*a*f)*x*Sqrt[a + b*x^2])/(2*b) + ((8*b^2*d - 4*a*b*e + 3*a^2*f)*Arc
Tanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b))/a
```

3.154.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.154. $\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1473
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[c^p*x^(4*p - 1)*((d + e*x^2)^(q + 1)/(e*(4*p + 2*q + 1)))
, x] + Simp[1/(e*(4*p + 2*q + 1))   Int[(d + e*x^2)^q*ExpandToSum[e*(4*p +
2*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
 2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] &&  !LtQ[q, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.154.4 Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-2abf\,x^4+3a^2f\,x^2-4abe\,x^2+8b^2c\right)}{8b^2ax}+\frac{\left(3a^2f-4aeb+8b^2d\right)\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{8b^{\frac{5}{2}}}$
pseudoelliptic	$\frac{3b^2x\left(a^2f-\frac{4}{3}aeb+\frac{8}{3}b^2d\right)a\,\operatorname{arctanh}\left(\frac{\sqrt{b\,x^2+a}}{x\sqrt{b}}\right)-8\sqrt{b\,x^2+a}\left(b^2c-\frac{x^2a\left(\frac{f\,x^2}{2}+e\right)b}{2}+\frac{3a^2f\,x^2}{8}\right)b^{\frac{5}{2}}}{8x\,b^{\frac{9}{2}}\,a}$
default	$\frac{d\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}+f\left(\frac{x^3\sqrt{b\,x^2+a}}{4b}-\frac{3a\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{2b^{\frac{3}{2}}}\right)}{4b}\right)+e\left(\frac{x\sqrt{b\,x^2+a}}{2b}-\frac{a\ln\left(x\sqrt{b}+\right.}{2b^{\frac{3}{2}}}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)^(1/2)*(-2*a*b*f*x^4+3*a^2*f*x^2-4*a*b*e*x^2+8*b^2*c)/b^2/a/
x+1/8*(3*a^2*f-4*a*b*e+8*b^2*d)/b^(5/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.154.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.85

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$$

$$= \left[\frac{(8\,ab^2d-4\,a^2be+3\,a^3f)\sqrt{b}x\log\left(-2\,bx^2-2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)+2\,(2\,ab^2fx^4-8\,b^3c+(4\,ab^2e-3\,a^2b}{16\,ab^3x}\right.$$

$$\left.-\frac{(8\,ab^2d-4\,a^2be+3\,a^3f)\sqrt{-b}x\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(2\,ab^2fx^4-8\,b^3c+(4\,ab^2e-3\,a^2bf)x^2)\sqrt{bx^2+a}}{8\,ab^3x}\right]$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/16*((8*a*b^2*d - 4*a^2*b*e + 3*a^3*f)*sqrt(b)*x*log(-2*b*x^2 - 2*sqrt(b
*x^2 + a)*sqrt(b)*x - a) + 2*(2*a*b^2*f*x^4 - 8*b^3*c + (4*a*b^2*e - 3*a^2
*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x), -1/8*((8*a*b^2*d - 4*a^2*b*e + 3*a^
3*f)*sqrt(-b)*x*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (2*a*b^2*f*x^4 - 8*b^
3*c + (4*a*b^2*e - 3*a^2*b*f)*x^2)*sqrt(b*x^2 + a))/(a*b^3*x)]
```

3.154.6 Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.10

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx$$
$$= d\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \wedge b\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0 \\ \frac{x}{\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$+ e\left(\begin{cases} -\frac{a\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{2b} + \frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0 \\ \frac{x^3}{3\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$+ f\left(\begin{cases} \frac{3a^2\left(\begin{cases} \frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0 \\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise} \end{cases}\right)}{8b^2} - \frac{3ax\sqrt{a+bx^2}}{8b^2} + \frac{x^3\sqrt{a+bx^2}}{4b} & \text{for } b\neq 0 \\ \frac{x^5}{5\sqrt{a}} & \text{otherwise} \end{cases}\right)$$
$$- \frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{a}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**2/(b*x**2+a)**(1/2),x)
```

output
```
d*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & N
e(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + e*Piecew
ise((-a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b
, 0)), (x**3/(3*sqrt(a)), True)) + f*Piecewise((3*a**2*Piecewise((log(2*sq
rt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0)), (x*log(x)/sqrt(b*x**2)
, True))/(8*b**2) - 3*a*x*sqrt(a + b*x**2)/(8*b**2) + x**3*sqrt(a + b*x**2
)/(4*b), Ne(b, 0)), (x**5/(5*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2)
+ 1)/a
```

3.154.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.01

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=\frac{\sqrt{bx^2+a}fx^3}{4\,b}+\frac{\sqrt{bx^2+a}ex}{2\,b}-\frac{3\,\sqrt{bx^2+a}afx}{8\,b^2}+\frac{d\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{ae\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{3\,a^2f\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{8\,b^{\frac{5}{2}}}-\frac{\sqrt{bx^2+a}c}{ax}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(b*x^2 + a)*f*x^3/b + 1/2*sqrt(b*x^2 + a)*e*x/b - 3/8*sqrt(b*x^2 +
 a)*a*f*x/b^2 + d*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*e*arcsinh(b*x/sqr
t(a*b))/b^(3/2) + 3/8*a^2*f*arcsinh(b*x/sqrt(a*b))/b^(5/2) - sqrt(b*x^2 +
a)*c/(a*x)
```

3.154.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.97

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=\frac{1}{8}\,\sqrt{bx^2+a}\left(\frac{2\,fx^2}{b}+\frac{4\,b^2e-3\,abf}{b^3}\right)x+\frac{2\,\sqrt{b}c}{\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a}-\frac{\left(8\,b^2d-4\,abe+3\,a^2f\right)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{16\,b^{\frac{5}{2}}}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^2/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/8*sqrt(b*x^2 + a)*(2*f*x^2/b + (4*b^2*e - 3*a*b*f)/b^3)*x + 2*sqrt(b)*c/
((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a) - 1/16*(8*b^2*d - 4*a*b*e + 3*a^2*f)
*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/b^(5/2)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^2\sqrt{a+bx^2}}\,dx=\int \frac{f\,x^6+e\,x^4+d\,x^2+c}{x^2\,\sqrt{b\,x^2+a}}\,dx$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)),x)
```

output
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^2*(a + b*x^2)^(1/2)), x)
```

3.155 $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

3.155.1 Optimal result

Integrand size = 32, antiderivative size = 110

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{3ax^3}+\frac{(2bc-3ad)\sqrt{a+bx^2}}{3a^2x}+\frac{fx\sqrt{a+bx^2}}{2b}+\frac{(2be-af)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}$$

output
```
1/2*(-a*f+2*b*e)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(3/2)-1/3*c*(b*x^2+a
)^(1/2)/a/x^3+1/3*(-3*a*d+2*b*c)*(b*x^2+a)^(1/2)/a^2/x+1/2*f*x*(b*x^2+a)^(
1/2)/b
```

3.155.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-2abc+4b^2cx^2-6abdx^2+3a^2fx^4)}{6a^2bx^3}+\frac{(-2be+af)\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{2b^{3/2}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(-2*a*b*c + 4*b^2*c*x^2 - 6*a*b*d*x^2 + 3*a^2*f*x^4))/(6*
a^2*b*x^3) + ((-2*b*e + a*f)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(2*b^(3/
2))
```

3.155.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2338, 9, 1588, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{-3afx^5-3aex^3+(2bc-3ad)x}{x^3\sqrt{bx^2+a}}dx}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int \frac{-3afx^4-3aex^2+2bc-3ad}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{3a^2(fx^2+e)}{\sqrt{bx^2+a}}dx}{a}-\frac{\sqrt{a+bx^2}(2bc-3ad)}{ax}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-3a\int \frac{fx^2+e}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}(2bc-3ad)}{ax}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{299}$$

$$-\frac{-3a\left(\frac{(2be-af)\int \frac{1}{\sqrt{bx^2+a}}dx}{2b}+\frac{fx\sqrt{a+bx^2}}{2b}\right)-\frac{\sqrt{a+bx^2}(2bc-3ad)}{ax}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-3a\left(\frac{(2be-af)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}+\frac{fx\sqrt{a+bx^2}}{2b}\right)-\frac{\sqrt{a+bx^2}(2bc-3ad)}{ax}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-3a\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2be-af)}{2b^{3/2}}+\frac{fx\sqrt{a+bx^2}}{2b}\right)-\frac{\sqrt{a+bx^2}(2bc-3ad)}{ax}}{3a}-\frac{c\sqrt{a+bx^2}}{3ax^3}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^4*Sqrt[a + b*x^2]),x]
```

output
```
-1/3*(c*Sqrt[a + b*x^2])/(a*x^3) - (-(((2*b*c - 3*a*d)*Sqrt[a + b*x^2])/(a
*x)) - 3*a*((f*x*Sqrt[a + b*x^2])/(2*b) + ((2*b*e - a*f)*ArcTanh[(Sqrt[b]*
x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(3*a)
```

3.155.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

3.155. $\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.155.4 Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.75

method	result
risch	$\frac{\sqrt{b\,x^{2}+a}\,\left(3 a^{2} f\,x^{4}-6 x^{2} a b d+4 b^{2} c\,x^{2}-2 a b c\right)}{6 b\,a^{2} x^{3}}-\frac{\left(a f-2 b e\right) \ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}$
pseudoelliptic	$-\frac{x^{3} a^{2} \left(a f-2 b e\right) \operatorname{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)-\left(-\frac{2 a \left(3 d\,x^{2}+c\right) b^{\frac{3}{2}}}{3}+x^{2} \left(\sqrt{b}\,a^{2} f\,x^{2}+\frac{4 b^{\frac{5}{2}} c}{3}\right)\right) \sqrt{b\,x^{2}+a}}{2 b^{\frac{3}{2}} x^{3} a^{2}}$
default	$\frac{e \ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+f\left(\frac{x\sqrt{b\,x^{2}+a}}{2 b}-\frac{a \ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{2 b^{\frac{3}{2}}}\right)+c\left(-\frac{\sqrt{b\,x^{2}+a}}{3 a\,x^{3}}+\frac{2 b\sqrt{b\,x^{2}+a}}{3 a^{2} x}\right)-\frac{d\sqrt{b\,x^{2}+a}}{a x}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*(b*x^2+a)^(1/2)*(3*a^2*f*x^4-6*a*b*d*x^2+4*b^2*c*x^2-2*a*b*c)/b/a^2/x^
3-1/2*(a*f-2*b*e)/b^(3/2)*ln(x*b^(1/2)+(b*x^2+a)^(1/2))
```

3.155.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.91

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= \left[-\frac{3\,(2\,a^2be-a^3f)\sqrt{b}x^3\log\left(-2\,bx^2+2\,\sqrt{bx^2+a}\sqrt{b}x-a\right)-2\,(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2}{12\,a^2b^2x^3}\right.$$
$$\left.-\frac{3\,(2\,a^2be-a^3f)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)-(3\,a^2bfx^4-2\,ab^2c+2\,(2\,b^3c-3\,ab^2d)x^2)\sqrt{bx^2+a}}{6\,a^2b^2x^3}\right]$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[-1/12*(3*(2*a^2*b*e - a^3*f)*sqrt(b)*x^3*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)
*sqrt(b)*x - a) - 2*(3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*b^3*c - 3*a*b^2*d)*x
^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3), -1/6*(3*(2*a^2*b*e - a^3*f)*sqrt(-b)*x
^3*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - (3*a^2*b*f*x^4 - 2*a*b^2*c + 2*(2*
b^3*c - 3*a*b^2*d)*x^2)*sqrt(b*x^2 + a))/(a^2*b^2*x^3)]
```

3.155.6 Sympy [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.78

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx$$
$$= e\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$+f\left(\begin{cases}-\frac{a\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{otherwise}\end{cases}\right)}{2b}+\frac{x\sqrt{a+bx^2}}{2b} & \text{for } b\neq 0\\ \frac{x^3}{3\sqrt{a}} & \text{otherwise}\end{cases}\right)$$
$$-\frac{\sqrt{b}c\sqrt{\frac{a}{bx^2}+1}}{3ax^2}-\frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{a}+\frac{2b^{\frac{3}{2}}c\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**4/(b*x**2+a)**(1/2),x)
```

output
```
e*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0) & N
e(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) + f*Piecew
ise((-a*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a,
0)), (x*log(x)/sqrt(b*x**2), True))/(2*b) + x*sqrt(a + b*x**2)/(2*b), Ne(b
, 0)), (x**3/(3*sqrt(a)), True)) - sqrt(b)*c*sqrt(a/(b*x**2) + 1)/(3*a*x**
2) - sqrt(b)*d*sqrt(a/(b*x**2) + 1)/a + 2*b**(3/2)*c*sqrt(a/(b*x**2) + 1)/
(3*a**2)
```

3.155.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx=\frac{\sqrt{bx^2+a}fx}{2\,b}+\frac{e\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{af\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{2\,b^{\frac{3}{2}}}+\frac{2\,\sqrt{bx^2+a}bc}{3\,a^2x}-\frac{\sqrt{bx^2+a}d}{ax}-\frac{\sqrt{bx^2+a}c}{3\,ax^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(b*x^2 + a)*f*x/b + e*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 1/2*a*f*arc
sinh(b*x/sqrt(a*b))/b^(3/2) + 2/3*sqrt(b*x^2 + a)*b*c/(a^2*x) - sqrt(b*x^2
 + a)*d/(a*x) - 1/3*sqrt(b*x^2 + a)*c/(a*x^3)
```

3.155.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.55

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx=\frac{\sqrt{bx^2+a}fx}{2\,b}-\frac{(2\,be-af)\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{4\,b^{\frac{3}{2}}}+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4\sqrt{b}d+6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2b^{\frac{3}{2}}c-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2a\sqrt{b}d-2\,ab^{\frac{3}{2}}c+3\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^4/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(b*x^2 + a)*f*x/b - 1/4*(2*b*e - a*f)*log((sqrt(b)*x - sqrt(b*x^2
+ a))^2)/b^(3/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2 + a))^4*sqrt(b)*d + 6*(s
qrt(b)*x - sqrt(b*x^2 + a))^2*b^(3/2)*c - 6*(sqrt(b)*x - sqrt(b*x^2 + a))^
2*a*sqrt(b)*d - 2*a*b^(3/2)*c + 3*a^2*sqrt(b)*d)/((sqrt(b)*x - sqrt(b*x^2
+ a))^2 - a)^3
```

3.155.9 Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{c+dx^2+ex^4+fx^6}{x^4\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{-f\,x^6-3\,e\,x^4+3\,d\,x^2+c}{3\,\sqrt{a}\,x^3} & \text{if } b=0 \\ \frac{e\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}}-\frac{d\,\sqrt{b\,x^2+a}}{a\,x}-\frac{a\,f\,\ln\left(2\,\sqrt{b}\,x+2\,\sqrt{b\,x^2+a}\right)}{2\,b^{3/2}}+\frac{f\,x\,\sqrt{b\,x^2+a}}{2\,b}-\frac{c\,\sqrt{b\,x^2+a}\,\left(a-2\,b\,x^2\right)}{3\,a^2\,x^3} & \text{if } b\neq 0 \end{cases}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^4*(a + b*x^2)^(1/2)),x)
```

output
```
piecewise(b == 0, -(c + 3*d*x^2 - 3*e*x^4 - f*x^6)/(3*a^(1/2)*x^3), b ~= 0
, (e*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (d*(a + b*x^2)^(1/2))/(
a*x) - (a*f*log(2*b^(1/2)*x + 2*(a + b*x^2)^(1/2)))/(2*b^(3/2)) + (f*x*(a
+ b*x^2)^(1/2))/(2*b) - (c*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3))
```

3.156 $\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx$

3.156.1 Optimal result

Integrand size = 32, antiderivative size = 118

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{5ax^5}+\frac{(4bc-5ad)\sqrt{a+bx^2}}{15a^2x^3} - \frac{(8b^2c-10abd+15a^2e)\sqrt{a+bx^2}}{15a^3x}+\frac{f\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}$$

output
```
f*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(1/2)-1/5*c*(b*x^2+a)^(1/2)/a/x^5+1
/15*(-5*a*d+4*b*c)*(b*x^2+a)^(1/2)/a^2/x^3-1/15*(15*a^2*e-10*a*b*d+8*b^2*c
)*(b*x^2+a)^(1/2)/a^3/x
```

3.156.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.83

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(8b^2cx^4-2abx^2(2c+5dx^2)+a^2(3c+5dx^2+15ex^4))}{15a^3x^5} - \frac{f\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{\sqrt{b}}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/15*(Sqrt[a + b*x^2]*(8*b^2*c*x^4 - 2*a*b*x^2*(2*c + 5*d*x^2) + a^2*(3*c
 + 5*d*x^2 + 15*e*x^4)))/(a^3*x^5) - (f*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]
])/Sqrt[b]
```

3.156.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2338, 9, 1588, 358, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2338}$$

$$-\frac{\int \frac{-5afx^5-5aex^3+(4bc-5ad)x}{x^5\sqrt{bx^2+a}}dx}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\int \frac{-5afx^4-5aex^2+4bc-5ad}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{15fx^2a^2+15ea^2-10bda+8b^2c}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}(4bc-5ad)}{3ax^3}}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{358}$$

$$-\frac{-\frac{15a^2f\int \frac{1}{\sqrt{bx^2+a}}dx-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4bc-5ad)}{3ax^3}}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{224}$$

$$-\frac{-\frac{15a^2f\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4bc-5ad)}{3ax^3}}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\frac{15a^2f\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}-\frac{\sqrt{a+bx^2}\left(15a^2e-10abd+8b^2c\right)}{ax}}{3a}-\frac{\sqrt{a+bx^2}(4bc-5ad)}{3ax^3}}{5a}-\frac{c\sqrt{a+bx^2}}{5ax^5}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^6*Sqrt[a + b*x^2]),x]
```

output
```
-1/5*(c*Sqrt[a + b*x^2])/(a*x^5) - (-1/3*((4*b*c - 5*a*d)*Sqrt[a + b*x^2])
/(a*x^3) - (-(((8*b^2*c - 10*a*b*d + 15*a^2*e)*Sqrt[a + b*x^2])/(a*x)) + (
15*a^2*f*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a))/(5*a)
```

3.156.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 358
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] + S
imp[d/e^2   Int[(e*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && NeQ[b*c - a*d, 0] && EqQ[Simplify[m + 2*p + 3], 0] && NeQ[m,
 -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2338
```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[R*(c*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] + Simp[1/(a*c*(
m + 1))   Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(
m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && Lt
Q[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

3.156.4 Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(15a^{2}e\,x^{4}-10abd\,x^{4}+8b^{2}c\,x^{4}+5a^{2}d\,x^{2}-4abc\,x^{2}+3a^{2}c\right)}{15a^{3}x^{5}}+\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}$
pseudoelliptic	$\frac{f\,a^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{b\,x^{2}+a}}{x\sqrt{b}}\right)x^{5}-\frac{\sqrt{b\,x^{2}+a}\,\left(-\frac{4x^{2}\left(\frac{5d\,x^{2}}{2}+c\right)a\,b^{\frac{3}{2}}}{3}+\frac{8b^{\frac{5}{2}}c\,x^{4}}{3}+a^{2}\sqrt{b}\,\left(5e\,x^{4}+\frac{5}{3}d\,x^{2}+c\right)\right)}{5}}{\sqrt{b}\,x^{5}a^{3}}$
default	$\frac{f\ln\left(x\sqrt{b}+\sqrt{b\,x^{2}+a}\right)}{\sqrt{b}}+d\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{e\sqrt{b\,x^{2}+a}}{ax}+c\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}}}{3a^{2}x}\right.}{5a}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(b*x^2+a)^(1/2)*(15*a^2*e*x^4-10*a*b*d*x^4+8*b^2*c*x^4+5*a^2*d*x^2-4
*a*b*c*x^2+3*a^2*c)/a^3/x^5+f*ln(x*b^(1/2)+(b*x^2+a)^(1/2))/b^(1/2)
```

3.156.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.87

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^6\sqrt{a + bx^2}}\,dx$$
$$= \left[\frac{15\,a^3\sqrt{b}fx^5\log\left(-2\,bx^2 - 2\,\sqrt{bx^2 + a}\sqrt{b}x - a\right) - 2\,((8\,b^3c - 10\,ab^2d + 15\,a^2be)x^4 + 3\,a^2bc - (4\,ab^2c - \right.}{30\,a^3bx^5}$$
$$-\frac{15\,a^3\sqrt{-b}fx^5\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right) + ((8\,b^3c - 10\,ab^2d + 15\,a^2be)x^4 + 3\,a^2bc - (4\,ab^2c - 5\,a^2bd)x^2)\sqrt{bx^2 +}{15\,a^3bx^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
[1/30*(15*a^3*sqrt(b)*f*x^5*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a
) - 2*((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)*x^4 + 3*a^2*b*c - (4*a*b^2*c -
5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^5), -1/15*(15*a^3*sqrt(-b)*f*x^5
*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((8*b^3*c - 10*a*b^2*d + 15*a^2*b*e)
*x^4 + 3*a^2*b*c - (4*a*b^2*c - 5*a^2*b*d)*x^2)*sqrt(b*x^2 + a))/(a^3*b*x^
5)]
```

3.156.6 Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 427, normalized size of antiderivative = 3.62

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{3a^4b^{\frac{9}{2}}c\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{2a^3b^{\frac{11}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{3a^2b^{\frac{13}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{12ab^{\frac{15}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{8b^{\frac{17}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + f\left(\begin{cases}\frac{\log\left(2\sqrt{b}\sqrt{a+bx^2}+2bx\right)}{\sqrt{b}} & \text{for } a\neq 0\wedge b\neq 0\\ \frac{x\log(x)}{\sqrt{bx^2}} & \text{for } b\neq 0\\ \frac{x}{\sqrt{a}} & \text{otherwise}\end{cases}\right) - \frac{\sqrt{b}d\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}d\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**6/(b*x**2+a)**(1/2),x)
```

output
```
-3*a**4*b**(9/2)*c*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*
x**6 + 15*a**3*b**6*x**8) - 2*a**3*b**(11/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(
15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 3*a**2*b**(13
/2)*c*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) - 12*a*b**(15/2)*c*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b*
*4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 8*b**(17/2)*c*x**8*sqrt
(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**
8) + f*Piecewise((log(2*sqrt(b)*sqrt(a + b*x**2) + 2*b*x)/sqrt(b), Ne(a, 0
) & Ne(b, 0)), (x*log(x)/sqrt(b*x**2), Ne(b, 0)), (x/sqrt(a), True)) - sqr
t(b)*d*sqrt(a/(b*x**2) + 1)/(3*a*x**2) - sqrt(b)*e*sqrt(a/(b*x**2) + 1)/a
+ 2*b**(3/2)*d*sqrt(a/(b*x**2) + 1)/(3*a**2)
```

3.156.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{f\,\operatorname{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{b}} - \frac{8\,\sqrt{bx^2+a}b^2c}{15\,a^3x} + \frac{2\,\sqrt{bx^2+a}bd}{3\,a^2x} - \frac{\sqrt{bx^2+a}e}{ax} + \frac{4\,\sqrt{bx^2+a}bc}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}d}{3\,ax^3} - \frac{\sqrt{bx^2+a}c}{5\,ax^5}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
f*arcsinh(b*x/sqrt(a*b))/sqrt(b) - 8/15*sqrt(b*x^2 + a)*b^2*c/(a^3*x) + 2/
3*sqrt(b*x^2 + a)*b*d/(a^2*x) - sqrt(b*x^2 + a)*e/(a*x) + 4/15*sqrt(b*x^2
+ a)*b*c/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*d/(a*x^3) - 1/5*sqrt(b*x^2 + a)*c
/(a*x^5)
```

3.156.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(100) = 200$.

Time = 0.33 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.70

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = -\frac{f\log\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2\right)}{2\,\sqrt{b}} + \frac{2\left(15\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^8\sqrt{b}e + 30\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 b^{\frac{3}{2}}d - 60\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^6 a\sqrt{b}e + 80\left(\sqrt{b}\right.\right.}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^6/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-1/2*f*log((sqrt(b)*x - sqrt(b*x^2 + a))^2)/sqrt(b) + 2/15*(15*(sqrt(b)*x
- sqrt(b*x^2 + a))^8*sqrt(b)*e + 30*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(3/2
)*d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*sqrt(b)*e + 80*(sqrt(b)*x - sqr
t(b*x^2 + a))^4*b^(5/2)*c - 70*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(3/2)*d
 + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^2*sqrt(b)*e - 40*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a*b^(5/2)*c + 50*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^2*b^(3/2
)*d - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3*sqrt(b)*e + 8*a^2*b^(5/2)*c -
 10*a^3*b^(3/2)*d + 15*a^4*sqrt(b)*e)/((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a
)^5
```

3.156.9 Mupad [B] (verification not implemented)

Time = 6.65 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89

$$\int \frac{c+dx^2+ex^4+fx^6}{x^6\sqrt{a+bx^2}}\,dx = \frac{f\,\ln\left(\sqrt{b}\,x+\sqrt{b\,x^2+a}\right)}{\sqrt{b}} - \frac{e\,\sqrt{b\,x^2+a}}{a\,x} - \frac{d\,\sqrt{b\,x^2+a}\,(a-2\,b\,x^2)}{3\,a^2\,x^3} - \frac{c\,\sqrt{b\,x^2+a}\,(3\,a^2-4\,a\,b\,x^2+8\,b^2\,x^4)}{15\,a^3\,x^5}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^6*(a + b*x^2)^(1/2)),x)
```

output
```
(f*log(b^(1/2)*x + (a + b*x^2)^(1/2)))/b^(1/2) - (e*(a + b*x^2)^(1/2))/(a*
x) - (d*(a + b*x^2)^(1/2)*(a - 2*b*x^2))/(3*a^2*x^3) - (c*(a + b*x^2)^(1/2
)*(3*a^2 + 8*b^2*x^4 - 4*a*b*x^2))/(15*a^3*x^5)
```

3.157 $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

3.157.1 Optimal result

Integrand size = 32, antiderivative size = 140

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{c\sqrt{a+bx^2}}{7ax^7}+\frac{(6bc-7ad)\sqrt{a+bx^2}}{35a^2x^5} - \frac{(24b^2c-28abd+35a^2e)\sqrt{a+bx^2}}{105a^3x^3} + \frac{(48b^3c-56ab^2d+70a^2be-105a^3f)\sqrt{a+bx^2}}{105a^4x}$$

output
```
-1/7*c*(b*x^2+a)^(1/2)/a/x^7+1/35*(-7*a*d+6*b*c)*(b*x^2+a)^(1/2)/a^2/x^5-1
/105*(35*a^2*e-28*a*b*d+24*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^3+1/105*(-105*a^3*
f+70*a^2*b*e-56*a*b^2*d+48*b^3*c)*(b*x^2+a)^(1/2)/a^4/x
```

3.157.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.74

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(48b^3cx^6-8ab^2x^4(3c+7dx^2)+2a^2bx^2(9c+14dx^2+35ex^4)-a^3(15c+21dx^2+35x^4(e+3fx}{105a^4x^7}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
(Sqrt[a + b*x^2]*(48*b^3*c*x^6 - 8*a*b^2*x^4*(3*c + 7*d*x^2) + 2*a^2*b*x^2
*(9*c + 14*d*x^2 + 35*e*x^4) - a^3*(15*c + 21*d*x^2 + 35*x^4*(e + 3*f*x^2)
)))/(105*a^4*x^7)
```

3.157.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {2334, 2089, 1588, 359, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{6bc-7a(fx^4+ex^2+d)}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-7afx^4-7aex^2+6bc-7ad}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{35fx^2a^2+35ea^2-28bda+24b^2c}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2}(6bc-7ad)}{5ax^5}}{7a}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{359}$$

$$-\frac{-\frac{-\frac{\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)\int \frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{3ax^3}}{5a}-\frac{\sqrt{a+bx^2}(6bc-7ad)}{5ax^5}}{7a}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

$$\Big\downarrow \text{242}$$

$$-\frac{-\frac{\frac{\sqrt{a+bx^2}\left(-105a^3f+70a^2be-56ab^2d+48b^3c\right)}{3a^2x}-\frac{\sqrt{a+bx^2}\left(35a^2e-28abd+24b^2c\right)}{3ax^3}}{5a}-\frac{\sqrt{a+bx^2}(6bc-7ad)}{5ax^5}}{7a}-\frac{c\sqrt{a+bx^2}}{7ax^7}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^8*Sqrt[a + b*x^2]),x]
```

output
```
-1/7*(c*Sqrt[a + b*x^2])/(a*x^7) - (-1/5*((6*b*c - 7*a*d)*Sqrt[a + b*x^2])
/(a*x^5) - (-1/3*((24*b^2*c - 28*a*b*d + 35*a^2*e)*Sqrt[a + b*x^2])/(a*x^3
) + ((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*Sqrt[a + b*x^2])/(3*
a^2*x))/(5*a))/(7*a)
```

3.157.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.157.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(\left(7f\,x^{6}+\frac{7}{3}e\,x^{4}+\frac{7}{5}d\,x^{2}+c\right)a^{3}-\frac{6b\,x^{2}\left(\frac{35}{9}e\,x^{4}+\frac{14}{9}d\,x^{2}+c\right)a^{2}}{5}+\frac{8b^{2}\left(\frac{7d\,x^{2}}{3}+c\right)x^{4}a}{5}-\frac{16b^{3}c\,x^{6}}{5}\right)\sqrt{b\,x^{2}+a}}{7x^{7}a^{4}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(105a^{3}f\,x^{6}-70a^{2}be\,x^{6}+56a\,b^{2}d\,x^{6}-48b^{3}c\,x^{6}+35a^{3}e\,x^{4}-28a^{2}bd\,x^{4}+24a\,b^{2}c\,x^{4}+21a^{3}d\,x^{2}-18a^{2}bc\,x^{2}+15c\,a^{3}\right)}{105x^{7}a^{4}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(105a^{3}f\,x^{6}-70a^{2}be\,x^{6}+56a\,b^{2}d\,x^{6}-48b^{3}c\,x^{6}+35a^{3}e\,x^{4}-28a^{2}bd\,x^{4}+24a\,b^{2}c\,x^{4}+21a^{3}d\,x^{2}-18a^{2}bc\,x^{2}+15c\,a^{3}\right)}{105x^{7}a^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(105a^{3}f\,x^{6}-70a^{2}be\,x^{6}+56a\,b^{2}d\,x^{6}-48b^{3}c\,x^{6}+35a^{3}e\,x^{4}-28a^{2}bd\,x^{4}+24a\,b^{2}c\,x^{4}+21a^{3}d\,x^{2}-18a^{2}bc\,x^{2}+15c\,a^{3}\right)}{105x^{7}a^{4}}$
default	$c\left(-\frac{\sqrt{b\,x^{2}+a}}{7a\,x^{7}}-\frac{6b\left(-\frac{\sqrt{b\,x^{2}+a}}{5a\,x^{5}}-\frac{4b\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)}{5a}\right)}{7a}\right)+e\left(-\frac{\sqrt{b\,x^{2}+a}}{3a\,x^{3}}+\frac{2b\sqrt{b\,x^{2}+a}}{3a^{2}x}\right)-\frac{f\sqrt{b\,x^{2}+a}}{ax}$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7*((7*f*x^6+7/3*e*x^4+7/5*d*x^2+c)*a^3-6/5*b*x^2*(35/9*e*x^4+14/9*d*x^2
+c)*a^2+8/5*b^2*(7/3*d*x^2+c)*x^4*a-16/5*b^3*c*x^6)*(b*x^2+a)^(1/2)/x^7/a^
4
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$$
$$=\frac{((48\,b^3c-56\,ab^2d+70\,a^2be-105\,a^3f)x^6-(24\,ab^2c-28\,a^2bd+35\,a^3e)x^4-15\,a^3c+3\,(6\,a^2bc-7\,a^3d)x}{105\,a^4x^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*((48*b^3*c - 56*a*b^2*d + 70*a^2*b*e - 105*a^3*f)*x^6 - (24*a*b^2*c
- 28*a^2*b*d + 35*a^3*e)*x^4 - 15*a^3*c + 3*(6*a^2*b*c - 7*a^3*d)*x^2)*sqr
t(b*x^2 + a)/(a^4*x^7)
```

3.157.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. $2(136) = 272$.

Time = 2.12 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.36

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = -\frac{5a^6b^{\frac{19}{2}}c\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{9a^5b^{\frac{21}{2}}cx^2\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{5a^4b^{\frac{23}{2}}cx^4\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{3a^4b^{\frac{9}{2}}d\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{5a^3b^{\frac{25}{2}}cx^6\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{2a^3b^{\frac{11}{2}}dx^2\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{30a^2b^{\frac{27}{2}}cx^8\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{3a^2b^{\frac{13}{2}}dx^4\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{40ab^{\frac{29}{2}}cx^{10}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{12ab^{\frac{15}{2}}dx^6\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} + \frac{16b^{\frac{31}{2}}cx^{12}\sqrt{\frac{a}{bx^2}+1}}{35a^7b^9x^6+105a^6b^{10}x^8+105a^5b^{11}x^{10}+35a^4b^{12}x^{12}} - \frac{8b^{\frac{17}{2}}dx^8\sqrt{\frac{a}{bx^2}+1}}{15a^5b^4x^4+30a^4b^5x^6+15a^3b^6x^8} - \frac{\sqrt{b}e\sqrt{\frac{a}{bx^2}+1}}{3ax^2} - \frac{\sqrt{b}f\sqrt{\frac{a}{bx^2}+1}}{a} + \frac{2b^{\frac{3}{2}}e\sqrt{\frac{a}{bx^2}+1}}{3a^2}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**8/(b*x**2+a)**(1/2),x)
```

output
```
-5*a**6*b**(19/2)*c*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**
10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 9*a**5*b**(21/2)*c
*x**2*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*
a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 5*a**4*b**(23/2)*c*x**4*sqrt(a/(
b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**
10 + 35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*d*sqrt(a/(b*x**2) + 1)/(15*a**
5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) + 5*a**3*b**(25/2)*c*
x**6*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a
**5*b**11*x**10 + 35*a**4*b**12*x**12) - 2*a**3*b**(11/2)*d*x**2*sqrt(a/(b
*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) +
30*a**2*b**(27/2)*c*x**8*sqrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**
6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**12) - 3*a**2*b**(13
/2)*d*x**4*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 1
5*a**3*b**6*x**8) + 40*a*b**(29/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(35*a**7*b
**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a**4*b**12*x**1
2) - 12*a*b**(15/2)*d*x**6*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x**4 + 30*a*
*4*b**5*x**6 + 15*a**3*b**6*x**8) + 16*b**(31/2)*c*x**12*sqrt(a/(b*x**2) +
 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x**10 + 35*a
**4*b**12*x**12) - 8*b**(17/2)*d*x**8*sqrt(a/(b*x**2) + 1)/(15*a**5*b**4*x
**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - sqrt(b)*e*sqrt(a/(b*x**2...
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.38

$$\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx = \frac{16\sqrt{bx^2+a}b^3c}{35\,a^4x} - \frac{8\sqrt{bx^2+a}b^2d}{15\,a^3x} + \frac{2\sqrt{bx^2+a}be}{3\,a^2x} - \frac{\sqrt{bx^2+a}f}{ax} - \frac{8\sqrt{bx^2+a}b^2c}{35\,a^3x^3} + \frac{4\sqrt{bx^2+a}bd}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}e}{3\,ax^3} + \frac{6\sqrt{bx^2+a}bc}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}d}{5\,ax^5} - \frac{\sqrt{bx^2+a}c}{7\,ax^7}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
16/35*sqrt(b*x^2 + a)*b^3*c/(a^4*x) - 8/15*sqrt(b*x^2 + a)*b^2*d/(a^3*x) +
 2/3*sqrt(b*x^2 + a)*b*e/(a^2*x) - sqrt(b*x^2 + a)*f/(a*x) - 8/35*sqrt(b*x
^2 + a)*b^2*c/(a^3*x^3) + 4/15*sqrt(b*x^2 + a)*b*d/(a^2*x^3) - 1/3*sqrt(b*
x^2 + a)*e/(a*x^3) + 6/35*sqrt(b*x^2 + a)*b*c/(a^2*x^5) - 1/5*sqrt(b*x^2 +
 a)*d/(a*x^5) - 1/7*sqrt(b*x^2 + a)*c/(a*x^7)
```

3.157.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(124) = 248$.

Time = 0.32 (sec) , antiderivative size = 548, normalized size of antiderivative = 3.91

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8\sqrt{a + bx^2}}\, dx$$
$$= \frac{2\left(105\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12}\sqrt{b}f + 210\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{10}b^{\frac{3}{2}}e - 630\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{10}a\sqrt{b}f + 560\right.}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^8/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*sqrt(b)*f + 210*(sqrt(b)*x - s
qrt(b*x^2 + a))^10*b^(3/2)*e - 630*(sqrt(b)*x - sqrt(b*x^2 + a))^10*a*sqrt
(b)*f + 560*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(5/2)*d - 910*(sqrt(b)*x - s
qrt(b*x^2 + a))^8*a*b^(3/2)*e + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^2*s
qrt(b)*f + 1680*(sqrt(b)*x - sqrt(b*x^2 + a))^6*b^(7/2)*c - 1400*(sqrt(b)*
x - sqrt(b*x^2 + a))^6*a*b^(5/2)*d + 1540*(sqrt(b)*x - sqrt(b*x^2 + a))^6*
a^2*b^(3/2)*e - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*sqrt(b)*f - 1008*
(sqrt(b)*x - sqrt(b*x^2 + a))^4*a*b^(7/2)*c + 1176*(sqrt(b)*x - sqrt(b*x^2
 + a))^4*a^2*b^(5/2)*d - 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^3*b^(3/2)*
e + 1575*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*sqrt(b)*f + 336*(sqrt(b)*x -
sqrt(b*x^2 + a))^2*a^2*b^(7/2)*c - 392*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^3
*b^(5/2)*d + 490*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(3/2)*e - 630*(sqrt
(b)*x - sqrt(b*x^2 + a))^2*a^5*sqrt(b)*f - 48*a^3*b^(7/2)*c + 56*a^4*b^(5/
2)*d - 70*a^5*b^(3/2)*e + 105*a^6*sqrt(b)*f)/((sqrt(b)*x - sqrt(b*x^2 + a)
)^2 - a)^7
```

3.157.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.89

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^8\sqrt{a + bx^2}}\, dx = \frac{\sqrt{b\,x^2 + a}\,(-105\,f\,a^3 + 70\,e\,a^2\,b - 56\,d\,a\,b^2 + 48\,c\,b^3)}{105\,a^4\,x} - \frac{\sqrt{b\,x^2 + a}\,(7\,a\,d - 6\,b\,c)}{35\,a^2\,x^5} - \frac{\sqrt{b\,x^2 + a}\,(35\,e\,a^2 - 28\,d\,a\,b + 24\,c\,b^2)}{105\,a^3\,x^3} - \frac{c\,\sqrt{b\,x^2 + a}}{7\,a\,x^7}$$

input
```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^8*(a + b*x^2)^(1/2)),x)
```

output
```
((a + b*x^2)^(1/2)*(48*b^3*c - 105*a^3*f - 56*a*b^2*d + 70*a^2*b*e))/(105*
a^4*x) - ((a + b*x^2)^(1/2)*(7*a*d - 6*b*c))/(35*a^2*x^5) - ((a + b*x^2)^(
1/2)*(24*b^2*c + 35*a^2*e - 28*a*b*d))/(105*a^3*x^3) - (c*(a + b*x^2)^(1/2
))/(7*a*x^7)
```

3.157. $\int \frac{c+dx^2+ex^4+fx^6}{x^8\sqrt{a+bx^2}}\,dx$

3.158 $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

3.158.1 Optimal result

Integrand size = 32, antiderivative size = 189

$$\begin{aligned}\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx =& -\frac{c\sqrt{a+bx^2}}{9ax^9}+\frac{(8bc-9ad)\sqrt{a+bx^2}}{63a^2x^7}\\&-\frac{(16b^2c-18abd+21a^2e)\sqrt{a+bx^2}}{105a^3x^5}\\&+\frac{(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^4x^3}\\&-\frac{2b(64b^3c-72ab^2d+84a^2be-105a^3f)\sqrt{a+bx^2}}{315a^5x}\end{aligned}$$

output
```
-1/9*c*(b*x^2+a)^(1/2)/a/x^9+1/63*(-9*a*d+8*b*c)*(b*x^2+a)^(1/2)/a^2/x^7-1
/105*(21*a^2*e-18*a*b*d+16*b^2*c)*(b*x^2+a)^(1/2)/a^3/x^5+1/315*(-105*a^3*
f+84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^4/x^3-2/315*b*(-105*a^
3*f+84*a^2*b*e-72*a*b^2*d+64*b^3*c)*(b*x^2+a)^(1/2)/a^5/x
```

3.158.2 Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.71

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{a+bx^2}(128b^4cx^8-16ab^3x^6(4c+9dx^2)+24a^2b^2x^4(2c+3dx^2+7ex^4)-2a^3bx^2(20c+27dx^2+42ex}{315a^5x^9}$$

input
```
Integrate[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

output
```
-1/315*(Sqrt[a + b*x^2]*(128*b^4*c*x^8 - 16*a*b^3*x^6*(4*c + 9*d*x^2) + 24
*a^2*b^2*x^4*(2*c + 3*d*x^2 + 7*e*x^4) - 2*a^3*b*x^2*(20*c + 27*d*x^2 + 42
*e*x^4 + 105*f*x^6) + a^4*(35*c + 45*d*x^2 + 63*e*x^4 + 105*f*x^6)))/(a^5*
x^9)
```

3.158.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.219$, Rules used = {2334, 2089, 1588, 27, 359, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{8bc-9a(fx^4+ex^2+d)}{x^8\sqrt{bx^2+a}}dx}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-9afx^4-9aex^2+8bc-9ad}{x^8\sqrt{bx^2+a}}dx}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \text{1588}$$

$$-\frac{-\frac{\int \frac{3\left(21fx^2a^2+21ea^2-18bda+16b^2c\right)}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{\sqrt{a+bx^2}(8bc-9ad)}{7ax^7}}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{3\int \frac{21fx^2a^2+21ea^2-18bda+16b^2c}{x^6\sqrt{bx^2+a}}dx}{7a}-\frac{\sqrt{a+bx^2}(8bc-9ad)}{7ax^7}}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \; 359$$

$$-\frac{-\frac{3\left(-\frac{\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)\int\frac{1}{x^4\sqrt{bx^2+a}}dx}{5a}-\frac{\sqrt{a+bx^2}\left(21a^2e-18abd+16b^2c\right)}{5ax^5}\right)}{7a}-\frac{\sqrt{a+bx^2}(8bc-9ad)}{7ax^7}}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \; 245$$

$$-\frac{-\frac{3\left(-\frac{\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)\left(-\frac{2b\int\frac{1}{x^2\sqrt{bx^2+a}}dx}{3a}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)}{5a}-\frac{\sqrt{a+bx^2}\left(21a^2e-18abd+16b^2c\right)}{5ax^5}\right)}{7a}-\frac{\sqrt{a+bx^2}(8bc-9ad)}{7ax^7}}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

$$\Big\downarrow \; 242$$

$$-\frac{-\frac{3\left(-\frac{\sqrt{a+bx^2}\left(21a^2e-18abd+16b^2c\right)}{5ax^5}-\frac{\left(\frac{2b\sqrt{a+bx^2}}{3a^2x}-\frac{\sqrt{a+bx^2}}{3ax^3}\right)\left(-105a^3f+84a^2be-72ab^2d+64b^3c\right)}{5a}\right)}{7a}-\frac{\sqrt{a+bx^2}(8bc-9ad)}{7ax^7}}{9a}-\frac{c\sqrt{a+bx^2}}{9ax^9}$$

input
```
Int[(c + d*x^2 + e*x^4 + f*x^6)/(x^10*Sqrt[a + b*x^2]),x]
```

output
```
-1/9*(c*Sqrt[a + b*x^2])/(a*x^9) - (-1/7*((8*b*c - 9*a*d)*Sqrt[a + b*x^2])
/(a*x^7) - (3*(-1/5*((16*b^2*c - 18*a*b*d + 21*a^2*e)*Sqrt[a + b*x^2])/(a*
x^5) - ((64*b^3*c - 72*a*b^2*d + 84*a^2*b*e - 105*a^3*f)*(-1/3*Sqrt[a + b*
x^2]/(a*x^3) + (2*b*Sqrt[a + b*x^2])/(3*a^2*x)))/(5*a)))/(7*a))/(9*a)
```

3.158. $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

3.158. $\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx$

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.158.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{\left(\left(3f\,x^6+\frac{9}{5}e\,x^4+\frac{9}{7}d\,x^2+c\right)a^4-\frac{8\left(\frac{21}{4}f\,x^6+\frac{21}{10}e\,x^4+\frac{27}{20}d\,x^2+c\right)b\,x^2a^3}{7}+\frac{48b^2x^4\left(\frac{7}{2}e\,x^4+\frac{3}{2}d\,x^2+c\right)a^2}{35}-\frac{64b^3x^6\left(\frac{9d\,x^2}{4}+c\right)a}{35}+\frac{128}{}\right.}{9x^9a^5}$
gosper	$-\frac{\sqrt{b\,x^2+a}\,\left(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\right.}{315x^9a^5}$
trager	$-\frac{\sqrt{b\,x^2+a}\,\left(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\right.}{315x^9a^5}$
risch	$-\frac{\sqrt{b\,x^2+a}\,\left(-210a^3bf\,x^8+168a^2b^2e\,x^8-144a\,b^3d\,x^8+128b^4c\,x^8+105a^4f\,x^6-84a^3be\,x^6+72a^2b^2d\,x^6-64a\,b^3c\,x^6+63a^4e\,x^4\right.}{315x^9a^5}$
default	$d\left(-\frac{\sqrt{b\,x^2+a}}{7a\,x^7}-\frac{6b\left(-\frac{\sqrt{b\,x^2+a}}{5a\,x^5}-\frac{4b\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)}{5a}\right)}{7a}\right)+f\left(-\frac{\sqrt{b\,x^2+a}}{3a\,x^3}+\frac{2b\sqrt{b\,x^2+a}}{3a^2x}\right)+e\left(-\frac{\sqrt{b}}{5}\right.$

input
```
int((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((3*f*x^6+9/5*e*x^4+9/7*d*x^2+c)*a^4-8/7*(21/4*f*x^6+21/10*e*x^4+27/2
0*d*x^2+c)*b*x^2*a^3+48/35*b^2*x^4*(7/2*e*x^4+3/2*d*x^2+c)*a^2-64/35*b^3*x
^6*(9/4*d*x^2+c)*a+128/35*b^4*c*x^8)*(b*x^2+a)^(1/2)/x^9/a^5
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.75

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{(2\,(64\,b^4c-72\,ab^3d+84\,a^2b^2e-105\,a^3bf)x^8-(64\,ab^3c-72\,a^2b^2d+84\,a^3be-105\,a^4f)x^6+35\,a^4c+3}{315\,a^5x^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
-1/315*(2*(64*b^4*c - 72*a*b^3*d + 84*a^2*b^2*e - 105*a^3*b*f)*x^8 - (64*a
*b^3*c - 72*a^2*b^2*d + 84*a^3*b*e - 105*a^4*f)*x^6 + 35*a^4*c + 3*(16*a^2
*b^2*c - 18*a^3*b*d + 21*a^4*e)*x^4 - 5*(8*a^3*b*c - 9*a^4*d)*x^2)*sqrt(b*
x^2 + a)/(a^5*x^9)
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1642 vs. 2(190) = 380.

Time = 2.92 (sec) , antiderivative size = 1642, normalized size of antiderivative = 8.69

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((f*x**6+e*x**4+d*x**2+c)/x**10/(b*x**2+a)**(1/2),x)
```

output
```
-35*a**8*b**(33/2)*c*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8
*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b*
*20*x**16) - 100*a**7*b**(35/2)*c*x**2*sqrt(a/(b*x**2) + 1)/(315*a**9*b**1
6*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x
**14 + 315*a**5*b**20*x**16) - 98*a**6*b**(37/2)*c*x**4*sqrt(a/(b*x**2) +
1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 +
1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**6*b**(19/2)*d*sqrt(a/
(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11*x*
*10 + 35*a**4*b**12*x**12) - 28*a**5*b**(39/2)*c*x**6*sqrt(a/(b*x**2) + 1)
/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 + 12
60*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 9*a**5*b**(21/2)*d*x**2*sqrt
(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b**11
*x**10 + 35*a**4*b**12*x**12) - 35*a**4*b**(41/2)*c*x**8*sqrt(a/(b*x**2) +
 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17*x**10 + 1890*a**7*b**18*x**12 +
 1260*a**6*b**19*x**14 + 315*a**5*b**20*x**16) - 5*a**4*b**(23/2)*d*x**4*s
qrt(a/(b*x**2) + 1)/(35*a**7*b**9*x**6 + 105*a**6*b**10*x**8 + 105*a**5*b*
*11*x**10 + 35*a**4*b**12*x**12) - 3*a**4*b**(9/2)*e*sqrt(a/(b*x**2) + 1)/
(15*a**5*b**4*x**4 + 30*a**4*b**5*x**6 + 15*a**3*b**6*x**8) - 280*a**3*b**
(43/2)*c*x**10*sqrt(a/(b*x**2) + 1)/(315*a**9*b**16*x**8 + 1260*a**8*b**17
*x**10 + 1890*a**7*b**18*x**12 + 1260*a**6*b**19*x**14 + 315*a**5*b**20...
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.46

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = -\frac{128\sqrt{bx^2+a}b^4c}{315\,a^5x} + \frac{16\sqrt{bx^2+a}b^3d}{35\,a^4x} - \frac{8\sqrt{bx^2+a}b^2e}{15\,a^3x} + \frac{2\sqrt{bx^2+a}bf}{3\,a^2x} + \frac{64\sqrt{bx^2+a}b^3c}{315\,a^4x^3} - \frac{8\sqrt{bx^2+a}b^2d}{35\,a^3x^3} + \frac{4\sqrt{bx^2+a}be}{15\,a^2x^3} - \frac{\sqrt{bx^2+a}f}{3\,ax^3} - \frac{16\sqrt{bx^2+a}b^2c}{105\,a^3x^5} + \frac{6\sqrt{bx^2+a}bd}{35\,a^2x^5} - \frac{\sqrt{bx^2+a}e}{5\,ax^5} + \frac{8\sqrt{bx^2+a}bc}{63\,a^2x^7} - \frac{\sqrt{bx^2+a}d}{7\,ax^7} - \frac{\sqrt{bx^2+a}c}{9\,ax^9}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
-128/315*sqrt(b*x^2 + a)*b^4*c/(a^5*x) + 16/35*sqrt(b*x^2 + a)*b^3*d/(a^4*
x) - 8/15*sqrt(b*x^2 + a)*b^2*e/(a^3*x) + 2/3*sqrt(b*x^2 + a)*b*f/(a^2*x)
+ 64/315*sqrt(b*x^2 + a)*b^3*c/(a^4*x^3) - 8/35*sqrt(b*x^2 + a)*b^2*d/(a^3
*x^3) + 4/15*sqrt(b*x^2 + a)*b*e/(a^2*x^3) - 1/3*sqrt(b*x^2 + a)*f/(a*x^3)
 - 16/105*sqrt(b*x^2 + a)*b^2*c/(a^3*x^5) + 6/35*sqrt(b*x^2 + a)*b*d/(a^2*
x^5) - 1/5*sqrt(b*x^2 + a)*e/(a*x^5) + 8/63*sqrt(b*x^2 + a)*b*c/(a^2*x^7)
- 1/7*sqrt(b*x^2 + a)*d/(a*x^7) - 1/9*sqrt(b*x^2 + a)*c/(a*x^9)
```

3.158.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(169) = 338$.

Time = 0.34 (sec) , antiderivative size = 660, normalized size of antiderivative = 3.49

$$\int \frac{c + dx^2 + ex^4 + fx^6}{x^{10}\sqrt{a + bx^2}}\,dx$$
$$= \frac{4\left(315\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{14} b^{\frac{3}{2}} f + 840\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} b^{\frac{5}{2}} e - 1995\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} a b^{\frac{3}{2}} f + 252\right.}{}$$

input
```
integrate((f*x^6+e*x^4+d*x^2+c)/x^10/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
4/315*(315*(sqrt(b)*x - sqrt(b*x^2 + a))^14*b^(3/2)*f + 840*(sqrt(b)*x - s
qrt(b*x^2 + a))^12*b^(5/2)*e - 1995*(sqrt(b)*x - sqrt(b*x^2 + a))^12*a*b^(
3/2)*f + 2520*(sqrt(b)*x - sqrt(b*x^2 + a))^10*b^(7/2)*d - 3780*(sqrt(b)*x
 - sqrt(b*x^2 + a))^10*a*b^(5/2)*e + 5355*(sqrt(b)*x - sqrt(b*x^2 + a))^10
*a^2*b^(3/2)*f + 8064*(sqrt(b)*x - sqrt(b*x^2 + a))^8*b^(9/2)*c - 6552*(sq
rt(b)*x - sqrt(b*x^2 + a))^8*a*b^(7/2)*d + 6804*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*a^2*b^(5/2)*e - 7875*(sqrt(b)*x - sqrt(b*x^2 + a))^8*a^3*b^(3/2)*f -
 5376*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a*b^(9/2)*c + 6048*(sqrt(b)*x - sqrt
(b*x^2 + a))^6*a^2*b^(7/2)*d - 6216*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^3*b^
(5/2)*e + 6825*(sqrt(b)*x - sqrt(b*x^2 + a))^6*a^4*b^(3/2)*f + 2304*(sqrt(
b)*x - sqrt(b*x^2 + a))^4*a^2*b^(9/2)*c - 2592*(sqrt(b)*x - sqrt(b*x^2 + a
))^4*a^3*b^(7/2)*d + 3024*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^4*b^(5/2)*e -
3465*(sqrt(b)*x - sqrt(b*x^2 + a))^4*a^5*b^(3/2)*f - 576*(sqrt(b)*x - sqrt
(b*x^2 + a))^2*a^3*b^(9/2)*c + 648*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^4*b^(
7/2)*d - 756*(sqrt(b)*x - sqrt(b*x^2 + a))^2*a^5*b^(5/2)*e + 945*(sqrt(b)*
x - sqrt(b*x^2 + a))^2*a^6*b^(3/2)*f + 64*a^4*b^(9/2)*c - 72*a^5*b^(7/2)*d
 + 84*a^6*b^(5/2)*e - 105*a^7*b^(3/2)*f)/((sqrt(b)*x - sqrt(b*x^2 + a))^2
- a)^9
```

3.158.9 Mupad [B] (verification not implemented)

Time = 6.14 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.90

$$\int \frac{c+dx^2+ex^4+fx^6}{x^{10}\sqrt{a+bx^2}}\,dx = \frac{\sqrt{b\,x^2+a}\,\left(-105\,f\,a^3+84\,e\,a^2\,b-72\,d\,a\,b^2+64\,c\,b^3\right)}{315\,a^4\,x^3} - \frac{\sqrt{b\,x^2+a}\,\left(9\,a\,d-8\,b\,c\right)}{63\,a^2\,x^7} - \frac{\sqrt{b\,x^2+a}\,\left(21\,e\,a^2-18\,d\,a\,b+16\,c\,b^2\right)}{105\,a^3\,x^5} - \frac{\sqrt{b\,x^2+a}\,\left(-210\,f\,a^3\,b+168\,e\,a^2\,b^2-144\,d\,a\,b^3+128\,c\,b^4\right)}{315\,a^5\,x} - \frac{c\,\sqrt{b\,x^2+a}}{9\,a\,x^9}$$

input

```
int((c + d*x^2 + e*x^4 + f*x^6)/(x^10*(a + b*x^2)^(1/2)),x)
```

output

```
((a + b*x^2)^(1/2)*(64*b^3*c - 105*a^3*f - 72*a*b^2*d + 84*a^2*b*e))/(315*
a^4*x^3) - ((a + b*x^2)^(1/2)*(9*a*d - 8*b*c))/(63*a^2*x^7) - ((a + b*x^2)
^(1/2)*(16*b^2*c + 21*a^2*e - 18*a*b*d))/(105*a^3*x^5) - ((a + b*x^2)^(1/2
)*(128*b^4*c + 168*a^2*b^2*e - 144*a*b^3*d - 210*a^3*b*f))/(315*a^5*x) - (
c*(a + b*x^2)^(1/2))/(9*a*x^9)
```

3.159 $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.159.1 Optimal result

Integrand size = 32, antiderivative size = 381

$$\begin{aligned}\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx &= \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^9}{7a\,(a+bx^2)^{7/2}}\\
&-\frac{(2Ab^3-a(9b^2B-16abC+23a^2D))\,x^9}{35a^2b^3\,(a+bx^2)^{5/2}}\\
&-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^7}{210a^2b^4\,(a+bx^2)^{3/2}}\\
&+\frac{Dx^9}{6b^3\,(a+bx^2)^{3/2}}-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^5}{30a^2b^5\sqrt{a+bx^2}}\\
&-\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x\sqrt{a+bx^2}}{16ab^7}\\
&+\frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,x^3\sqrt{a+bx^2}}{24a^2b^6}\\
&+\frac{(16Ab^3-72ab^2B+198a^2bC-429a^3D)\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{16b^{15/2}}\end{aligned}$$

output
```
1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^9/a/(b*x^2+a)^(7/2)-1/35*(2*A*b^3-a*(9
*B*b^2-16*C*a*b+23*D*a^2))*x^9/a^2/b^3/(b*x^2+a)^(5/2)-1/210*(16*A*b^3-3*a
*(24*B*b^2-66*C*a*b+143*D*a^2))*x^7/a^2/b^4/(b*x^2+a)^(3/2)+1/6*D*x^9/b^3/
(b*x^2+a)^(3/2)+1/16*(16*A*b^3-72*B*a*b^2+198*C*a^2*b-429*D*a^3)*arctanh(x
*b^(1/2)/(b*x^2+a)^(1/2))/b^(15/2)-1/30*(16*A*b^3-3*a*(24*B*b^2-66*C*a*b+1
43*D*a^2))*x^5/a^2/b^5/(b*x^2+a)^(1/2)-1/16*(16*A*b^3-3*a*(24*B*b^2-66*C*a
*b+143*D*a^2))*x*(b*x^2+a)^(1/2)/a/b^7+1/24*(16*A*b^3-3*a*(24*B*b^2-66*C*a
*b+143*D*a^2))*x^3*(b*x^2+a)^(1/2)/a^2/b^6
```

3.159.2 Mathematica [A] (verified)

Time = 1.26 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.68

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(45045a^6D-2310a^5b(9C-65Dx^2)+42a^4b^2(180B-1650Cx^2+41\cdots}{} + \frac{(16Ab^3-3a(24b^2B-66abC+143a^2D))\,\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{8b^{15/2}}$$

input
```
Integrate[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(45045*a^6*D - 2310*a^5*b*(9*C - 65*D*x^2) + 42*a^4*b^2*(180*B - 1650*C
*x^2 + 4147*D*x^4) - 12*a^3*b^3*(140*A - 2100*B*x^2 + 6699*C*x^4 - 6292*D*
x^6) - 2*a*b^5*x^4*(3248*A - 6336*B*x^2 + 1155*C*x^4 + 455*D*x^6) + a^2*b^
4*x^2*(-5600*A + 29232*B*x^2 - 34848*C*x^4 + 5005*D*x^6) + 4*b^6*x^6*(-704
*A + 35*(6*B*x^2 + 3*C*x^4 + 2*D*x^6))))/(1680*b^7*(a + b*x^2)^(7/2)) + ((
16*A*b^3 - 3*a*(24*b^2*B - 66*a*b*C + 143*a^2*D))*ArcTanh[(Sqrt[b]*x)/(-Sq
rt[a] + Sqrt[a + b*x^2])])/(8*b^(15/2))
```

3.159.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2335, 9, 1586, 9, 27, 363, 252, 252, 262, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \; 2335$$

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^7\left(-7aDx^5-7a\left(C-\frac{aD}{b}\right)x^3+\left(2Ab-\frac{9a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int \frac{x^8\left(-7aDx^4-7a\left(C-\frac{aD}{b}\right)x^2+2Ab-\frac{9a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \; 1586$$

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{x^7\left(\frac{35a^2Dx^3}{b}+\left(8Ab-\frac{9a\left(18Da^2-11bCa+4b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}$$

$$\Big\downarrow \; 9$$

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{x^8\left(35a^2Dx^2+b\left(8Ab-\frac{9a\left(18Da^2-11bCa+4b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}$$

$$\Big\downarrow \; 27$$

$$\frac{x^9\left(A-\frac{a\left(a^2D-abC+b^2B\right)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a\left(a+bx^2\right)^{5/2}}-\frac{\int \frac{x^8\left(8Ab^2+35a^2Dx^2-9a\left(\frac{18Da^2}{b}-11Ca+4bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}}{7ab}$$

$$\Big\downarrow \; 363$$

3.159. $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\int\frac{x^8}{\left(bx^2+a\right)^{5/2}}dx}{2b}+\frac{35a^2Dx^9}{6b\left(a+bx^2\right)^{3/2}}}{5ab}}{7ab}$$

$\downarrow$ 252

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\left(\frac{7\int\frac{x^6}{\left(bx^2+a\right)^{3/2}}dx}{3b}-\frac{x^7}{3b\left(a+bx^2\right)^{3/2}}\right)}{2b}+\frac{35a^2Dx^9}{6b\left(a+bx^2\right)^{3/2}}}{5ab}}{7ab}$$

$\downarrow$ 252

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\left(\frac{7\left(\frac{5\int\frac{x^4}{\sqrt{bx^2+a}}dx}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^7}{3b\left(a+bx^2\right)^{3/2}}\right)}{2b}+\frac{35a^2Dx^9}{6b\left(a+bx^2\right)^{3/2}}}{5ab}}{7ab}$$

$\downarrow$ 262

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\left(\frac{7\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\int\frac{x^2}{\sqrt{bx^2+a}}dx}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^7}{3b\left(a+bx^2\right)^{3/2}}\right)}{2b}+}{5ab}}{7ab}$$

3.159. $\displaystyle\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

$\downarrow$ 262

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\left(\frac{7\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{}{3}\right)}{2b}}{5ab}}{7ab}$$

$\downarrow$ 224

3.159. $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\left(-429a^3D+198a^2bC-72ab^2B+16Ab^3\right)\left(\frac{7\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+\cdots}}\right)}{3b}\cdots\right)}{2b}}{5ab}}{7ab}$$

$\downarrow$ 219

3.159. $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{x^9\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-$$

$$\frac{\frac{x^9\left(2Ab-\frac{a\left(23a^2D-16abC+9b^2B\right)}{b^2}\right)}{5a(a+bx^2)^{5/2}}-\frac{\frac{35a^2Dx^9}{6b\left(a+bx^2\right)^{3/2}}+\frac{\left(\frac{7\left(\frac{5\left(\frac{x^3\sqrt{a+bx^2}}{4b}-\frac{3a\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{4b}\right)}{b}-\frac{x^5}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^7}{3b\left(a+bx^2\right)^{3/}}}{2b}}{5ab}}{7ab}$$

input
```
Int[(x^8*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^9)/(7*a*(a + b*x^2)^(7/2)) - (((2
*A*b - (a*(9*b^2*B - 16*a*b*C + 23*a^2*D))/b^2)*x^9)/(5*a*(a + b*x^2)^(5/2
)) - ((35*a^2*D*x^9)/(6*b*(a + b*x^2)^(3/2)) + ((16*A*b^3 - 72*a*b^2*B + 1
98*a^2*b*C - 429*a^3*D)*(-1/3*x^7/(b*(a + b*x^2)^(3/2)) + (7*(-(x^5/(b*Sqr
t[a + b*x^2])) + (5*((x^3*Sqrt[a + b*x^2])/(4*b) - (3*a*((x*Sqrt[a + b*x^2
])/(2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/(4*b)))/
b))/(3*b)))/(2*b))/(5*a*b))/(7*a*b)
```

3.159.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

3.159. $\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.159.4 Maple [A] (verified)

Time = 3.75 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{58a\,x^5\left(\frac{65}{464}Dx^6+\frac{165}{464}C\,x^4-\frac{396}{203}x^2B+A\right)b^{\frac{11}{2}}}{15}+\left(\frac{1}{6}Dx^{13}+\frac{1}{4}C\,x^{11}+\frac{1}{2}B\,x^9-\frac{176}{105}A\,x^7\right)b^{\frac{13}{2}}-\frac{99\left(-\frac{65Dx^2}{9}+C\right)a^5x\,b^{\frac{3}{2}}}{8}+\frac{9a^4x\left(\frac{414}{180}\right.}{}$
default	$A\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+D\left(\frac{x^{13}}{6b(b\,x^2+a)^{\frac{7}{2}}}-\right.$ 13

3.159. $\int \frac{x^8\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

input
```
int(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(7/2)/b^(15/2)*(-58/15*a*x^5*(65/464*D*x^6+165/464*C*x^4-396/2
03*x^2*B+A)*b^(11/2)+(1/6*D*x^13+1/4*C*x^11+1/2*B*x^9-176/105*A*x^7)*b^(13
/2)-99/8*(-65/9*D*x^2+C)*a^5*x*b^(3/2)+9/2*a^4*x*(4147/180*D*x^4-55/6*C*x^
2+B)*b^(5/2)-a^3*(-1573/35*D*x^6+957/20*C*x^4-15*x^2*B+A)*x*b^(7/2)-10/3*a
^2*x^3*(-143/160*D*x^6+1089/175*C*x^4-261/50*x^2*B+A)*b^(9/2)+429/16*D*b^(
1/2)*a^6*x+(b^3*A-9/2*a*b^2*B+99/8*C*a^2*b-429/16*D*a^3)*(b*x^2+a)^(7/2)*a
rctanh((b*x^2+a)^(1/2)/x/b^(1/2)))
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 987, normalized size of antiderivative = 2.59

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,((429\,Da^3b^4-198\,Ca^2b^5+72\,Bab^6-16\,Ab^7)x^8+429\,Da^7-198\right.$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/3360*(105*((429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8
+ 429*D*a^7 - 198*C*a^6*b + 72*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3
 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198
*C*a^4*b^3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5
*b^2 + 72*B*a^4*b^3 - 16*A*a^3*b^4)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x
^2 + a)*sqrt(b)*x - a) + 2*(280*D*b^7*x^13 - 70*(13*D*a*b^6 - 6*C*b^7)*x^1
1 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^7)*x^9 + 176*(429*D*a^3*b^4 -
198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 + 406*(429*D*a^4*b^3 - 198*C*a^
3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + 350*(429*D*a^5*b^2 - 198*C*a^4*b^
3 + 72*B*a^3*b^4 - 16*A*a^2*b^5)*x^3 + 105*(429*D*a^6*b - 198*C*a^5*b^2 +
72*B*a^4*b^3 - 16*A*a^3*b^4)*x)*sqrt(b*x^2 + a))/(b^12*x^8 + 4*a*b^11*x^6
+ 6*a^2*b^10*x^4 + 4*a^3*b^9*x^2 + a^4*b^8), 1/1680*(105*((429*D*a^3*b^4 -
 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^8 + 429*D*a^7 - 198*C*a^6*b + 72
*B*a^5*b^2 - 16*A*a^4*b^3 + 4*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^
5 - 16*A*a*b^6)*x^6 + 6*(429*D*a^5*b^2 - 198*C*a^4*b^3 + 72*B*a^3*b^4 - 16
*A*a^2*b^5)*x^4 + 4*(429*D*a^6*b - 198*C*a^5*b^2 + 72*B*a^4*b^3 - 16*A*a^3
*b^4)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (280*D*b^7*x^13 -
 70*(13*D*a*b^6 - 6*C*b^7)*x^11 + 35*(143*D*a^2*b^5 - 66*C*a*b^6 + 24*B*b^
7)*x^9 + 176*(429*D*a^3*b^4 - 198*C*a^2*b^5 + 72*B*a*b^6 - 16*A*b^7)*x^7 +
 406*(429*D*a^4*b^3 - 198*C*a^3*b^4 + 72*B*a^2*b^5 - 16*A*a*b^6)*x^5 + ...
```

3.159.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**8*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1221 vs. 2(345) = 690.

Time = 0.22 (sec) , antiderivative size = 1221, normalized size of antiderivative = 3.20

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/6*D*x^13/((b*x^2 + a)^(7/2)*b) - 13/24*D*a*x^11/((b*x^2 + a)^(7/2)*b^2)
+ 1/4*C*x^11/((b*x^2 + a)^(7/2)*b) + 143/48*D*a^2*x^9/((b*x^2 + a)^(7/2)*b
^3) - 11/8*C*a*x^9/((b*x^2 + a)^(7/2)*b^2) + 1/2*B*x^9/((b*x^2 + a)^(7/2)*
b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2)
 + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*A*
x + 429/560*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^
2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*
D*a^3*x/b^3 - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)
^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/
2)*b^4))*C*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x
^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 +
 a)^(7/2)*b^4))*B*a*x/b + 143/80*D*a^3*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 2
0*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^4 - 33/
40*C*a^2*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2
) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^3 + 3/10*B*a*x*(15*x^4/((b*x^2 + a)^(
5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3)
)/b^2 - 1/15*A*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/
2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b + 143/16*D*a^3*x*(3*x^2/((b*x^2
 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^5 - 33/8*C*a^2*x*(3*x^2/((
b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*B*a*x*(3*x...
```

3.159.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.90

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(35\left(2\left(\frac{4\,Dx^2}{b}-\frac{13\,Da^4b^{11}-6\,Ca^3b^{12}}{a^3b^{13}}\right)x^2+\frac{143\,Da^5b^{10}-66\,Ca^4b^{11}+24\,Ba^3b^{12}}{a^3b^{13}}\right)\right.\right.\right.\right.}{}$$

$$+\frac{(429\,Da^3-198\,Ca^2b+72\,Bab^2-16\,Ab^3)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{16\,b^{\frac{15}{2}}}$$

input
```
integrate(x^8*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/1680*((((35*(2*(4*D*x^2/b - (13*D*a^4*b^11 - 6*C*a^3*b^12)/(a^3*b^13))*x
^2 + (143*D*a^5*b^10 - 66*C*a^4*b^11 + 24*B*a^3*b^12)/(a^3*b^13))*x^2 + 17
6*(429*D*a^6*b^9 - 198*C*a^5*b^10 + 72*B*a^4*b^11 - 16*A*a^3*b^12)/(a^3*b^
13))*x^2 + 406*(429*D*a^7*b^8 - 198*C*a^6*b^9 + 72*B*a^5*b^10 - 16*A*a^4*b
^11)/(a^3*b^13))*x^2 + 350*(429*D*a^8*b^7 - 198*C*a^7*b^8 + 72*B*a^6*b^9 -
 16*A*a^5*b^10)/(a^3*b^13))*x^2 + 105*(429*D*a^9*b^6 - 198*C*a^8*b^7 + 72*
B*a^7*b^8 - 16*A*a^6*b^9)/(a^3*b^13))*x/(b*x^2 + a)^(7/2) + 1/16*(429*D*a^
3 - 198*C*a^2*b + 72*B*a*b^2 - 16*A*b^3)*log(abs(-sqrt(b)*x + sqrt(b*x^2 +
 a)))/b^(15/2)
```

3.159.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^8(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \int \frac{x^8\left(A+B\,x^2+C\,x^4+x^6\,D\right)}{\left(b\,x^2+a\right)^{9/2}}\,dx$$

input
```
int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^8*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.160 $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.160.1 Optimal result

Integrand size = 32, antiderivative size = 279

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^7}{7a(a+bx^2)^{7/2}} + \frac{(b^2B-2abC+3a^2D)x^7}{5ab^3(a+bx^2)^{5/2}} + \frac{(8b^2B-36abC+99a^2D)x^5}{60ab^4(a+bx^2)^{3/2}} + \frac{Dx^7}{4b^3(a+bx^2)^{3/2}} + \frac{(8b^2B-36abC+99a^2D)x^3}{12ab^5\sqrt{a+bx^2}} - \frac{(8b^2B-36abC+99a^2D)x\sqrt{a+bx^2}}{8ab^6} + \frac{(8b^2B-36abC+99a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{8b^{13/2}}$$

output
```
1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^7/a/(b*x^2+a)^(7/2)+1/5*(B*b^2-2*C*a*b
+3*D*a^2)*x^7/a/b^3/(b*x^2+a)^(5/2)+1/60*(8*B*b^2-36*C*a*b+99*D*a^2)*x^5/a
/b^4/(b*x^2+a)^(3/2)+1/4*D*x^7/b^3/(b*x^2+a)^(3/2)+1/8*(8*B*b^2-36*C*a*b+9
9*D*a^2)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b^(13/2)+1/12*(8*B*b^2-36*C*a*
b+99*D*a^2)*x^3/a/b^5/(b*x^2+a)^(1/2)-1/8*(8*B*b^2-36*C*a*b+99*D*a^2)*x*(b
*x^2+a)^(1/2)/a/b^6
```

3.160.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.77

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{x(-10395a^6D+120Ab^6x^6+630a^5b(6C-55Dx^2)+a^2b^4x^4(-3248B-}{}$$
$$+\frac{(8b^2B-36abC+99a^2D)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{4b^{13/2}}$$

input
```
Integrate[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(-10395*a^6*D + 120*A*b^6*x^6 + 630*a^5*b*(6*C - 55*D*x^2) + a^2*b^4*x^
4*(-3248*B + 6336*C*x^2 - 1155*D*x^4) - 42*a^4*b^2*(20*B - 300*C*x^2 + 957
*D*x^4) - 8*a^3*b^3*x^2*(350*B - 1827*C*x^2 + 2178*D*x^4) + 2*a*b^5*x^6*(-
704*B + 105*(2*C*x^2 + D*x^4))))/(840*a*b^6*(a + b*x^2)^(7/2)) + ((8*b^2*B
 - 36*a*b*C + 99*a^2*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])
/(4*b^(13/2))
```

3.160.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.88, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2335, 9, 27, 1586, 9, 27, 363, 252, 252, 262, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{7x^5\left(aDx^5+a\left(C-\frac{aD}{b}\right)x^3+a\left(B-\frac{a(bC-aD)}{b^2}\right)x\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{7x^6\left(aDx^4+a\left(C-\frac{aD}{b}\right)x^2+\frac{a(Da^2-bCa+b^2B)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{x^6\left(aDx^4+a\left(C-\frac{aD}{b}\right)x^2+\frac{a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1586}$$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{x^5\left(\frac{a\left(16Da^2-9bCa+2b^2B\right)x}{b^2}-\frac{5a^2Dx^3}{b}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{ax^6\left(16Da^2-5bDx^2a-9bCa+2b^2B\right)}{b^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\int \frac{x^6\left(16Da^2-5bDx^2a-9bCa+2b^2B\right)}{\left(bx^2+a\right)^{5/2}}dx}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 363}$$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2D-36abC+8b^2B)\int \frac{x^6}{\left(bx^2+a\right)^{5/2}}dx-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 252}$$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2D-36abC+8b^2B)\left(\frac{5\int \frac{x^4}{\left(bx^2+a\right)^{3/2}}dx}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 252}$$

3.160. $\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2D-36abC+8b^2B)\left(\frac{5\left(\frac{3\int\frac{x^2}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$\Big\downarrow$ 262

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2D-36abC+8b^2B)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{\sqrt{bx^2+a}}dx}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}(99a^2D-36abC+8b^2B)\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{2b}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}+\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}$$

$\Big\downarrow$ 219

3.160. $\displaystyle\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{x^7\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}+\frac{\frac{x^7(3a^2D-2abC+b^2B)}{5b^2(a+bx^2)^{5/2}}-\frac{\frac{1}{4}\left(\frac{5\left(\frac{3\left(\frac{x\sqrt{a+bx^2}}{2b}-\frac{a\,\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{3/2}}\right)}{b}-\frac{x^3}{b\sqrt{a+bx^2}}\right)}{3b}-\frac{x^5}{3b\left(a+bx^2\right)^{3/2}}\right)(99a^2D-36abC+8b^2B)-\frac{5aDx^7}{4\left(a+bx^2\right)^{3/2}}}{5b^2}}{ab}$$

input
```
Int[(x^6*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^7)/(7*a*(a + b*x^2)^(7/2)) + (((b
^2*B - 2*a*b*C + 3*a^2*D)*x^7)/(5*b^2*(a + b*x^2)^(5/2)) - ((-5*a*D*x^7)/(
4*(a + b*x^2)^(3/2)) + ((8*b^2*B - 36*a*b*C + 99*a^2*D)*(-1/3*x^5/(b*(a +
b*x^2)^(3/2)) + (5*(-(x^3/(b*Sqrt[a + b*x^2])) + (3*((x*Sqrt[a + b*x^2])/(
2*b) - (a*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*b^(3/2))))/b))/(3*b)))/
4)/(5*b^2))/(a*b)
```

3.160.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] && !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.160. $\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.160.4 Maple [A] (verified)

Time = 3.67 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{7a(bx^2+a)^{\frac{7}{2}}\left(Bb^2-\frac{9}{2}Cab+\frac{99}{8}Da^2\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+x\left(-\frac{176a\left(-\frac{105}{704}Dx^4-\frac{105}{352}Cx^2+B\right)x^6b^{\frac{11}{2}}}{15}-7a^4\left(\frac{957}{20}Dx^4-15Cx^2\right.\right.}{7(bx^2}$
default	$B\left(-\frac{x^7}{7b(bx^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+A\left(-\frac{x^5}{2b(bx^2+a)^{\frac{7}{2}}}+\right.$

3.160. $\int \frac{x^6\left(A+Bx^2+Cx^4+Dx^6\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

input
```
int(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/7/(b*x^2+a)^(7/2)*(7*a*(b*x^2+a)^(7/2)*(B*b^2-9/2*C*a*b+99/8*D*a^2)*arct
anh((b*x^2+a)^(1/2)/x/b^(1/2))+x*(-176/15*a*(-105/704*D*x^4-105/352*C*x^2+
B)*x^6*b^(11/2)-7*a^4*(957/20*D*x^4-15*C*x^2+B)*b^(5/2)-70/3*a^3*x^2*(1089
/175*D*x^4-261/50*C*x^2+B)*b^(7/2)-406/15*a^2*x^4*(165/464*D*x^4-396/203*C
*x^2+B)*b^(9/2)+63/2*a^5*(-55/6*D*x^2+C)*b^(3/2)+A*b^(13/2)*x^6-693/8*D*b^
(1/2)*a^6))/b^(13/2)/a
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.92

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\left((99\,Da^3b^4-36\,Ca^2b^5+8\,Bab^6)x^8+99\,Da^7-36\,Ca^6b+8\,Ba^5b\right.}{}\right.$$

$$-\frac{105\left((99\,Da^3b^4-36\,Ca^2b^5+8\,Bab^6)x^8+99\,Da^7-36\,Ca^6b+8\,Ba^5b^2+4\left(99\,Da^4b^3-36\,Ca^3b^4+8\,Ba^2\right.\right.}{}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/1680*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^8 + 99*D*a^7 - 3
6*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^
6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*(99*D*a^6*b - 36
*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sq
rt(b)*x - a) + 2*(210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9 -
8*(2178*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(99*
D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a^4
*b^3 + 8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x)
*sqrt(b*x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b^8
*x^2 + a^5*b^7), -1/840*(105*((99*D*a^3*b^4 - 36*C*a^2*b^5 + 8*B*a*b^6)*x^
8 + 99*D*a^7 - 36*C*a^6*b + 8*B*a^5*b^2 + 4*(99*D*a^4*b^3 - 36*C*a^3*b^4 +
 8*B*a^2*b^5)*x^6 + 6*(99*D*a^5*b^2 - 36*C*a^4*b^3 + 8*B*a^3*b^4)*x^4 + 4*
(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/
sqrt(b*x^2 + a)) - (210*D*a*b^6*x^11 - 105*(11*D*a^2*b^5 - 4*C*a*b^6)*x^9
- 8*(2178*D*a^3*b^4 - 792*C*a^2*b^5 + 176*B*a*b^6 - 15*A*b^7)*x^7 - 406*(9
9*D*a^4*b^3 - 36*C*a^3*b^4 + 8*B*a^2*b^5)*x^5 - 350*(99*D*a^5*b^2 - 36*C*a
^4*b^3 + 8*B*a^3*b^4)*x^3 - 105*(99*D*a^6*b - 36*C*a^5*b^2 + 8*B*a^4*b^3)*
x)*sqrt(b*x^2 + a))/(a*b^11*x^8 + 4*a^2*b^10*x^6 + 6*a^3*b^9*x^4 + 4*a^4*b
^8*x^2 + a^5*b^7)]
```

3.160.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**6*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(247) = 494$.

Time = 0.23 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.53

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/4*D*x^11/((b*x^2 + a)^(7/2)*b) - 11/8*D*a*x^9/((b*x^2 + a)^(7/2)*b^2) +
1/2*C*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*B*x - 99/280*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70
*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a
^3/((b*x^2 + a)^(7/2)*b^4))*D*a^2*x/b^2 + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*
b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3)
 + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*C*a*x/b - 33/40*D*a^2*x*(15*x^4/((b*x^2
 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/
2)*b^3))/b^3 + 3/10*C*a*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2
 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2 - 1/15*B*x*(15*x^4/(
(b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 +
a)^(5/2)*b^3))/b - 1/2*A*x^5/((b*x^2 + a)^(7/2)*b) - 33/8*D*a^2*x*(3*x^2/(
(b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^4 + 3/2*C*a*x*(3*x^2
/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3 - 1/3*B*x*(3*x^2
/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - 99/8*D*a^3*x^3
/((b*x^2 + a)^(5/2)*b^5) + 9/2*C*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - B*a*x^3
/((b*x^2 + a)^(5/2)*b^3) - 5/8*A*a*x^3/((b*x^2 + a)^(7/2)*b^2) + 4587/280*
D*a^2*x/(sqrt(b*x^2 + a)*b^6) + 561/280*D*a^3*x/((b*x^2 + a)^(3/2)*b^6) -
2871/280*D*a^4*x/((b*x^2 + a)^(5/2)*b^6) - 417/70*C*a*x/(sqrt(b*x^2 + a...
```

3.160.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 265, normalized size of antiderivative = 0.95

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(105\left(\frac{2\,Dx^2}{b}-\frac{11\,Da^4b^9-4\,Ca^3b^{10}}{a^3b^{11}}\right)x^2-\frac{8\,(2178\,Da^5b^8-792\,Ca^4b^9+176\,Ba^3b^{10}-}{a^3b^{11}}\right.\right.\right.\right.}{}$$
$$-\frac{(99\,Da^2-36\,Cab+8\,Bb^2)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{8\,b^{\frac{13}{2}}}$$

input
```
integrate(x^6*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/840*((((105*(2*D*x^2/b - (11*D*a^4*b^9 - 4*C*a^3*b^10)/(a^3*b^11))*x^2 -
 8*(2178*D*a^5*b^8 - 792*C*a^4*b^9 + 176*B*a^3*b^10 - 15*A*a^2*b^11)/(a^3*
b^11))*x^2 - 406*(99*D*a^6*b^7 - 36*C*a^5*b^8 + 8*B*a^4*b^9)/(a^3*b^11))*x
^2 - 350*(99*D*a^7*b^6 - 36*C*a^6*b^7 + 8*B*a^5*b^8)/(a^3*b^11))*x^2 - 105
*(99*D*a^8*b^5 - 36*C*a^7*b^6 + 8*B*a^6*b^7)/(a^3*b^11))*x/(b*x^2 + a)^(7/
2) - 1/8*(99*D*a^2 - 36*C*a*b + 8*B*b^2)*log(abs(-sqrt(b)*x + sqrt(b*x^2 +
 a)))/b^(13/2)
```

3.160.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^6(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^6\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^6*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.161 $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.161.1 Optimal result

Integrand size = 32, antiderivative size = 210

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(A-\frac{a(b^2B-abC+a^2D)}{b^3}\right)x^5}{7a(a+bx^2)^{7/2}} + \frac{(2Ab^3+a(5b^2B-12abC+19a^2D))x^5}{35a^2b^3(a+bx^2)^{5/2}} + \frac{a(bC-3aD)x}{3b^5(a+bx^2)^{3/2}} - \frac{(4bC-15aD)x}{3b^5\sqrt{a+bx^2}} + \frac{Dx\sqrt{a+bx^2}}{2b^5} + \frac{(2bC-9aD)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

output
```
1/7*(A-a*(B*b^2-C*a*b+D*a^2)/b^3)*x^5/a/(b*x^2+a)^(7/2)+1/35*(2*A*b^3+a*(5
*B*b^2-12*C*a*b+19*D*a^2))*x^5/a^2/b^3/(b*x^2+a)^(5/2)+1/3*a*(C*b-3*D*a)*x
/b^5/(b*x^2+a)^(3/2)+1/2*(2*C*b-9*D*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/
b^(11/2)-1/3*(4*C*b-15*D*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*D*x*(b*x^2+a)^(1/2)/
b^5
```

3.161.2 Mathematica [A] (verified)

Time = 1.01 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.85

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\frac{x(945a^6D+12Ab^6x^6+6ab^5x^4(7A+5Bx^2)-210a^5b(C-15Dx^2)+a}{21}$$
$$+\frac{(2bC-9aD)\text{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(945*a^6*D + 12*A*b^6*x^6 + 6*a*b^5*x^4*(7*A + 5*B*x^2) - 210*a^5*b*(C
- 15*D*x^2) + a^2*b^4*x^6*(-352*C + 105*D*x^2) + 14*a^4*b^2*x^2*(-50*C + 2
61*D*x^2) + 4*a^3*b^3*x^4*(-203*C + 396*D*x^2)))/(210*a^2*b^5*(a + b*x^2)^
(7/2)) + ((2*b*C - 9*a*D)*ArcTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])
])/b^(11/2)
```

3.161.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {2335, 9, 25, 1586, 9, 27, 360, 1471, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-\frac{\int -\frac{x^3\left(7aDx^5+7a\left(C-\frac{aD}{b}\right)x^3+\left(2Ab+\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)x\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \ 9$$

$$\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\,(a+bx^2)^{7/2}}-\frac{\int -\frac{x^4\left(7aDx^4+7a\left(C-\frac{aD}{b}\right)x^2+2Ab+\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^4\left(7aDx^4+7a\left(C-\frac{aD}{b}\right)x^2+2Ab+\frac{5a\left(Da^2-bCa+b^2B\right)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{35x^3\left(\frac{a^2Dx^3}{b}+\frac{a^2(bC-2aD)x}{b^2}\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{9}$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{35a^2x^4\left(bDx^2+bC-2aD\right)}{b^2\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{7a\int \frac{x^4\left(bDx^2+bC-2aD\right)}{\left(bx^2+a\right)^{5/2}}dx}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\int \frac{-3b^3Dx^4-3b^2(bC-3aD)x^2+ab(bC-3aD)}{\left(bx^2+a\right)^{3/2}}dx}{3b^3}\right)}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(4bC-15aD)}{\sqrt{a+bx^2}}-\frac{\int \frac{3ab\left(bDx^2+bC-4aD\right)}{\sqrt{bx^2+a}}dx}{a}}{3b^3}\right)}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

3.161. $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

$$\Big\downarrow \ 27$$

$$\frac{\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(4bC-15aD)}{\sqrt{a+bx^2}}-3b\int \frac{bDx^2+bC-4aD}{\sqrt{bx^2+a}}dx}{3b^3}\right)}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 299$$

$$\frac{\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(4bC-15aD)}{\sqrt{a+bx^2}}-3b\left(\frac{1}{2}(2bC-9aD)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 224$$

$$\frac{\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(4bC-15aD)}{\sqrt{a+bx^2}}-3b\left(\frac{1}{2}(2bC-9aD)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}+\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{x^5\left(\frac{a\left(19a^2D-12abC+5b^2B\right)}{b^2}+2Ab\right)}{5a(a+bx^2)^{5/2}}+\frac{7a\left(\frac{ax(bC-3aD)}{3b^2\left(a+bx^2\right)^{3/2}}-\frac{\frac{bx(4bC-15aD)}{\sqrt{a+bx^2}}-3b\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bC-9aD)}{2\sqrt{b}}+\frac{1}{2}Dx\sqrt{a+bx^2}\right)}{3b^3}\right)}{b^2}}{7ab}+\frac{x^5\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^4*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

3.161. $\int \frac{x^4\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^5)/(7*a*(a + b*x^2)^(7/2)) + (((2
*A*b + (a*(5*b^2*B - 12*a*b*C + 19*a^2*D))/b^2)*x^5)/(5*a*(a + b*x^2)^(5/2
)) + (7*a*((a*(b*C - 3*a*D)*x)/(3*b^2*(a + b*x^2)^(3/2)) - ((b*(4*b*C - 15
*a*D)*x)/Sqrt[a + b*x^2] - 3*b*((D*x*Sqrt[a + b*x^2])/2 + ((2*b*C - 9*a*D)
*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b])))/(3*b^3)))/b^2)/(7*a*b
)
```

3.161.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

3.161. $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.161. $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.161.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{a\left(\frac{5x^2B}{7}+A\right)x^5 b^{\frac{11}{2}}}{5}+\frac{2A\,b^{\frac{13}{2}}x^7}{35}+a^2\left(-x\,a^3\left(-15Dx^2+C\right)b^{\frac{3}{2}}-\frac{10a^2\left(-\frac{261Dx^2}{50}+C\right)x^3b^{\frac{5}{2}}}{3}-\frac{58a\,x^5\left(-\frac{396Dx^2}{203}+C\right)b^{\frac{7}{2}}}{15}+\left(\frac{1}{2}Dx^9\right.\right.}{(b\,x^2+a)^{\frac{7}{2}}b^{\frac{11}{2}}a^2}$
default	$C\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{b\,x^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{b\,x^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+B\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\right.$

3.161. $\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

input
```
int(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(7/2)*(1/5*a*(5/7*x^2*B+A)*x^5*b^(11/2)+2/35*A*b^(13/2)*x^7+a^
2*(-x*a^3*(-15*D*x^2+C)*b^(3/2)-10/3*a^2*(-261/50*D*x^2+C)*x^3*b^(5/2)-58/
15*a*x^5*(-396/203*D*x^2+C)*b^(7/2)+(1/2*D*x^9-176/105*C*x^7)*b^(9/2)+9/2*
D*b^(1/2)*a^4*x+arctanh((b*x^2+a)^(1/2)/x/b^(1/2))*(b*x^2+a)^(7/2)*(C*b-9/
2*D*a)))/b^(11/2)/a^2
```

3.161.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 653, normalized size of antiderivative = 3.11

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \Bigg[\frac{105\,((9\,Da^3b^4-2\,Ca^2b^5)x^8+9\,Da^7-2\,Ca^6b+4\,(9\,Da^4b^3-2\,Ca^3b^4}{}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/420*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*a^6*b + 4*(9*
D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^4 + 4*(9*D*
a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)
*x - a) + 2*(105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a
*b^6 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 +
350*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqr
t(b*x^2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^
2 + a^6*b^6), 1/210*(105*((9*D*a^3*b^4 - 2*C*a^2*b^5)*x^8 + 9*D*a^7 - 2*C*
a^6*b + 4*(9*D*a^4*b^3 - 2*C*a^3*b^4)*x^6 + 6*(9*D*a^5*b^2 - 2*C*a^4*b^3)*
x^4 + 4*(9*D*a^6*b - 2*C*a^5*b^2)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x
^2 + a)) + (105*D*a^2*b^5*x^9 + 2*(792*D*a^3*b^4 - 176*C*a^2*b^5 + 15*B*a*
b^6 + 6*A*b^7)*x^7 + 14*(261*D*a^4*b^3 - 58*C*a^3*b^4 + 3*A*a*b^6)*x^5 + 3
50*(9*D*a^5*b^2 - 2*C*a^4*b^3)*x^3 + 105*(9*D*a^6*b - 2*C*a^5*b^2)*x)*sqrt
(b*x^2 + a))/(a^2*b^10*x^8 + 4*a^3*b^9*x^6 + 6*a^4*b^8*x^4 + 4*a^5*b^7*x^2
 + a^6*b^6)]
```

3.161.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6467 vs. $2(199) = 398$.

Time = 94.78 (sec) , antiderivative size = 6467, normalized size of antiderivative = 30.80

$$\int \frac{x^4(A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x**4*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1
+ b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3
*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(35*a**(11/2)*sqrt(1 + b*x**2/a) + 10
5*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sqrt(1 + b*x
**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a))) + B*x**7/(7*a**(9/2)*s
qrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*sqrt(1 + b*x**2/a) + 21*a**(5/2)*b*
*2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + C*
(105*a**(205/2)*b**45*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**
(205/2)*b**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt
(1 + b*x**2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100
*a**(199/2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2
)*x**8*sqrt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**
2/a) + 105*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 630*a**(203/2
)*b**46*x**2*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b
**(99/2)*sqrt(1 + b*x**2/a) + 630*a**(203/2)*b**(101/2)*x**2*sqrt(1 + b*x*
*2/a) + 1575*a**(201/2)*b**(103/2)*x**4*sqrt(1 + b*x**2/a) + 2100*a**(199/
2)*b**(105/2)*x**6*sqrt(1 + b*x**2/a) + 1575*a**(197/2)*b**(107/2)*x**8*sq
rt(1 + b*x**2/a) + 630*a**(195/2)*b**(109/2)*x**10*sqrt(1 + b*x**2/a) + 10
5*a**(193/2)*b**(111/2)*x**12*sqrt(1 + b*x**2/a)) + 1575*a**(201/2)*b**47*
x**4*sqrt(1 + b*x**2/a)*asinh(sqrt(b)*x/sqrt(a))/(105*a**(205/2)*b**(99...
```

3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(183) = 366$.

Time = 0.23 (sec) , antiderivative size = 753, normalized size of antiderivative = 3.59

$$\int \frac{x^4(A + Bx^2 + Cx^4 + Dx^6)}{(a + bx^2)^{9/2}}\, dx = \text{Too large to display}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
1/2*D*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*C*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3
/((b*x^2 + a)^(7/2)*b^4))*D*a*x/b + 3/10*D*a*x*(15*x^4/((b*x^2 + a)^(5/2)*
b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2
 - 1/15*C*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^
2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*B*x^5/((b*x^2 + a)^(7/2)*b) +
3/2*D*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3
- 1/3*C*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2
+ 9/2*D*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - C*a*x^3/((b*x^2 + a)^(5/2)*b^3)
- 5/8*B*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*A*x^3/((b*x^2 + a)^(7/2)*b) -
417/70*D*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*D*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*D*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*C*x/(sqrt(b*x^2 + a)*b
^4) + 17/105*C*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*C*a^2*x/((b*x^2 + a)^(5
/2)*b^4) + 1/14*B*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*B*x/(sqrt(b*x^2 + a)*a*b
^3) + 3/56*B*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*B*a^2*x/((b*x^2 + a)^(7/2
)*b^3) + 3/140*A*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*A*x/(sqrt(b*x^2 + a)*a^2
*b^2) + 1/35*A*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*A*a*x/((b*x^2 + a)^(7/2)
*b^2) - 9/2*D*a*arcsinh(b*x/sqrt(a*b))/b^(11/2) + C*arcsinh(b*x/sqrt(a*...
```

3.161.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.97

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(\frac{105\,Dx^2}{b}+\frac{2\,(792\,Da^4b^7-176\,Ca^3b^8+15\,Ba^2b^9+6\,Aab^{10})}{a^3b^9}\right)x^2+\frac{14\,(261\,Da^5b^6-58\,C}{a^3b^9}\right.\right.\right.\right.}{210\,(bx^2+}$$
$$+\frac{(9\,Da-2\,Cb)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^4*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/210*((((105*D*x^2/b + 2*(792*D*a^4*b^7 - 176*C*a^3*b^8 + 15*B*a^2*b^9 +
6*A*a*b^10)/(a^3*b^9))*x^2 + 14*(261*D*a^5*b^6 - 58*C*a^4*b^7 + 3*A*a^2*b^
9)/(a^3*b^9))*x^2 + 350*(9*D*a^6*b^5 - 2*C*a^5*b^6)/(a^3*b^9))*x^2 + 105*(
9*D*a^7*b^4 - 2*C*a^6*b^5)/(a^3*b^9))*x/(b*x^2 + a)^(7/2) + 1/2*(9*D*a - 2
*C*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(11/2)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^4\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^4*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.162 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.162.1 Optimal result

Integrand size = 32, antiderivative size = 179

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{a^3Dx}{b^4(a+bx^2)^{7/2}} + \frac{(Ab^3-10a^3D)x^3}{3ab^3(a+bx^2)^{7/2}} + \frac{(4Ab^3+3ab^2B-58a^3D)x^5}{15a^2b^2(a+bx^2)^{7/2}} + \frac{(8Ab^3+6ab^2B+15a^2bC-176a^3D)x^7}{105a^3b(a+bx^2)^{7/2}} + \frac{D\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
-a^3*D*x/b^4/(b*x^2+a)^(7/2)+1/3*(A*b^3-10*D*a^3)*x^3/a/b^3/(b*x^2+a)^(7/2
)+1/15*(4*A*b^3+3*B*a*b^2-58*D*a^3)*x^5/a^2/b^2/(b*x^2+a)^(7/2)+1/105*(8*A
*b^3+6*B*a*b^2+15*C*a^2*b-176*D*a^3)*x^7/a^3/b/(b*x^2+a)^(7/2)+D*arctanh(x
*b^(1/2)/(b*x^2+a)^(1/2))/b^(9/2)
```

3.162.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.82

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \frac{-105a^6Dx-350a^5bDx^3-406a^4b^2Dx^5+8Ab^6x^7-176a^3b^3Dx^7+2ab}{105a^3b^4(a+bx^2)^{7/2}} - \frac{D\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
(-105*a^6*D*x - 350*a^5*b*D*x^3 - 406*a^4*b^2*D*x^5 + 8*A*b^6*x^7 - 176*a^
3*b^3*D*x^7 + 2*a*b^5*x^5*(14*A + 3*B*x^2) + a^2*b^4*x^3*(35*A + 21*B*x^2
+ 15*C*x^4))/(105*a^3*b^4*(a + b*x^2)^(7/2)) - (D*Log[-(Sqrt[b]*x) + Sqrt[
a + b*x^2]])/b^(9/2)
```

3.162.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2335, 9, 25, 1586, 9, 25, 27, 357, 252, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2335}$$

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x\left(7aDx^5+7a\left(C-\frac{aD}{b}\right)x^3+\left(4Ab+\frac{3a(Da^2-bCa+b^2B)}{b^2}\right)x\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{9}$$

$$\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x^2\left(7aDx^4+7a\left(C-\frac{aD}{b}\right)x^2+4Ab+\frac{3a(Da^2-bCa+b^2B)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{x^2\left(7aDx^4+7a\left(C-\frac{aD}{b}\right)x^2+4Ab+\frac{3a(Da^2-bCa+b^2B)}{b^2}\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$\frac{\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x\left(\frac{35a^2Dx^3}{b}+\left(8Ab+\frac{3a\left(-12Da^2+5bCa+2b^2B\right)}{b^2}\right)x\right)}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x^2\left(35a^2Dx^2+b\left(8Ab+\frac{3a\left(-12Da^2+5bCa+2b^2B\right)}{b^2}\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{x^2\left(8Ab^2+35a^2Dx^2+3a\left(-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{b\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{x^2\left(8Ab^2+35a^2Dx^2+3a\left(-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 357$$

$$\frac{\frac{\frac{35a^2D\int \frac{x^2}{\left(bx^2+a\right)^{3/2}}dx}{b}+\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 252$$

3.162. $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{35a^2D\left(\frac{\int \frac{1}{\sqrt{bx^2+a}}dx}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 224

$$\frac{\frac{\frac{35a^2D\left(\frac{\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}}{b}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}+\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{3ab\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{x^3\left(a\left(-71a^2D+15abC+6b^2B\right)+8Ab^3\right)}{3ab\left(a+bx^2\right)^{3/2}}+\frac{35a^2D\left(\frac{\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}-\frac{x}{b\sqrt{a+bx^2}}\right)}{b}}{5ab}+\frac{x^3\left(\frac{a\left(17a^2D-10abC+3b^2B\right)}{b^2}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(A-\frac{a(a^2D-abC+b^2B)}{b^3}\right)}{7a\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6))/(a + b*x^2)^(9/2),x]
```

output
```
((A - (a*(b^2*B - a*b*C + a^2*D))/b^3)*x^3)/(7*a*(a + b*x^2)^(7/2)) + (((4
*A*b + (a*(3*b^2*B - 10*a*b*C + 17*a^2*D))/b^2)*x^3)/(5*a*(a + b*x^2)^(5/2
)) + (((8*A*b^3 + a*(6*b^2*B + 15*a*b*C - 71*a^2*D))*x^3)/(3*a*b*(a + b*x^
2)^(3/2)) + (35*a^2*D*(-(x/(b*Sqrt[a + b*x^2])) + ArcTanh[(Sqrt[b]*x)/Sqrt
[a + b*x^2]]/b^(3/2)))/b)/(5*a*b))/(7*a*b)
```

3.162. $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

3.162.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 357
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_
Symbol] :> Simp[(b*c - a*d)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*b*e*(m +
1))), x] + Simp[d/b   Int[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b*c - a*d, 0] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

3.162. $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.162.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{\frac{a^{2} x^{3} \left(\frac{3}{7} C\, x^{4}+\frac{3}{5} x^{2} B+A\right) b^{\frac{9}{2}}}{3}+\frac{4 a \left(\frac{3 x^{2} B}{14}+A\right) x^{5} b^{\frac{11}{2}}}{15}+\frac{8 A\, b^{\frac{13}{2}} x^{7}}{105}+a^{3} \left(\left(b\, x^{2}+a\right)^{\frac{7}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b\, x^{2}+a}}{x \sqrt{b}}\right)-\frac{176 x^{7} b^{\frac{7}{2}}}{105}-\frac{58 b^{\frac{5}{2}} a\, x^{5}}{15}-\frac{10}{}\right.}{\left(b\, x^{2}+a\right)^{\frac{7}{2}} b^{\frac{9}{2}} a^{3}}$
default	$D \left(-\frac{x^{7}}{7 b (b\, x^{2}+a)^{\frac{7}{2}}}+\frac{-\frac{x^{5}}{5 b \left(b\, x^{2}+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^{3}}{3 b \left(b\, x^{2}+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b \sqrt{b\, x^{2}+a}}+\frac{\ln\left(x \sqrt{b}+\sqrt{b\, x^{2}+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+C \left(-\frac{x^{5}}{2 b (b\, x^{2}+a)^{\frac{7}{2}}}+\right.$

3.162. $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx$

input
```
int(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*x^2+a)^(7/2)*(1/3*a^2*x^3*(3/7*C*x^4+3/5*x^2*B+A)*b^(9/2)+4/15*a*(3/1
4*x^2*B+A)*x^5*b^(11/2)+8/105*A*b^(13/2)*x^7+a^3*((b*x^2+a)^(7/2)*arctanh(
(b*x^2+a)^(1/2)/x/b^(1/2))-176/105*x^7*b^(7/2)-58/15*b^(5/2)*a*x^5-10/3*b^
(3/2)*a^2*x^3-b^(1/2)*a^3*x)*D)/b^(9/2)/a^3
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.74

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7)\sqrt{b}\log\left(-\right.}{}\right.$$

$$-\frac{105\,(Da^3b^4x^8+4\,Da^4b^3x^6+6\,Da^5b^2x^4+4\,Da^6bx^2+Da^7)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+(105\,Da^6bx+(176\,D}{105\,(a^3b^9x^8+4\,a^4b^8x^6}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
[1/210*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 + 6*D*a^5*b^2*x^4 + 4*D*a^6*b
*x^2 + D*a^7)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
(105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 - 8*A*b^7)*x^7
+ 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*a^5*b^2 - A*a^
2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 + 6*a^5*b^7*x^4
+ 4*a^6*b^6*x^2 + a^7*b^5), -1/105*(105*(D*a^3*b^4*x^8 + 4*D*a^4*b^3*x^6 +
 6*D*a^5*b^2*x^4 + 4*D*a^6*b*x^2 + D*a^7)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(
b*x^2 + a)) + (105*D*a^6*b*x + (176*D*a^3*b^4 - 15*C*a^2*b^5 - 6*B*a*b^6 -
 8*A*b^7)*x^7 + 7*(58*D*a^4*b^3 - 3*B*a^2*b^5 - 4*A*a*b^6)*x^5 + 35*(10*D*
a^5*b^2 - A*a^2*b^5)*x^3)*sqrt(b*x^2 + a))/(a^3*b^9*x^8 + 4*a^4*b^8*x^6 +
6*a^5*b^7*x^4 + 4*a^6*b^6*x^2 + a^7*b^5)]
```

3.162.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3803 vs. 2(178) = 356.

Time = 66.25 (sec) , antiderivative size = 3803, normalized size of antiderivative = 21.25

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x*
*2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(3
5*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) +
105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))) + C*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*
sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)
*b**3*x**6*sqrt(1 + b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**...
```

3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. $2(160) = 320$.

Time = 0.25 (sec) , antiderivative size = 533, normalized size of antiderivative = 2.98

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Dx - \frac{Dx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b} - \frac{Cx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{Dx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2} - \frac{Dax^3}{(bx^2+a)^{\frac{5}{2}}b^3} - \frac{5\,Cax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{139\,Dx}{105\,\sqrt{bx^2+a}b^4} + \frac{17\,Dax}{105\,(bx^2+a)^{\frac{3}{2}}b^4} - \frac{29\,Da^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4} + \frac{Cx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Cx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Cax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Ca^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Bx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Bx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Bx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Bax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Ax}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Ax}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Ax}{35\,(bx^2+a)^{\frac{5}{2}}ab} + \frac{D\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*D*x -
1/15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2)
+ 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) - 1/3
*D*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - D*a
*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B
*x^3/((b*x^2 + a)^(7/2)*b) + 139/105*D*x/(sqrt(b*x^2 + a)*b^4) + 17/105*D*
a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14
*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*C*a*
x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*
B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*B*
x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*A*x
/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*A*x/((b
*x^2 + a)^(3/2)*a^2*b) + 1/35*A*x/((b*x^2 + a)^(5/2)*a*b) + D*arcsinh(b*x/
sqrt(a*b))/b^(9/2)
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.89

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx = -\frac{\left(\left(x^2\left(\frac{(176\,Da^3b^6-15\,Ca^2b^7-6\,Bab^8-8\,Ab^9)x^2}{a^3b^7}+\frac{7\,(58\,Da^4b^5-3\,Ba^2b^7-4\,Aab^8)}{a^3b^7}\right)+\frac{35\,(10\,Da^5b^4-Aa^2b^7)}{a^3b^7}\right)x^2+\frac{105\,Da^3}{b^4}\right)x}{105\,(bx^2+a)^{\frac{7}{2}}} - \frac{D\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate(x^2*(D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((x^2*((176*D*a^3*b^6 - 15*C*a^2*b^7 - 6*B*a*b^8 - 8*A*b^9)*x^2/(a^
3*b^7) + 7*(58*D*a^4*b^5 - 3*B*a^2*b^7 - 4*A*a*b^8)/(a^3*b^7)) + 35*(10*D*
a^5*b^4 - A*a^2*b^7)/(a^3*b^7))*x^2 + 105*D*a^3/b^4)*x/(b*x^2 + a)^(7/2) -
 D*log(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.162.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.163 $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$

3.163.1 Optimal result

Integrand size = 29, antiderivative size = 134

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{Ax}{a(a+bx^2)^{7/2}}+\frac{(6Ab+aB)x^3}{3a^2(a+bx^2)^{7/2}} + \frac{(24Ab^2+a(4bB+3aC))x^5}{15a^3(a+bx^2)^{7/2}}+\frac{(48Ab^3+a(8b^2B+6abC+15a^2D))x^7}{105a^4(a+bx^2)^{7/2}}$$

output
```
A*x/a/(b*x^2+a)^(7/2)+1/3*(6*A*b+B*a)*x^3/a^2/(b*x^2+a)^(7/2)+1/15*(24*A*b
^2+a*(4*B*b+3*C*a))*x^5/a^3/(b*x^2+a)^(7/2)+1/105*(48*A*b^3+a*(8*B*b^2+6*C
*a*b+15*D*a^2))*x^7/a^4/(b*x^2+a)^(7/2)
```

3.163.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.73

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{48Ab^3x^7+8ab^2x^5(21A+Bx^2)+2a^2bx^3(105A+14Bx^2+3Cx^4)+a^3(105}{105a^4(a+bx^2)^{7/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(48*A*b^3*x^7 + 8*a*b^2*x^5*(21*A + B*x^2) + 2*a^2*b*x^3*(105*A + 14*B*x^2
 + 3*C*x^4) + a^3*(105*A*x + 35*B*x^3 + 21*C*x^5 + 15*D*x^7))/(105*a^4*(a
+ b*x^2)^(7/2))
```

3.163.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.37, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {2344, 2089, 1586, 9, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2344}$$

$$\frac{\int \frac{x^2(6Ab+a(Dx^4+Cx^2+B))}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$\frac{\int \frac{x^2(aDx^4+aCx^2+6Ab+aB)}{(bx^2+a)^{9/2}}dx}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$\frac{\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x\left(\frac{7a^2Dx^3}{b}+\left(24Ab+\frac{a(-3Da^2+3bCa+4b^2B)}{b^2}\right)x\right)}{(bx^2+a)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{9}$$

$$\frac{\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x^2\left(7a^2Dx^2+b\left(24Ab+\frac{a(-3Da^2+3bCa+4b^2B)}{b^2}\right)\right)}{b(bx^2+a)^{7/2}}dx}{7a}}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Dx^2+a\left(-\frac{3Da^2}{b}+3Ca+4bB\right)\right)}{b(bx^2+a)^{7/2}}dx}{7a}+\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{x^2\left(24Ab^2+7a^2Dx^2+a\left(-\frac{3Da^2}{b}+3Ca+4bB\right)\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 362$$

$$\frac{\frac{\frac{\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)\int \frac{x^2}{(bx^2+a)^{5/2}}dx}{5ab}+\frac{x^3\left(a\left(-10a^2D+3abC+4b^2B\right)+24Ab^3\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}+\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

$$\Big\downarrow \; 242$$

$$\frac{\frac{x^3\left(\frac{a(a^2D-abC+b^2B)}{b^2}+6Ab\right)}{7a(a+bx^2)^{7/2}}+\frac{\frac{x^3\left(a\left(15a^2D+6abC+8b^2B\right)+48Ab^3\right)}{15a^2b\left(a+bx^2\right)^{3/2}}+\frac{x^3\left(a\left(-10a^2D+3abC+4b^2B\right)+24Ab^3\right)}{5ab\left(a+bx^2\right)^{5/2}}}{7ab}}{a}+\frac{Ax}{a\,(a+bx^2)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(a + b*x^2)^(9/2),x]
```

output
```
(A*x)/(a*(a + b*x^2)^(7/2)) + (((6*A*b + (a*(b^2*B - a*b*C + a^2*D))/b^2)*
x^3)/(7*a*(a + b*x^2)^(7/2)) + (((24*A*b^3 + a*(4*b^2*B + 3*a*b*C - 10*a^2
*D))*x^3)/(5*a*b*(a + b*x^2)^(5/2)) + ((48*A*b^3 + a*(8*b^2*B + 6*a*b*C +
15*a^2*D))*x^3)/(15*a^2*b*(a + b*x^2)^(3/2)))/(7*a*b))/a
```

3.163.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

3.163. $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$

3.163.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{\left(\left(\frac{1}{7}Dx^6+\frac{1}{5}Cx^4+\frac{1}{3}x^2B+A\right)a^3+2\left(\frac{1}{35}Cx^4+\frac{2}{15}x^2B+A\right)bx^2a^2+\frac{8\left(\frac{x^2B}{21}+A\right)b^2x^4a}{5}+\frac{16x^6b^3A}{35}\right)x}{(bx^2+a)^{\frac{7}{2}}a^4}$
gosper	$\frac{x(48x^6b^3A+8Bab^2x^6+6a^2bCx^6+15Da^3x^6+168aAb^2x^4+28Ba^2bx^4+21a^3Cx^4+210a^2Abx^2+35Ba^3x^2+105a^3A)}{105(bx^2+a)^{\frac{7}{2}}a^4}$
trager	$\frac{x(48x^6b^3A+8Bab^2x^6+6a^2bCx^6+15Da^3x^6+168aAb^2x^4+28Ba^2bx^4+21a^3Cx^4+210a^2Abx^2+35Ba^3x^2+105a^3A)}{105(bx^2+a)^{\frac{7}{2}}a^4}$
default	$A\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a(bx^2+a)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a(bx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)+D\left(-\frac{x^5}{2b(bx^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b(bx^2+a)}\right.}{}\right.$

3.163. $\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx$

input
```
int((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
((1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*a^3+2*(1/35*C*x^4+2/15*x^2*B+A)*b*x^2*a
^2+8/5*(1/21*x^2*B+A)*b^2*x^4*a+16/35*x^6*b^3*A)*x/(b*x^2+a)^(7/2)/a^4
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{((15\,Da^3+6\,Ca^2b+8\,Bab^2+48\,Ab^3)x^7+7\,(3\,Ca^3+4\,Ba^2b+24\,Aab^2)x^5}{105\,(a^4b^4x^8+4\,a^5b^3x^6+6\,a^6b^2x^4+4\,a^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*((15*D*a^3 + 6*C*a^2*b + 8*B*a*b^2 + 48*A*b^3)*x^7 + 7*(3*C*a^3 + 4*
B*a^2*b + 24*A*a*b^2)*x^5 + 105*A*a^3*x + 35*(B*a^3 + 6*A*a^2*b)*x^3)*sqrt
(b*x^2 + a)/(a^4*b^4*x^8 + 4*a^5*b^3*x^6 + 6*a^6*b^2*x^4 + 4*a^7*b*x^2 + a
^8)
```

3.163.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2088 vs. 2(129) = 258.

Time = 47.67 (sec) , antiderivative size = 2088, normalized size of antiderivative = 15.58

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

3.163.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. 2(120) = 240.

Time = 0.20 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.50

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = -\frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Dx}{7\,\sqrt{bx^2+a}ab^3} + \frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1
/4*C*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*
x/((b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b
*x^2 + a)^(7/2)*a) + 1/14*D*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^
2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2
 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^
2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 +
a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5
/2)*a*b)
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(15\,Da^3b^3+6\,Ca^2b^4+8\,Bab^5+48\,Ab^6)x^2}{a^4b^3}+\frac{7\,(3\,Ca^3b^3+4\,Ba^2b^4+24\,Aab^5)}{a^4b^3}\right)+\frac{35\,(Ba^3b^3 \cdots}{a^4 \cdots}\right.\right.}{105\,(bx^2+a)^{\frac{7}{2}}}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((15*D*a^3*b^3 + 6*C*a^2*b^4 + 8*B*a*b^5 + 48*A*b^6)*x^2/(a^4*
b^3) + 7*(3*C*a^3*b^3 + 4*B*a^2*b^4 + 24*A*a*b^5)/(a^4*b^3)) + 35*(B*a^3*b
^3 + 6*A*a^2*b^4)/(a^4*b^3))*x^2 + 105*A/a)*x/(b*x^2 + a)^(7/2)
```

3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{(a+bx^2)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.164 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx$

3.164.1 Optimal result

Integrand size = 32, antiderivative size = 185

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = -\frac{A}{ax\left(a+bx^2\right)^{7/2}}-\frac{(8Ab-aB)x}{a^2\left(a+bx^2\right)^{7/2}} -\frac{\left(48Ab^2-a(6bB+aC)\right)x^3}{3a^3\left(a+bx^2\right)^{7/2}}-\frac{\left(4b(48Ab^2-a(6bB+aC))-3a^3D\right)x^5}{15a^4\left(a+bx^2\right)^{7/2}} -\frac{2b\left(4b(48Ab^2-a(6bB+aC))-3a^3D\right)x^7}{105a^5\left(a+bx^2\right)^{7/2}}$$

output
```
-A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(
6*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*D
*a^3)*x^5/a^4/(b*x^2+a)^(7/2)-2/105*b*(4*b*(48*A*b^2-a*(6*B*b+C*a))-3*D*a^
3)*x^7/a^5/(b*x^2+a)^(7/2)
```

3.164.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+Cx^4)-}{105a^5}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 2
1*B*x^2 + C*x^4) - 7*a^4*(15*A - 15*B*x^2 - 5*C*x^4 - 3*D*x^6) + 2*a^3*b*x
^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6))/(105*a^5*x*(a + b*x^2)^(7/2)
)
```

3.164.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2334, 2087, 1469, 362, 245, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{8Ab-a\left(Dx^4+Cx^2+B\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2087}$$

$$-\frac{\int \frac{-aDx^4-aCx^2+8Ab-aB}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1469}$$

$$-\frac{\frac{\int \frac{x^2\left(-Dx^2a^2-Ca^2+6b(8Ab-aB)\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{362}$$

$$-\frac{\frac{\frac{\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)\int \frac{x^2}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x^3\left(48Ab^3-a\left(a^2(-D)+abC+6b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a\left(a+bx^2\right)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{245}$$

$$-\frac{\frac{\frac{\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)\left(\frac{2b\int\frac{x^4}{\left(bx^2+a\right)^{7/2}}dx}{3a}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x^3\left(48Ab^3-a\left(a^2(-D)+abC+6b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 242$$

$$-\frac{\frac{\frac{x^3\left(48Ab^3-a\left(a^2(-D)+abC+6b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{\left(\frac{2bx^5}{15a^2\left(a+bx^2\right)^{5/2}}+\frac{x^3}{3a\left(a+bx^2\right)^{5/2}}\right)\left(-3a^3D-4ab(aC+6bB)+192Ab^3\right)}{7ab}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
-(A/(a*x*(a + b*x^2)^(7/2))) - (((8*A*b - a*B)*x)/(a*(a + b*x^2)^(7/2)) +
(((48*A*b^3 - a*(6*b^2*B + a*b*C - a^2*D))*x^3)/(7*a*b*(a + b*x^2)^(7/2))
+ ((192*A*b^3 - 4*a*b*(6*b*B + a*C) - 3*a^3*D)*(x^3/(3*a*(a + b*x^2)^(5/2)
) + (2*b*x^5)/(15*a^2*(a + b*x^2)^(5/2))))/(7*a*b))/a)/a
```

3.164.3.1 Defintions of rubi rules used

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1469
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[a^p*x*((d + e*x^2)^(q + 1)/d), x] + Simp[1/d   Int[x^2*(d
 + e*x^2)^q*(d*PolynomialQuotient[(a + b*x^2 + c*x^4)^p - a^p, x^2, x] - e*
a^p*(2*q + 3)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && ILtQ[q + 1/2, 0] && LtQ[4
*p + 2*q + 1, 0]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.164.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(-\frac{1}{5}Dx^6-\frac{1}{3}Cx^4-x^2B+A\right)a^4+8b\left(-\frac{1}{140}Dx^6-\frac{1}{30}Cx^4-\frac{1}{4}x^2B+A\right)x^2a^3+16\left(-\frac{1}{210}Cx^4-\frac{1}{10}x^2B+A\right)b^2x^4a^2+\frac{64b^3x^6\left(-\right.}{5}}{(bx^2+a)^{\frac{7}{2}}xa^5}$
gosper	$-\frac{384Ab^4x^8-48Bab^3x^8-8Ca^2b^2x^8-6Da^3bx^8+1344Aab^3x^6-168Ba^2b^2x^6-28Ca^3bx^6-21Da^4x^6+1680Aa^2b^2x^4-210}{105x(bx^2+a)^{\frac{7}{2}}a^5}$
trager	$-\frac{384Ab^4x^8-48Bab^3x^8-8Ca^2b^2x^8-6Da^3bx^8+1344Aab^3x^6-168Ba^2b^2x^6-28Ca^3bx^6-21Da^4x^6+1680Aa^2b^2x^4-210}{105x(bx^2+a)^{\frac{7}{2}}a^5}$
default	$B\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a(bx^2+a)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a(bx^2+a)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)+D\left(-\frac{x^3}{4b(bx^2+a)^{\frac{7}{2}}}+\frac{3a\left(-\frac{x}{6b\left(bx^2+a\right)}\right.}{}\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-((-1/5*D*x^6-1/3*C*x^4-x^2*B+A)*a^4+8*b*(-1/140*D*x^6-1/30*C*x^4-1/4*x^2*
B+A)*x^2*a^3+16*(-1/210*C*x^4-1/10*x^2*B+A)*b^2*x^4*a^2+64/5*b^3*x^6*(-1/2
8*x^2*B+A)*a+128/35*A*b^4*x^8)/(b*x^2+a)^(7/2)/x/a^5
```

3.164. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}\,dx$

3.164.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{(2\left(3\,Da^3b+4\,Ca^2b^2+24\,Bab^3-192\,Ab^4\right)x^8+7\left(3\,Da^4+4\,Ca^3b+24\,Ba\right.}{105\left(a^5b^4x^9+\right.}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(2*(3*D*a^3*b + 4*C*a^2*b^2 + 24*B*a*b^3 - 192*A*b^4)*x^8 + 7*(3*D*a
^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35*(C*a^4 +
 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqrt(b*x^2 +
 a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 + a^9*x)
```

3.164.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2392 vs. 2(170) = 340.

Time = 77.57 (sec) , antiderivative size = 2392, normalized size of antiderivative = 12.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqr
t(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/
a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12
*sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b
**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
+ 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + ...
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{Dx^3}{4\,(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Bx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}a} + \frac{3\,Dx}{140\,(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Dx}{35\,\sqrt{bx^2+a}a^2b^2} + \frac{Dx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Dax}{28\,(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx}{7\,(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\,\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35\,(bx^2+a)^{\frac{5}{2}}ab} - \frac{128\,Abx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-1/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*
B*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/(
(b*x^2 + a)^(7/2)*a) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(
b*x^2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*
x^2 + a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^
2 + a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a
)^(5/2)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 +
a)^(3/2)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 +
a)^(7/2)*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7)x^2}{a^{14}b^3}+\frac{7\,(3\,Da^{13}b^3+4\,Ca^{12}b^4+24\,Ba^{11}b^5-132\cdots}{a^{14}b^3}\cdots\right.\right.\right.}{105\,(bx^2+a)\cdots} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6 - 279*A*a^9*b^7)
*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^11*b^5 - 132*A*a
^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*A*a^11*b^5)/(a^1
4*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*x/(b*x^2 + a)^(7
/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a^4)
```

3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^2\,\left(b\,x^2+a\right)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

3.165 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$

3.165.1 Optimal result

Integrand size = 32, antiderivative size = 242

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx = -\frac{A}{3ax^3(a+bx^2)^{7/2}}+\frac{10Ab-3aB}{3a^2x(a+bx^2)^{7/2}} + \frac{(80Ab^2-3a(8bB-aC))x}{3a^3(a+bx^2)^{7/2}}+\frac{(160Ab^3-a(48b^2B-6abC-a^2D))x^3}{3a^4(a+bx^2)^{7/2}} + \frac{4b(160Ab^3-a(48b^2B-6abC-a^2D))x^5}{15a^5(a+bx^2)^{7/2}} + \frac{8b^2(160Ab^3-a(48b^2B-6abC-a^2D))x^7}{105a^6(a+bx^2)^{7/2}}$$

output
```
-1/3*A/a/x^3/(b*x^2+a)^(7/2)+1/3*(10*A*b-3*B*a)/a^2/x/(b*x^2+a)^(7/2)+1/3*
(80*A*b^2-3*a*(8*B*b-C*a))*x/a^3/(b*x^2+a)^(7/2)+1/3*(160*A*b^3-a*(48*B*b^
2-6*C*a*b-D*a^2))*x^3/a^4/(b*x^2+a)^(7/2)+4/15*b*(160*A*b^3-a*(48*B*b^2-6*
C*a*b-D*a^2))*x^5/a^5/(b*x^2+a)^(7/2)+8/105*b^2*(160*A*b^3-a*(48*B*b^2-6*C
*a*b-D*a^2))*x^7/a^6/(b*x^2+a)^(7/2)
```

3.165.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.68

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{1280Ab^5x^{10}+128ab^4x^8(35A-3Bx^2)+16a^2b^3x^6(350A-84Bx^2+3Cx^4)}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
(1280*A*b^5*x^10 + 128*a*b^4*x^8*(35*A - 3*B*x^2) + 16*a^2*b^3*x^6*(350*A
- 84*B*x^2 + 3*C*x^4) - 35*a^5*(A + 3*B*x^2 - 3*C*x^4 - D*x^6) + 8*a^3*b^2
*x^4*(350*A - 210*B*x^2 + 21*C*x^4 + D*x^6) + 14*a^4*b*x^2*(25*A - 60*B*x^
2 + 15*C*x^4 + 2*D*x^6))/(105*a^6*x^3*(a + b*x^2)^(7/2))
```

3.165.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {2334, 2089, 1588, 298, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$$

$$\Big\downarrow \; 2334$$

$$-\frac{\int \frac{10Ab-3a\left(Dx^4+Cx^2+B\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2089$$

$$-\frac{\int \frac{-3aDx^4-3aCx^2+10Ab-3aB}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 1588$$

$$-\frac{-\frac{\int \frac{80Ab^2+3a^2Dx^2-3a(8bB-aC)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{10Ab-3aB}{ax\left(a+bx^2\right)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 298$$

$$-\frac{-\frac{\frac{3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2D-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$-\frac{-\frac{\frac{3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2D-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$-\frac{-\frac{\frac{3\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)\left(\frac{4\left(\frac{2\int \frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7ab}+\frac{x\left(80Ab^3-3a\left(a^2D-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}}{a}-\frac{10Ab-3}{ax(a+bx^2}}{3a}$$

$$-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 208$$

$$-\frac{-\frac{\frac{x\left(80Ab^3-3a\left(a^2D-abC+8b^2B\right)\right)}{7ab\left(a+bx^2\right)^{7/2}}+\frac{3\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)\left(160Ab^3-a\left(a^2(-D)-6abC+48b^2B\right)\right)}{7ab}}{a}-\frac{10Ab-3aB}{ax(a+bx^2)^{7/2}}}{3a}-\frac{A}{3ax^3\left(a+bx^2\right)^{7/2}}$$

3.165. $\displaystyle\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^4*(a + b*x^2)^(9/2)),x]
```

output
```
-1/3*A/(a*x^3*(a + b*x^2)^(7/2)) - (-((10*A*b - 3*a*B)/(a*x*(a + b*x^2)^(7
/2))) - (((80*A*b^3 - 3*a*(8*b^2*B - a*b*C + a^2*D))*x)/(7*a*b*(a + b*x^2)
^(7/2)) + (3*(160*A*b^3 - a*(48*b^2*B - 6*a*b*C - a^2*D))*(x/(5*a*(a + b*x
^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])
))/(5*a)))/(7*a*b))/a)/(3*a)
```

3.165.3.1 Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.165.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{7(Dx^6+3Cx^4-3x^2B-A)a^5+70bx^2(\frac{2}{25}Dx^6+\frac{3}{5}Cx^4-\frac{12}{5}x^2B+A)a^4+560b^2x^4(\frac{1}{350}Dx^6+\frac{3}{50}Cx^4-\frac{3}{5}x^2B+A)a^3+1120(\frac{3}{350} \ldots}{21(bx^2+a)^{\frac{7}{2}}x^3a^6}$
gosper	$-\frac{-1280Ab^5x^{10}+384Bab^4x^{10}-48Ca^2b^3x^{10}-8Da^3b^2x^{10}-4480aAb^4x^8+1344Ba^2b^3x^8-168Ca^3b^2x^8-28Da^4bx^8-5600 \ldots}{105x^3(\ldots}$
trager	$-\frac{-1280Ab^5x^{10}+384Bab^4x^{10}-48Ca^2b^3x^{10}-8Da^3b^2x^{10}-4480aAb^4x^8+1344Ba^2b^3x^8-168Ca^3b^2x^8-28Da^4bx^8-5600 \ldots}{105x^3(\ldots}$
default	$C\left(\frac{x}{7a(bx^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{bx^2+a}}\right)}{7a}}{a}\right)+D\left(-\frac{x}{6b(bx^2+a)^{\frac{7}{2}}}+\frac{a\left(\frac{x}{7a\left(bx^2+a\right)^{\frac{7}{2}}} \ldots\right)}{\ldots} \ldots\right.$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*(7*(D*x^6+3*C*x^4-3*B*x^2-A)*a^5+70*b*x^2*(2/25*D*x^6+3/5*C*x^4-12/5*
x^2*B+A)*a^4+560*b^2*x^4*(1/350*D*x^6+3/50*C*x^4-3/5*x^2*B+A)*a^3+1120*(3/
350*C*x^4-6/25*x^2*B+A)*b^3*x^6*a^2+896*(-3/35*x^2*B+A)*b^4*x^8*a+256*A*b^
5*x^10)/(b*x^2+a)^(7/2)/x^3/a^6
```

3.165. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx$

3.165.5 Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{(8\,(Da^3b^2+6\,Ca^2b^3-48\,Bab^4+160\,Ab^5)x^{10}+28\,(Da^4b+6\,Ca^3b^2-48\,B$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(8*(D*a^3*b^2 + 6*C*a^2*b^3 - 48*B*a*b^4 + 160*A*b^5)*x^10 + 28*(D*a
^4*b + 6*C*a^3*b^2 - 48*B*a^2*b^3 + 160*A*a*b^4)*x^8 + 35*(D*a^5 + 6*C*a^4
*b - 48*B*a^3*b^2 + 160*A*a^2*b^3)*x^6 - 35*A*a^5 + 35*(3*C*a^5 - 24*B*a^4
*b + 80*A*a^3*b^2)*x^4 - 35*(3*B*a^5 - 10*A*a^4*b)*x^2)*sqrt(b*x^2 + a)/(a
^6*b^4*x^11 + 4*a^7*b^3*x^9 + 6*a^8*b^2*x^7 + 4*a^9*b*x^5 + a^10*x^3)
```

3.165.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2861 vs. 2(224) = 448.

Time = 120.75 (sec) , antiderivative size = 2861, normalized size of antiderivative = 11.82

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**4/(b*x**2+a)**(9/2),x)
```

output
```
A*(-7*a**6*b**(51/2)*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10
*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x
**10 + 21*a**6*b**30*x**12) + 63*a**5*b**(53/2)*x**2*sqrt(a/(b*x**2) + 1)/
(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a*
*8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 630*a**4*b**
(55/2)*x**4*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x*
*4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21
*a**6*b**30*x**12) + 1680*a**3*b**(57/2)*x**6*sqrt(a/(b*x**2) + 1)/(21*a**
11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**2
8*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 2016*a**2*b**(59/2)
*x**8*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 2
10*a**9*b**27*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*
b**30*x**12) + 1152*a*b**(61/2)*x**10*sqrt(a/(b*x**2) + 1)/(21*a**11*b**25
*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27*x**6 + 210*a**8*b**28*x**8 +
 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12) + 256*b**(63/2)*x**12*sqrt(a/
(b*x**2) + 1)/(21*a**11*b**25*x**2 + 105*a**10*b**26*x**4 + 210*a**9*b**27
*x**6 + 210*a**8*b**28*x**8 + 105*a**7*b**29*x**10 + 21*a**6*b**30*x**12))
 + B*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a*...
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.39

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^4 (a + bx^2)^{9/2}} \, dx = \frac{16 \, Cx}{35 \sqrt{bx^2 + a} a^4} + \frac{8 \, Cx}{35 \, (bx^2 + a)^{\frac{3}{2}} a^3} + \frac{6 \, Cx}{35 \, (bx^2 + a)^{\frac{5}{2}} a^2} + \frac{Cx}{7 \, (bx^2 + a)^{\frac{7}{2}} a} - \frac{Dx}{7 \, (bx^2 + a)^{\frac{7}{2}} b} + \frac{8 \, Dx}{105 \sqrt{bx^2 + a} a^3 b} + \frac{4 \, Dx}{105 \, (bx^2 + a)^{\frac{3}{2}} a^2 b} + \frac{Dx}{35 \, (bx^2 + a)^{\frac{5}{2}} ab} - \frac{128 \, Bbx}{35 \sqrt{bx^2 + a} a^5} - \frac{64 \, Bbx}{35 \, (bx^2 + a)^{\frac{3}{2}} a^4} - \frac{48 \, Bbx}{35 \, (bx^2 + a)^{\frac{5}{2}} a^3} - \frac{8 \, Bbx}{7 \, (bx^2 + a)^{\frac{7}{2}} a^2} + \frac{256 \, Ab^2 x}{21 \sqrt{bx^2 + a} a^6} + \frac{128 \, Ab^2 x}{21 \, (bx^2 + a)^{\frac{3}{2}} a^5} + \frac{32 \, Ab^2 x}{7 \, (bx^2 + a)^{\frac{5}{2}} a^4} + \frac{80 \, Ab^2 x}{21 \, (bx^2 + a)^{\frac{7}{2}} a^3} - \frac{B}{(bx^2 + a)^{\frac{7}{2}} ax} + \frac{10 \, Ab}{3 \, (bx^2 + a)^{\frac{7}{2}} a^2 x} - \frac{A}{3 \, (bx^2 + a)^{\frac{7}{2}} ax^3}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*C*x/(sqrt(b*x^2 + a)*a^4) + 8/35*C*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
C*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*C*x/((b*x^2 + a)^(7/2)*a) - 1/7*D*x/((b*
x^2 + a)^(7/2)*b) + 8/105*D*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*D*x/((b*x^2
+ a)^(3/2)*a^2*b) + 1/35*D*x/((b*x^2 + a)^(5/2)*a*b) - 128/35*B*b*x/(sqrt(
b*x^2 + a)*a^5) - 64/35*B*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*B*b*x/((b*x^
2 + a)^(5/2)*a^3) - 8/7*B*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*A*b^2*x/(sq
rt(b*x^2 + a)*a^6) + 128/21*A*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*A*b^2*x
/((b*x^2 + a)^(5/2)*a^4) + 80/21*A*b^2*x/((b*x^2 + a)^(7/2)*a^3) - B/((b*x
^2 + a)^(7/2)*a*x) + 10/3*A*b/((b*x^2 + a)^(7/2)*a^2*x) - 1/3*A/((b*x^2 +
a)^(7/2)*a*x^3)
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{(8\,Da^{15}b^5+48\,Ca^{14}b^6-279\,Ba^{13}b^7+790\,Aa^{12}b^8)x^2}{a^{18}b^3}+\frac{7\,(4\,Da^{16}b^4+24\,Ca^{15}b^5-132\,Ba^{14}b^6\cdots}{a^{18}b^3}\right.\right.\right.}{10\cdots}$$
$$+\frac{2\left(3\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ba\sqrt{b}-12\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^4Ab^{\frac{3}{2}}-6\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2Ba^2\sqrt{b}+30\left(\sqrt{b}x\cdots\right.\right.}{3\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)^3a^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^4/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((8*D*a^15*b^5 + 48*C*a^14*b^6 - 279*B*a^13*b^7 + 790*A*a^12*b
^8)*x^2/(a^18*b^3) + 7*(4*D*a^16*b^4 + 24*C*a^15*b^5 - 132*B*a^14*b^6 + 36
5*A*a^13*b^7)/(a^18*b^3)) + 35*(D*a^17*b^3 + 6*C*a^16*b^4 - 30*B*a^15*b^5
+ 80*A*a^14*b^6)/(a^18*b^3))*x^2 + 105*(C*a^17*b^3 - 4*B*a^16*b^4 + 10*A*a
^15*b^5)/(a^18*b^3))*x/(b*x^2 + a)^(7/2) + 2/3*(3*(sqrt(b)*x - sqrt(b*x^2
+ a))^4*B*a*sqrt(b) - 12*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*b^(3/2) - 6*(sq
rt(b)*x - sqrt(b*x^2 + a))^2*B*a^2*sqrt(b) + 30*(sqrt(b)*x - sqrt(b*x^2 +
a))^2*A*a*b^(3/2) + 3*B*a^3*sqrt(b) - 14*A*a^2*b^(3/2))/(((sqrt(b)*x - sqr
t(b*x^2 + a))^2 - a)^3*a^5)
```

3.165.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^4(a+bx^2)^{9/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^4\,(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^4*(a + b*x^2)^(9/2)), x)
```

3.166 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$

3.166.1 Optimal result

Integrand size = 32, antiderivative size = 281

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx=-\frac{A}{5ax^5(a+bx^2)^{7/2}}+\frac{12Ab-5aB}{15a^2x^3(a+bx^2)^{7/2}}-\frac{24Ab^2-a(10bB-3aC)}{3a^3x(a+bx^2)^{7/2}}-\frac{(192Ab^3-a(80b^2B-24abC+3a^2D))x}{21a^4(a+bx^2)^{7/2}}-\frac{2(192Ab^3-a(80b^2B-24abC+3a^2D))x}{35a^5(a+bx^2)^{5/2}}-\frac{8(192Ab^3-a(80b^2B-24abC+3a^2D))x}{105a^6(a+bx^2)^{3/2}}-\frac{16(192Ab^3-a(80b^2B-24abC+3a^2D))x}{105a^7\sqrt{a+bx^2}}$$

output
```
-1/5*A/a/x^5/(b*x^2+a)^(7/2)+1/15*(12*A*b-5*B*a)/a^2/x^3/(b*x^2+a)^(7/2)+1
/3*(-24*A*b^2+a*(10*B*b-3*C*a))/a^3/x/(b*x^2+a)^(7/2)-1/21*(192*A*b^3-a*(8
0*B*b^2-24*C*a*b+3*D*a^2))*x/a^4/(b*x^2+a)^(7/2)-2/35*(192*A*b^3-a*(80*B*b
^2-24*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(5/2)-8/105*(192*A*b^3-a*(80*B*b^2-2
4*C*a*b+3*D*a^2))*x/a^6/(b*x^2+a)^(3/2)-16/105*(192*A*b^3-a*(80*B*b^2-24*C
*a*b+3*D*a^2))*x/a^7/(b*x^2+a)^(1/2)
```

3.166.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{-3072Ab^6x^{12}+256ab^5x^{10}(-42A+5Bx^2)-128a^2b^4x^8(105A-35Bx^2+3}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
(-3072*A*b^6*x^12 + 256*a*b^5*x^10*(-42*A + 5*B*x^2) - 128*a^2*b^4*x^8*(10
5*A - 35*B*x^2 + 3*C*x^4) + 16*a^3*b^3*x^6*(-420*A + 350*B*x^2 - 84*C*x^4
+ 3*D*x^6) + 56*a^4*b^2*x^4*(-15*A + 50*B*x^2 - 30*C*x^4 + 3*D*x^6) + 14*a
^5*b*x^2*(6*A + 25*B*x^2 - 60*C*x^4 + 15*D*x^6) - 7*a^6*(3*A + 5*x^2*(B +
3*C*x^2 - 3*D*x^4)))/(105*a^7*x^5*(a + b*x^2)^(7/2))
```

3.166.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.83, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {2334, 2089, 1588, 27, 359, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \ 2334$$

$$-\frac{\int \frac{12Ab-5a\left(Dx^4+Cx^2+B\right)}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 2089$$

$$-\frac{\int \frac{-5aDx^4-5aCx^2+12Ab-5aB}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 1588$$

$$-\frac{-\frac{\int \frac{5\left(24Ab^2+3a^2Dx^2-a(10bB-3aC)\right)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{5\int \frac{24Ab^2+3a^2Dx^2-a(10bB-3aC)}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 359$$

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)\int \frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)\left(\frac{6\int \frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/2}}}{5a}-\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 209$$

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)\left(\frac{6\left(\frac{4\int \frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{12Ab-5aB}{3ax^3(a+bx^2)^{7/}}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \ 209$$

3.166. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx$

$$-\frac{-\frac{5\left(-\frac{\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 208$$

3.166. $\int\frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx$

$$-\frac{-\frac{5\left(-\frac{\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)\left(192Ab^3-a\left(3a^2D-24abC+80b^2B\right)\right)}{a}-\frac{24Ab^2-a(10bB-3aC)}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-}{5a}$$

$$\frac{A}{5ax^5\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^6*(a + b*x^2)^(9/2)),x]
```

output
```
-1/5*A/(a*x^5*(a + b*x^2)^(7/2)) - (-1/3*(12*A*b - 5*a*B)/(a*x^3*(a + b*x^
2)^(7/2)) - (5*(-((24*A*b^2 - a*(10*b*B - 3*a*C))/(a*x*(a + b*x^2)^(7/2)))
 - ((192*A*b^3 - a*(80*b^2*B - 24*a*b*C + 3*a^2*D))*(x/(7*a*(a + b*x^2)^(7
/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*
x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)))/a))/(3*a))/(5*a)
```

3.166.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.166.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{\left(-5Dx^{6}+5Cx^{4}+\frac{5}{3}x^{2}B+A\right)a^{6}-4\left(\frac{5}{2}Dx^{6}-10Cx^{4}+\frac{25}{6}x^{2}B+A\right)b\,x^{2}a^{5}+40\left(-\frac{1}{5}Dx^{6}+2Cx^{4}-\frac{10}{3}x^{2}B+A\right)b^{2}x^{4}a^{4}+320b^{3}x}{5\left(b\,x^{2}+a\right)^{\frac{7}{2}}x^{5}}$
gosper	$-\frac{3072Ab^{6}x^{12}-1280Bab^{5}x^{12}+384Ca^{2}b^{4}x^{12}-48Da^{3}b^{3}x^{12}+10752Aab^{5}x^{10}-4480Ba^{2}b^{4}x^{10}+1344Ca^{3}b^{3}x^{10}-168Da^{4}b^{2}}{}$
trager	$-\frac{3072Ab^{6}x^{12}-1280Bab^{5}x^{12}+384Ca^{2}b^{4}x^{12}-48Da^{3}b^{3}x^{12}+10752Aab^{5}x^{10}-4480Ba^{2}b^{4}x^{10}+1344Ca^{3}b^{3}x^{10}-168Da^{4}b^{2}}{}$
default	$D\left(\frac{x}{7a\left(b\,x^{2}+a\right)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^{2}+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^{2}+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^{2}\sqrt{b\,x^{2}+a}}\right)}{7a}}{a}\right)+B\left(-\frac{1}{3a\,x^{3}\left(b\,x^{2}+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{}{ax\left(bx\right.}\right.}{}\right.$

3.166. $\int \frac{A+Bx^{2}+Cx^{4}+Dx^{6}}{x^{6}\left(a+bx^{2}\right)^{9/2}}\,dx$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*((-5*D*x^6+5*C*x^4+5/3*x^2*B+A)*a^6-4*(5/2*D*x^6-10*C*x^4+25/6*x^2*B+
A)*b*x^2*a^5+40*(-1/5*D*x^6+2*C*x^4-10/3*x^2*B+A)*b^2*x^4*a^4+320*b^3*x^6*
(-1/140*D*x^6+1/5*C*x^4-5/6*x^2*B+A)*a^3+640*(1/35*C*x^4-1/3*x^2*B+A)*b^4*
x^8*a^2+512*(-5/42*x^2*B+A)*b^5*x^10*a+1024/7*A*b^6*x^12)/(b*x^2+a)^(7/2)/
x^5/a^7
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{(16\,(3\,Da^3b^3-24\,Ca^2b^4+80\,Bab^5-192\,Ab^6)x^{12}+56\,(3\,Da^4b^2-24\,Ca^3b^3}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
1/105*(16*(3*D*a^3*b^3 - 24*C*a^2*b^4 + 80*B*a*b^5 - 192*A*b^6)*x^12 + 56*
(3*D*a^4*b^2 - 24*C*a^3*b^3 + 80*B*a^2*b^4 - 192*A*a*b^5)*x^10 + 70*(3*D*a
^5*b - 24*C*a^4*b^2 + 80*B*a^3*b^3 - 192*A*a^2*b^4)*x^8 - 21*A*a^6 + 35*(3
*D*a^6 - 24*C*a^5*b + 80*B*a^4*b^2 - 192*A*a^3*b^3)*x^6 - 35*(3*C*a^6 - 10
*B*a^5*b + 24*A*a^4*b^2)*x^4 - 7*(5*B*a^6 - 12*A*a^5*b)*x^2)*sqrt(b*x^2 +
a)/(a^7*b^4*x^13 + 4*a^8*b^3*x^11 + 6*a^9*b^2*x^9 + 4*a^10*b*x^7 + a^11*x^
5)
```

3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**6/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6(a+bx^2)^{9/2}}\,dx = \frac{16\,Dx}{35\,\sqrt{bx^2+a}a^4} + \frac{8\,Dx}{35\,(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Dx}{35\,(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Dx}{7\,(bx^2+a)^{\frac{7}{2}}a} - \frac{128\,Cbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Cbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Cbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Cbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Bb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Bb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Bb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Bb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Ab^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Ab^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Ab^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Ab^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} - \frac{C}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Bb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Ab^2}{(bx^2+a)^{\frac{7}{2}}a^3x} - \frac{B}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{A}{5\,(bx^2+a)^{\frac{7}{2}}ax^5}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
16/35*D*x/(sqrt(b*x^2 + a)*a^4) + 8/35*D*x/((b*x^2 + a)^(3/2)*a^3) + 6/35*
D*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*D*x/((b*x^2 + a)^(7/2)*a) - 128/35*C*b*x
/(sqrt(b*x^2 + a)*a^5) - 64/35*C*b*x/((b*x^2 + a)^(3/2)*a^4) - 48/35*C*b*x
/((b*x^2 + a)^(5/2)*a^3) - 8/7*C*b*x/((b*x^2 + a)^(7/2)*a^2) + 256/21*B*b^
2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*B*b^2*x/((b*x^2 + a)^(3/2)*a^5) + 32/7*
B*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*B*b^2*x/((b*x^2 + a)^(7/2)*a^3) -
1024/35*A*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*A*b^3*x/((b*x^2 + a)^(3/2)*
a^6) - 384/35*A*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*A*b^3*x/((b*x^2 + a)^
(7/2)*a^4) - C/((b*x^2 + a)^(7/2)*a*x) + 10/3*B*b/((b*x^2 + a)^(7/2)*a^2*x
) - 8*A*b^2/((b*x^2 + a)^(7/2)*a^3*x) - 1/3*B/((b*x^2 + a)^(7/2)*a*x^3) +
4/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 1/5*A/((b*x^2 + a)^(7/2)*a*x^5)
```

3.166.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(252) = 504$.

Time = 0.30 (sec) , antiderivative size = 592, normalized size of antiderivative = 2.11

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^6 (a + bx^2)^{9/2}} \, dx = \frac{\left(\left(x^2 \left(\frac{(48\,Da^{18}b^6 - 279\,Ca^{17}b^7 + 790\,Ba^{16}b^8 - 1686\,Aa^{15}b^9)x^2}{a^{22}b^3} + \frac{7\,(24\,Da^{19}b^5 - 132\,Ca^{18}b^6 + 365\,B}{a^{22}b^3}\right.\right.\right.}{}$$

$$+ \frac{2\left(15\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Ca^2\sqrt{b} - 60\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Bab^{\frac{3}{2}} + 150\left(\sqrt{b}x - \sqrt{bx^2+a}\right)^8 Ab^{\frac{5}{2}} - 60\left(\sqrt{\right.\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^6/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((48*D*a^18*b^6 - 279*C*a^17*b^7 + 790*B*a^16*b^8 - 1686*A*a^1
5*b^9)*x^2/(a^22*b^3) + 7*(24*D*a^19*b^5 - 132*C*a^18*b^6 + 365*B*a^17*b^7
 - 768*A*a^16*b^8)/(a^22*b^3)) + 35*(6*D*a^20*b^4 - 30*C*a^19*b^5 + 80*B*a
^18*b^6 - 165*A*a^17*b^7)/(a^22*b^3))*x^2 + 105*(D*a^21*b^3 - 4*C*a^20*b^4
 + 10*B*a^19*b^5 - 20*A*a^18*b^6)/(a^22*b^3))*x/(b*x^2 + a)^(7/2) + 2/15*(
15*(sqrt(b)*x - sqrt(b*x^2 + a))^8*C*a^2*sqrt(b) - 60*(sqrt(b)*x - sqrt(b*
x^2 + a))^8*B*a*b^(3/2) + 150*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*b^(5/2) -
60*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*a^3*sqrt(b) + 270*(sqrt(b)*x - sqrt(b
*x^2 + a))^6*B*a^2*b^(3/2) - 720*(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a*b^(5/
2) + 90*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^4*sqrt(b) - 430*(sqrt(b)*x - s
qrt(b*x^2 + a))^4*B*a^3*b^(3/2) + 1260*(sqrt(b)*x - sqrt(b*x^2 + a))^4*A*a
^2*b^(5/2) - 60*(sqrt(b)*x - sqrt(b*x^2 + a))^2*C*a^5*sqrt(b) + 290*(sqrt(
b)*x - sqrt(b*x^2 + a))^2*B*a^4*b^(3/2) - 840*(sqrt(b)*x - sqrt(b*x^2 + a)
)^2*A*a^3*b^(5/2) + 15*C*a^6*sqrt(b) - 70*B*a^5*b^(3/2) + 198*A*a^4*b^(5/2
))/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)^5*a^6)
```

3.166.9 Mupad [B] (verification not implemented)

Time = 7.41 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.44

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^6\left(a+bx^2\right)^{9/2}}\,dx = \frac{\frac{61\,A\,b}{35\,a^3}+\frac{78\,A\,b^2\,x^2}{35\,a^4}}{x^3\,\left(b\,x^2+a\right)^{5/2}}+\frac{\frac{128\,B\,b}{21\,a^5}+\frac{256\,B\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}}+\frac{x\,D}{\left(b\,x^2+a\right)^{9/2}} - \frac{\frac{B}{3\,a^2}+\frac{19\,B\,b\,x^2}{21\,a^3}}{x^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{\frac{C}{a^4}+\frac{128\,C\,b\,x^2}{35\,a^5}}{x\,\sqrt{b\,x^2+a}}-\frac{\frac{512\,A\,b^2}{35\,a^6}+\frac{1024\,A\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}}-\frac{A\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5} + \frac{18\,b^2\,x^5\,D}{5\,a^2\,\left(b\,x^2+a\right)^{9/2}}+\frac{72\,b^3\,x^7\,D}{35\,a^3\,\left(b\,x^2+a\right)^{9/2}}+\frac{16\,b^4\,x^9\,D}{35\,a^4\,\left(b\,x^2+a\right)^{9/2}}-\frac{A\,b}{7\,a^2\,x^3\,\left(b\,x^2+a\right)^{7/2}} - \frac{32\,B\,b}{21\,a^4\,x\,\left(b\,x^2+a\right)^{3/2}}+\frac{B\,b^2\,x}{7\,a^3\,\left(b\,x^2+a\right)^{7/2}}+\frac{27\,A\,b^2}{7\,a^5\,x\,\left(b\,x^2+a\right)^{3/2}} + \frac{3\,b\,x^3\,D}{a\,\left(b\,x^2+a\right)^{9/2}}-\frac{29\,C\,b\,x}{35\,a^4\,\left(b\,x^2+a\right)^{3/2}}-\frac{13\,C\,b\,x}{35\,a^3\,\left(b\,x^2+a\right)^{5/2}}-\frac{C\,b\,x}{7\,a^2\,\left(b\,x^2+a\right)^{7/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^6*(a + b*x^2)^(9/2)),x)
```

output
```
((61*A*b)/(35*a^3) + (78*A*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((
128*B*b)/(21*a^5) + (256*B*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) + (x*D
)/(a + b*x^2)^(9/2) - (B/(3*a^2) + (19*B*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)
^(5/2)) - (C/a^4 + (128*C*b*x^2)/(35*a^5))/(x*(a + b*x^2)^(1/2)) - ((512*A
*b^2)/(35*a^6) + (1024*A*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a
+ b*x^2)^(1/2))/(5*a^5*x^5) + (18*b^2*x^5*D)/(5*a^2*(a + b*x^2)^(9/2)) + (
72*b^3*x^7*D)/(35*a^3*(a + b*x^2)^(9/2)) + (16*b^4*x^9*D)/(35*a^4*(a + b*x
^2)^(9/2)) - (A*b)/(7*a^2*x^3*(a + b*x^2)^(7/2)) - (32*B*b)/(21*a^4*x*(a +
 b*x^2)^(3/2)) + (B*b^2*x)/(7*a^3*(a + b*x^2)^(7/2)) + (27*A*b^2)/(7*a^5*x
*(a + b*x^2)^(3/2)) + (3*b*x^3*D)/(a*(a + b*x^2)^(9/2)) - (29*C*b*x)/(35*a
^4*(a + b*x^2)^(3/2)) - (13*C*b*x)/(35*a^3*(a + b*x^2)^(5/2)) - (C*b*x)/(7
*a^2*(a + b*x^2)^(7/2))
```

3.167 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

3.167.1 Optimal result

Integrand size = 32, antiderivative size = 334

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx =& -\frac{A}{7ax^7(a+bx^2)^{7/2}}+\frac{2Ab-aB}{5a^2x^5(a+bx^2)^{7/2}}\\&-\frac{24Ab^2-a(12bB-5aC)}{15a^3x^3(a+bx^2)^{7/2}}+\frac{48Ab^3-a(24b^2B-10abC+3a^2D)}{3a^4x(a+bx^2)^{7/2}}\\&+\frac{8b(48Ab^3-a(24b^2B-10abC+3a^2D))x}{21a^5(a+bx^2)^{7/2}}\\&+\frac{16b(48Ab^3-a(24b^2B-10abC+3a^2D))x}{35a^6(a+bx^2)^{5/2}}\\&+\frac{64b(48Ab^3-a(24b^2B-10abC+3a^2D))x}{105a^7(a+bx^2)^{3/2}}\\&+\frac{128b(48Ab^3-a(24b^2B-10abC+3a^2D))x}{105a^8\sqrt{a+bx^2}}\end{aligned}$$

output
```
-1/7*A/a/x^7/(b*x^2+a)^(7/2)+1/5*(2*A*b-B*a)/a^2/x^5/(b*x^2+a)^(7/2)+1/15*
(-24*A*b^2+a*(12*B*b-5*C*a))/a^3/x^3/(b*x^2+a)^(7/2)+1/3*(48*A*b^3-a*(24*B
*b^2-10*C*a*b+3*D*a^2))/a^4/x/(b*x^2+a)^(7/2)+8/21*b*(48*A*b^3-a*(24*B*b^2
-10*C*a*b+3*D*a^2))*x/a^5/(b*x^2+a)^(7/2)+16/35*b*(48*A*b^3-a*(24*B*b^2-10
*C*a*b+3*D*a^2))*x/a^6/(b*x^2+a)^(5/2)+64/105*b*(48*A*b^3-a*(24*B*b^2-10*C
*a*b+3*D*a^2))*x/a^7/(b*x^2+a)^(3/2)+128/105*b*(48*A*b^3-a*(24*B*b^2-10*C*
a*b+3*D*a^2))*x/a^8/(b*x^2+a)^(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx = \frac{6144Ab^7x^{14}-3072ab^6x^{12}(-7A+Bx^2)+256a^2b^5x^{10}(105A-42Bx^2+5C$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

output
```
(6144*A*b^7*x^14 - 3072*a*b^6*x^12*(-7*A + B*x^2) + 256*a^2*b^5*x^10*(105*
A - 42*B*x^2 + 5*C*x^4) + 14*a^6*b*x^2*(3*A + 6*B*x^2 + 25*C*x^4 - 60*D*x^
6) + 112*a^4*b^3*x^6*(15*A - 60*B*x^2 + 50*C*x^4 - 12*D*x^6) + 128*a^3*b^4
*x^8*(105*A - 105*B*x^2 + 35*C*x^4 - 3*D*x^6) - 56*a^5*b^2*x^4*(3*A + 15*B
*x^2 - 50*C*x^4 + 30*D*x^6) - a^7*(15*A + 21*B*x^2 + 35*x^4*(C + 3*D*x^2))
)/(105*a^8*x^7*(a + b*x^2)^(7/2))
```

3.167.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2334, 27, 2089, 1588, 359, 245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{7(2Ab-a(Dx^4+Cx^2+B))}{x^6(bx^2+a)^{9/2}}dx}{7a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{2Ab-a(Dx^4+Cx^2+B)}{x^6(bx^2+a)^{9/2}}dx}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{\int \frac{-aDx^4-aCx^2+2Ab-aB}{x^6(bx^2+a)^{9/2}}dx}{a}-\frac{A}{7ax^7(a+bx^2)^{7/2}}$$

$\Big\downarrow$ 1588

$$-\frac{-\frac{\int \frac{24Ab^2+5a^2Dx^2-a(12bB-5aC)}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 359

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)\int\frac{1}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 245

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)\left(-\frac{8b\int\frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 209

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)\left(-\frac{8b\left(\frac{6\int\frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-\frac{2Ab-aB}{5ax^5(a+bx^2)^{7/2}}}{a}-\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$\Big\downarrow$ 209

3.167. $\displaystyle\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)\left(-\frac{8b\left(\frac{6\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{24Ab^2-a(12bB-5aC)}{3ax^3\left(a+bx^2\right)^{7/2}}}{5a}-}{a}$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 209$$

$$-\frac{-\frac{-\frac{5\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-}{5a}}{a}$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

3.167. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx$

$\Big\downarrow$ 208

$$-\frac{-\frac{-\frac{5\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)\left(48Ab^3-a\left(3a^2D-10abC+24b^2B\right)\right)}{3a}-\frac{24Ab^2-\cdots}{3ax^3\cdots}}{5a}\cdots}{a}\cdots$$

$$\frac{A}{7ax^7\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^8*(a + b*x^2)^(9/2)),x]
```

output
```
-1/7*A/(a*x^7*(a + b*x^2)^(7/2)) - (-1/5*(2*A*b - a*B)/(a*x^5*(a + b*x^2)^
(7/2)) - (-1/3*(24*A*b^2 - a*(12*b*B - 5*a*C))/(a*x^3*(a + b*x^2)^(7/2)) -
 (5*(48*A*b^3 - a*(24*b^2*B - 10*a*b*C + 3*a^2*D))*(-(1/(a*x*(a + b*x^2)^(
7/2))) - (8*b*(x/(7*a*(a + b*x^2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) +
 (4*(x/(3*a*(a + b*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(
7*a)))/a))/(3*a))/(5*a))/a
```

3.167.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

3.167. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8\left(a+bx^2\right)^{9/2}}\,dx$

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.167. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx$

3.167.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\left(-105 D x^{6}-35 C x^{4}-21 x^{2} B-15 A\right) a^{7}+42 b x^{2}\left(-20 D x^{6}+\frac{25}{3} C x^{4}+2 x^{2} B+A\right) a^{6}-168 b^{2} x^{4}\left(10 D x^{6}-\frac{50}{3} C x^{4}+5 x^{2} B+A\right) a^{5}+}{}$
gosper	$-\frac{-6144 A b^{7} x^{14}+3072 B a b^{6} x^{14}-1280 C a^{2} b^{5} x^{14}+384 D a^{3} b^{4} x^{14}-21504 A a b^{6} x^{12}+10752 B a^{2} b^{5} x^{12}-4480 C a^{3} b^{4} x^{12}+1344}{}$
trager	$-\frac{-6144 A b^{7} x^{14}+3072 B a b^{6} x^{14}-1280 C a^{2} b^{5} x^{14}+384 D a^{3} b^{4} x^{14}-21504 A a b^{6} x^{12}+10752 B a^{2} b^{5} x^{12}-4480 C a^{3} b^{4} x^{12}+1344}{}$
	$2b\left(-\frac{1}{5a x^{5}\left(b x^{2}+a\right)^{\frac{7}{2}}}-\frac{12b\left(-\frac{1}{3a x^{3}\left(b x^{2}+a\right)^{\frac{7}{2}}}-\frac{10b\left(-\frac{1}{a x\left(b x^{2}+a\right)^{\frac{7}{2}}}-\frac{8b\left(\frac{x}{7a\left(b x^{2}+a\right)^{\frac{7}{2}}}+\frac{6}{35a\left(b\right.}\right.}{}\right)}{3}\right)}{5a}\right.$

3.167. $\int \frac{A+B x^{2}+C x^{4}+D x^{6}}{x^{8}\left(a+b x^{2}\right)^{9/2}}\, dx$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-105*D*x^6-35*C*x^4-21*B*x^2-15*A)*a^7+42*b*x^2*(-20*D*x^6+25/3*C*
x^4+2*x^2*B+A)*a^6-168*b^2*x^4*(10*D*x^6-50/3*C*x^4+5*x^2*B+A)*a^5+1680*(-
4/5*D*x^6+10/3*C*x^4-4*x^2*B+A)*b^3*x^6*a^4+13440*(-1/35*D*x^6+1/3*C*x^4-x
^2*B+A)*b^4*x^8*a^3+26880*(1/21*C*x^4-2/5*x^2*B+A)*b^5*x^10*a^2+21504*b^6*
x^12*(-1/7*x^2*B+A)*a+6144*A*b^7*x^14)/(b*x^2+a)^(7/2)/x^7/a^8
```

3.167.5 Fricas [A] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.93

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^{9/2}} \, dx =$$

$$-\frac{(128\,(3\,Da^3b^4 - 10\,Ca^2b^5 + 24\,Bab^6 - 48\,Ab^7)x^{14} + 448\,(3\,Da^4b^3 - 10\,Ca^3b^4 + 24\,Ba^2b^5 - 48\,Aab^6)x^{12} +}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="fricas")
```

output
```
-1/105*(128*(3*D*a^3*b^4 - 10*C*a^2*b^5 + 24*B*a*b^6 - 48*A*b^7)*x^14 + 44
8*(3*D*a^4*b^3 - 10*C*a^3*b^4 + 24*B*a^2*b^5 - 48*A*a*b^6)*x^12 + 560*(3*D
*a^5*b^2 - 10*C*a^4*b^3 + 24*B*a^3*b^4 - 48*A*a^2*b^5)*x^10 + 280*(3*D*a^6
*b - 10*C*a^5*b^2 + 24*B*a^4*b^3 - 48*A*a^3*b^4)*x^8 + 15*A*a^7 + 35*(3*D*
a^7 - 10*C*a^6*b + 24*B*a^5*b^2 - 48*A*a^4*b^3)*x^6 + 7*(5*C*a^7 - 12*B*a^
6*b + 24*A*a^5*b^2)*x^4 + 21*(B*a^7 - 2*A*a^6*b)*x^2)*sqrt(b*x^2 + a)/(a^8
*b^4*x^15 + 4*a^9*b^3*x^13 + 6*a^10*b^2*x^11 + 4*a^11*b*x^9 + a^12*x^7)
```

3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^{9/2}} \, dx = \text{Timed out}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**8/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.46

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx = -\frac{128\,Dbx}{35\,\sqrt{bx^2+a}a^5} - \frac{64\,Dbx}{35\,(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Dbx}{35\,(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Dbx}{7\,(bx^2+a)^{\frac{7}{2}}a^2} + \frac{256\,Cb^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Cb^2x}{21\,(bx^2+a)^{\frac{3}{2}}a^5} + \frac{32\,Cb^2x}{7\,(bx^2+a)^{\frac{5}{2}}a^4} + \frac{80\,Cb^2x}{21\,(bx^2+a)^{\frac{7}{2}}a^3} - \frac{1024\,Bb^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Bb^3x}{35\,(bx^2+a)^{\frac{3}{2}}a^6} - \frac{384\,Bb^3x}{35\,(bx^2+a)^{\frac{5}{2}}a^5} - \frac{64\,Bb^3x}{7\,(bx^2+a)^{\frac{7}{2}}a^4} + \frac{2048\,Ab^4x}{35\,\sqrt{bx^2+a}a^8} + \frac{1024\,Ab^4x}{35\,(bx^2+a)^{\frac{3}{2}}a^7} + \frac{768\,Ab^4x}{35\,(bx^2+a)^{\frac{5}{2}}a^6} + \frac{128\,Ab^4x}{7\,(bx^2+a)^{\frac{7}{2}}a^5} - \frac{D}{(bx^2+a)^{\frac{7}{2}}ax} + \frac{10\,Cb}{3\,(bx^2+a)^{\frac{7}{2}}a^2x} - \frac{8\,Bb^2}{(bx^2+a)^{\frac{7}{2}}a^3x} + \frac{16\,Ab^3}{(bx^2+a)^{\frac{7}{2}}a^4x} - \frac{C}{3\,(bx^2+a)^{\frac{7}{2}}ax^3} + \frac{4\,Bb}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^3} - \frac{8\,Ab^2}{5\,(bx^2+a)^{\frac{7}{2}}a^3x^3} - \frac{B}{5\,(bx^2+a)^{\frac{7}{2}}ax^5} + \frac{2\,Ab}{5\,(bx^2+a)^{\frac{7}{2}}a^2x^5} - \frac{A}{7\,(bx^2+a)^{\frac{7}{2}}ax^7}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="maxima")
```

output
```
-128/35*D*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*D*b*x/((b*x^2 + a)^(3/2)*a^4)
- 48/35*D*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*D*b*x/((b*x^2 + a)^(7/2)*a^2)
+ 256/21*C*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*C*b^2*x/((b*x^2 + a)^(3/2)
*a^5) + 32/7*C*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*C*b^2*x/((b*x^2 + a)^
(7/2)*a^3) - 1024/35*B*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*B*b^3*x/((b*x^
2 + a)^(3/2)*a^6) - 384/35*B*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*B*b^3*x/
((b*x^2 + a)^(7/2)*a^4) + 2048/35*A*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*
A*b^4*x/((b*x^2 + a)^(3/2)*a^7) + 768/35*A*b^4*x/((b*x^2 + a)^(5/2)*a^6) +
 128/7*A*b^4*x/((b*x^2 + a)^(7/2)*a^5) - D/((b*x^2 + a)^(7/2)*a*x) + 10/3*
C*b/((b*x^2 + a)^(7/2)*a^2*x) - 8*B*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*A*b
^3/((b*x^2 + a)^(7/2)*a^4*x) - 1/3*C/((b*x^2 + a)^(7/2)*a*x^3) + 4/5*B*b/(
(b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^3) - 1/5*B
/((b*x^2 + a)^(7/2)*a*x^5) + 2/5*A*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 1/7*A/(
(b*x^2 + a)^(7/2)*a*x^7)
```

3.167.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(300) = 600$.

Time = 0.32 (sec) , antiderivative size = 938, normalized size of antiderivative = 2.81

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6}{x^8 (a + bx^2)^{9/2}}\, dx =$$

$$-\frac{\left(\left(x^2 \left(\frac{(279\, Da^{21} b^7 - 790\, Ca^{20} b^8 + 1686\, Ba^{19} b^9 - 3072\, Aa^{18} b^{10}) x^2}{a^{26} b^3} + \frac{7\,(132\, Da^{22} b^6 - 365\, Ca^{21} b^7 + 768\, Ba^{20} b^8 - 1386\, Aa^{19} b^9)}{a^{26} b^3}\right) + \frac{35\,(30\, Da\ldots}{}\right.\right.}{105\,(bx^2 + a)^{\frac{7}{2}}}$$

$$+\frac{2\left(105\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Da^3\sqrt{b} - 420\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Ca^2 b^{\frac{3}{2}} + 1050\left(\sqrt{b}x - \sqrt{bx^2 + a}\right)^{12} Bab^{\frac{5}{2}}\ldots\right.}{}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^8/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((x^2*((279*D*a^21*b^7 - 790*C*a^20*b^8 + 1686*B*a^19*b^9 - 3072*A*
a^18*b^10)*x^2/(a^26*b^3) + 7*(132*D*a^22*b^6 - 365*C*a^21*b^7 + 768*B*a^2
0*b^8 - 1386*A*a^19*b^9)/(a^26*b^3)) + 35*(30*D*a^23*b^5 - 80*C*a^22*b^6 +
 165*B*a^21*b^7 - 294*A*a^20*b^8)/(a^26*b^3))*x^2 + 105*(4*D*a^24*b^4 - 10
*C*a^23*b^5 + 20*B*a^22*b^6 - 35*A*a^21*b^7)/(a^26*b^3))*x/(b*x^2 + a)^(7/
2) + 2/105*(105*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^3*sqrt(b) - 420*(sqrt
(b)*x - sqrt(b*x^2 + a))^12*C*a^2*b^(3/2) + 1050*(sqrt(b)*x - sqrt(b*x^2 +
 a))^12*B*a*b^(5/2) - 2100*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*b^(7/2) - 63
0*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^4*sqrt(b) + 2730*(sqrt(b)*x - sqrt(
b*x^2 + a))^10*C*a^3*b^(3/2) - 7140*(sqrt(b)*x - sqrt(b*x^2 + a))^10*B*a^2
*b^(5/2) + 14700*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a*b^(7/2) + 1575*(sqrt
(b)*x - sqrt(b*x^2 + a))^8*D*a^5*sqrt(b) - 7210*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*C*a^4*b^(3/2) + 19950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*B*a^3*b^(5/2)
- 42840*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^2*b^(7/2) - 2100*(sqrt(b)*x -
sqrt(b*x^2 + a))^6*D*a^6*sqrt(b) + 9940*(sqrt(b)*x - sqrt(b*x^2 + a))^6*C*
a^5*b^(3/2) - 28560*(sqrt(b)*x - sqrt(b*x^2 + a))^6*B*a^4*b^(5/2) + 64680*
(sqrt(b)*x - sqrt(b*x^2 + a))^6*A*a^3*b^(7/2) + 1575*(sqrt(b)*x - sqrt(b*x
^2 + a))^4*D*a^7*sqrt(b) - 7560*(sqrt(b)*x - sqrt(b*x^2 + a))^4*C*a^6*b^(3
/2) + 21966*(sqrt(b)*x - sqrt(b*x^2 + a))^4*B*a^5*b^(5/2) - 49812*(sqrt(b)
*x - sqrt(b*x^2 + a))^4*A*a^4*b^(7/2) - 630*(sqrt(b)*x - sqrt(b*x^2 + a...
```

3.167.9 Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.26

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^8(a+bx^2)^{9/2}}\,dx = \frac{\frac{61\,B\,b}{35\,a^3}+\frac{78\,B\,b^2\,x^2}{35\,a^4}}{x^3\,(b\,x^2+a)^{5/2}}+\frac{\frac{128\,C\,b}{21\,a^5}+\frac{256\,C\,b^2\,x^2}{21\,a^6}}{x\,\sqrt{b\,x^2+a}} - \frac{\frac{C}{3\,a^2}+\frac{19\,C\,b\,x^2}{21\,a^3}}{x^3\,(b\,x^2+a)^{5/2}} - \frac{\frac{167\,A\,b^2}{35\,a^4}+\frac{191\,A\,b^3\,x^2}{35\,a^5}}{x^3\,(b\,x^2+a)^{5/2}} + \frac{\frac{1024\,A\,b^3}{35\,a^7}+\frac{2048\,A\,b^4\,x^2}{35\,a^8}}{x\,\sqrt{b\,x^2+a}} - \frac{\frac{512\,B\,b^2}{35\,a^6}+\frac{1024\,B\,b^3\,x^2}{35\,a^7}}{x\,\sqrt{b\,x^2+a}} - \frac{A\,\sqrt{b\,x^2+a}}{7\,a^5\,x^7} - \frac{B\,\sqrt{b\,x^2+a}}{5\,a^5\,x^5} - \frac{\left(\frac{a}{b\,x^2}+1\right)^{9/2}D\,{}_2\mathrm{F}_1\left(\frac{9}{2},5;\ 6;\ -\frac{a}{b\,x^2}\right)}{10\,x\,(b\,x^2+a)^{9/2}} + \frac{34\,A\,b\,\sqrt{b\,x^2+a}}{35\,a^6\,x^5} - \frac{B\,b}{7\,a^2\,x^3\,(b\,x^2+a)^{7/2}} - \frac{32\,C\,b}{21\,a^4\,x\,(b\,x^2+a)^{3/2}} + \frac{C\,b^2\,x}{7\,a^3\,(b\,x^2+a)^{7/2}} - \frac{58\,A\,b^3}{7\,a^6\,x\,(b\,x^2+a)^{3/2}} + \frac{A\,b^2}{7\,a^3\,x^3\,(b\,x^2+a)^{7/2}} + \frac{27\,B\,b^2}{7\,a^5\,x\,(b\,x^2+a)^{3/2}}$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^8*(a + b*x^2)^(9/2)),x)
```

output
```
((61*B*b)/(35*a^3) + (78*B*b^2*x^2)/(35*a^4))/(x^3*(a + b*x^2)^(5/2)) + ((
128*C*b)/(21*a^5) + (256*C*b^2*x^2)/(21*a^6))/(x*(a + b*x^2)^(1/2)) - (C/(
3*a^2) + (19*C*b*x^2)/(21*a^3))/(x^3*(a + b*x^2)^(5/2)) - ((167*A*b^2)/(35
*a^4) + (191*A*b^3*x^2)/(35*a^5))/(x^3*(a + b*x^2)^(5/2)) + ((1024*A*b^3)/
(35*a^7) + (2048*A*b^4*x^2)/(35*a^8))/(x*(a + b*x^2)^(1/2)) - ((512*B*b^2)
/(35*a^6) + (1024*B*b^3*x^2)/(35*a^7))/(x*(a + b*x^2)^(1/2)) - (A*(a + b*x
^2)^(1/2))/(7*a^5*x^7) - (B*(a + b*x^2)^(1/2))/(5*a^5*x^5) - ((a/(b*x^2) +
 1)^(9/2)*D*hypergeom([9/2, 5], 6, -a/(b*x^2)))/(10*x*(a + b*x^2)^(9/2)) +
 (34*A*b*(a + b*x^2)^(1/2))/(35*a^6*x^5) - (B*b)/(7*a^2*x^3*(a + b*x^2)^(7
/2)) - (32*C*b)/(21*a^4*x*(a + b*x^2)^(3/2)) + (C*b^2*x)/(7*a^3*(a + b*x^2
)^(7/2)) - (58*A*b^3)/(7*a^6*x*(a + b*x^2)^(3/2)) + (A*b^2)/(7*a^3*x^3*(a
+ b*x^2)^(7/2)) + (27*B*b^2)/(7*a^5*x*(a + b*x^2)^(3/2))
```

3.168 $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$

3.168.1 Optimal result

Integrand size = 32, antiderivative size = 392

$$\begin{aligned}\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx &= -\frac{A}{9ax^9(a+bx^2)^{7/2}}+\frac{16Ab-9aB}{63a^2x^7(a+bx^2)^{7/2}}\\ &-\frac{32Ab^2-9a(2bB-aC)}{45a^3x^5(a+bx^2)^{7/2}}+\frac{128Ab^3-3a(24b^2B-12abC+5a^2D)}{45a^4x^3(a+bx^2)^{7/2}}\\ &-\frac{2b(128Ab^3-3a(24b^2B-12abC+5a^2D))}{9a^5x(a+bx^2)^{7/2}}\\ &-\frac{16b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))x}{63a^6(a+bx^2)^{7/2}}\\ &-\frac{32b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))x}{105a^7(a+bx^2)^{5/2}}\\ &-\frac{128b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))x}{315a^8(a+bx^2)^{3/2}}\\ &-\frac{256b^2(128Ab^3-3a(24b^2B-12abC+5a^2D))x}{315a^9\sqrt{a+bx^2}}\end{aligned}$$

output
```
-1/9*A/a/x^9/(b*x^2+a)^(7/2)+1/63*(16*A*b-9*B*a)/a^2/x^7/(b*x^2+a)^(7/2)+1
/45*(-32*A*b^2+9*a*(2*B*b-C*a))/a^3/x^5/(b*x^2+a)^(7/2)+1/45*(128*A*b^3-3*
a*(24*B*b^2-12*C*a*b+5*D*a^2))/a^4/x^3/(b*x^2+a)^(7/2)-2/9*b*(128*A*b^3-3*
a*(24*B*b^2-12*C*a*b+5*D*a^2))/a^5/x/(b*x^2+a)^(7/2)-16/63*b^2*(128*A*b^3-
3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^6/(b*x^2+a)^(7/2)-32/105*b^2*(128*A*b
^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^7/(b*x^2+a)^(5/2)-128/315*b^2*(128
*A*b^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^8/(b*x^2+a)^(3/2)-256/315*b^2*
(128*A*b^3-3*a*(24*B*b^2-12*C*a*b+5*D*a^2))*x/a^9/(b*x^2+a)^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx = \frac{-32768Ab^8x^{16}+2048ab^7x^{14}(-56A+9Bx^2)-1024a^2b^6x^{12}(140A-63Bx^2}{}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

output
```
(-32768*A*b^8*x^16 + 2048*a*b^7*x^14*(-56*A + 9*B*x^2) - 1024*a^2*b^6*x^12
*(140*A - 63*B*x^2 + 9*C*x^4) - 56*a^6*b^2*x^4*(4*A + 9*B*x^2 + 45*C*x^4 -
 150*D*x^6) + 4480*a^4*b^4*x^8*(-2*A + 9*B*x^2 - 9*C*x^4 + 3*D*x^6) + 256*
a^3*b^5*x^10*(-280*A + 315*B*x^2 - 126*C*x^4 + 15*D*x^6) - a^8*(35*A + 45*
B*x^2 + 63*C*x^4 + 105*D*x^6) + 112*a^5*b^3*x^6*(8*A + 45*B*x^2 - 180*C*x^
4 + 150*D*x^6) + 2*a^7*b*x^2*(40*A + 21*(3*B*x^2 + 6*C*x^4 + 25*D*x^6)))/(
315*a^9*x^9*(a + b*x^2)^(7/2))
```

3.168.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.74, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {2334, 2089, 1588, 27, 359, 245, 245, 209, 209, 209, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \ 2334$$

$$-\frac{\int \frac{16Ab-9a\left(Dx^4+Cx^2+B\right)}{x^8\left(bx^2+a\right)^{9/2}}dx}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2089}$$

$$-\frac{\int \frac{-9aDx^4-9aCx^2+16Ab-9aB}{x^8\left(bx^2+a\right)^{9/2}}dx}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1588}$$

$$-\frac{-\frac{\int \frac{7\left(32Ab^2+9a^2Dx^2-9a(2bB-aC)\right)}{x^6\left(bx^2+a\right)^{9/2}}dx}{7a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{-\frac{\int \frac{32Ab^2+9a^2Dx^2-9a(2bB-aC)}{x^6\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 359}$$

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\int \frac{1}{x^4\left(bx^2+a\right)^{9/2}}dx}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 245}$$

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\int \frac{1}{x^2\left(bx^2+a\right)^{9/2}}dx}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9aB}{7ax^7\left(a+bx^2\right)^{7/2}}}{9a}-\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 245}$$

3.168. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}dx$

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\int\frac{1}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB-aC)}{5ax^5\left(a+bx^2\right)^{7/2}}}{a}-\frac{16Ab-9a}{7ax^7(a+bx^2}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\int\frac{1}{\left(bx^2+a\right)^{7/2}}dx}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)}{5a}-\frac{32Ab^2-9a(2bB-}{5ax^5\left(a+bx^2\right)^{7}}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 209

3.168. $\int\frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\int\frac{1}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/\ldots}}\right.}{5a}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 209$$

3.168. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx$

$$-\frac{-\frac{-\frac{3\left(-15a^3D-36ab(2bB-aC)+128Ab^3\right)\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2\int\frac{1}{\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7\cdots}}\cdots\right)}{3a}\cdots\right)}{5a}\cdots}{a}\cdots}{9a}\cdots$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

3.168. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx$

$\Big\downarrow$ 208

$$-\frac{-\frac{-\frac{3\left(-\frac{10b\left(-\frac{8b\left(\frac{6\left(\frac{4\left(\frac{2x}{3a^2\sqrt{a+bx^2}}+\frac{x}{3a\left(a+bx^2\right)^{3/2}}\right)}{5a}+\frac{x}{5a\left(a+bx^2\right)^{5/2}}\right)}{7a}+\frac{x}{7a\left(a+bx^2\right)^{7/2}}\right)}{a}-\frac{1}{ax\left(a+bx^2\right)^{7/2}}\right)}{3a}-\frac{1}{3ax^3\left(a+bx^2\right)^{7/2}}\right)\left(-15a^3D-36ab(\right.}{5a}}{a}}{9a}$$

$$\frac{A}{9ax^9\left(a+bx^2\right)^{7/2}}$$

input

```
Int[(A + B*x^2 + C*x^4 + D*x^6)/(x^10*(a + b*x^2)^(9/2)),x]
```

3.168. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx$

output
```
-1/9*A/(a*x^9*(a + b*x^2)^(7/2)) - (-1/7*(16*A*b - 9*a*B)/(a*x^7*(a + b*x^
2)^(7/2)) - (-1/5*(32*A*b^2 - 9*a*(2*b*B - a*C))/(a*x^5*(a + b*x^2)^(7/2))
 - (3*(128*A*b^3 - 36*a*b*(2*b*B - a*C) - 15*a^3*D)*(-1/3*1/(a*x^3*(a + b*
x^2)^(7/2)) - (10*b*(-(1/(a*x*(a + b*x^2)^(7/2))) - (8*b*(x/(7*a*(a + b*x^
2)^(7/2)) + (6*(x/(5*a*(a + b*x^2)^(5/2)) + (4*(x/(3*a*(a + b*x^2)^(3/2))
+ (2*x)/(3*a^2*Sqrt[a + b*x^2])))/(5*a)))/(7*a)))/a))/(3*a)))/(5*a))/a)/(9
*a)
```

3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 209
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && ILtQ[p + 3/2, 0]
```

rule 245
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x^(m + 1)*((a +
 b*x^2)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + 2*(p + 1) + 1)/(a*(m + 1)))
   Int[x^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, m, p}, x] && ILtQ[Si
mplify[(m + 1)/2 + p + 1], 0] && NeQ[m, -1]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 1588
```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[R*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(d*f*(m + 1))), x] + Simp[1/(d*f
^2*(m + 1))   Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[d*f*(m + 1)*(Qx/x
) - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && Ne
Q[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

3.168.4 Maple [A] (verified)

Time = 3.65 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\left(-105Dx^6-63C\,x^4-45x^2B-35A\right)a^8+80b\,x^2\left(\frac{105}{8}Dx^6+\frac{63}{20}C\,x^4+\frac{63}{40}x^2B+A\right)a^7-224\left(-\frac{75}{2}Dx^6+\frac{45}{4}C\,x^4+\frac{9}{4}x^2B+A\right)b^2x^4a}{}$
gosper	$-\frac{32768A\,b^8x^{16}-18432Ba\,b^7x^{16}+9216C\,a^2b^6x^{16}-3840Da^3b^5x^{16}+114688Aa\,b^7x^{14}-64512B\,a^2b^6x^{14}+32256C\,a^3b^5x^{14}-1}{}$
trager	$-\frac{32768A\,b^8x^{16}-18432Ba\,b^7x^{16}+9216C\,a^2b^6x^{16}-3840Da^3b^5x^{16}+114688Aa\,b^7x^{14}-64512B\,a^2b^6x^{14}+32256C\,a^3b^5x^{14}-1}{}$

3.168. $\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx$

input
```
int((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-105*D*x^6-63*C*x^4-45*B*x^2-35*A)*a^8+80*b*x^2*(105/8*D*x^6+63/20
*C*x^4+63/40*x^2*B+A)*a^7-224*(-75/2*D*x^6+45/4*C*x^4+9/4*x^2*B+A)*b^2*x^4
*a^6+896*(75/4*D*x^6-45/2*C*x^4+45/8*x^2*B+A)*b^3*x^6*a^5-8960*b^4*x^8*(-3
/2*D*x^6+9/2*C*x^4-9/2*x^2*B+A)*a^4-71680*(-3/56*D*x^6+9/20*C*x^4-9/8*x^2*
B+A)*b^5*x^10*a^3-143360*(9/140*C*x^4-9/20*x^2*B+A)*b^6*x^12*a^2-114688*(-
9/56*x^2*B+A)*b^7*x^14*a-32768*A*b^8*x^16)/(b*x^2+a)^(7/2)/x^9/a^9
```

3.168.5 Fricas [A] (verification not implemented)

Time = 0.97 (sec) , antiderivative size = 354, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{(256\,(15\,Da^3b^5-36\,Ca^2b^6+72\,Bab^7-128\,Ab^8)x^{16}+896\,(15\,Da^4b^4-36\,C$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="fricas"
)
```

output
```
1/315*(256*(15*D*a^3*b^5 - 36*C*a^2*b^6 + 72*B*a*b^7 - 128*A*b^8)*x^16 + 8
96*(15*D*a^4*b^4 - 36*C*a^3*b^5 + 72*B*a^2*b^6 - 128*A*a*b^7)*x^14 + 1120*
(15*D*a^5*b^3 - 36*C*a^4*b^4 + 72*B*a^3*b^5 - 128*A*a^2*b^6)*x^12 + 560*(1
5*D*a^6*b^2 - 36*C*a^5*b^3 + 72*B*a^4*b^4 - 128*A*a^3*b^5)*x^10 - 35*A*a^8
 + 70*(15*D*a^7*b - 36*C*a^6*b^2 + 72*B*a^5*b^3 - 128*A*a^4*b^4)*x^8 - 7*(
15*D*a^8 - 36*C*a^7*b + 72*B*a^6*b^2 - 128*A*a^5*b^3)*x^6 - 7*(9*C*a^8 - 1
8*B*a^7*b + 32*A*a^6*b^2)*x^4 - 5*(9*B*a^8 - 16*A*a^7*b)*x^2)*sqrt(b*x^2 +
 a)/(a^9*b^4*x^17 + 4*a^10*b^3*x^15 + 6*a^11*b^2*x^13 + 4*a^12*b*x^11 + a^
13*x^9)
```

3.168.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((D*x**6+C*x**4+B*x**2+A)/x**10/(b*x**2+a)**(9/2),x)
```

output
```
Timed out
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 579, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx = \frac{256\,Db^2x}{21\,\sqrt{bx^2+a}a^6} + \frac{128\,Db^2x}{21\,\left(bx^2+a\right)^{\frac{3}{2}}a^5} + \frac{32\,Db^2x}{7\,\left(bx^2+a\right)^{\frac{5}{2}}a^4} + \frac{80\,Db^2x}{21\,\left(bx^2+a\right)^{\frac{7}{2}}a^3} - \frac{1024\,Cb^3x}{35\,\sqrt{bx^2+a}a^7} - \frac{512\,Cb^3x}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^6} - \frac{384\,Cb^3x}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^5} - \frac{64\,Cb^3x}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a^4} + \frac{2048\,Bb^4x}{35\,\sqrt{bx^2+a}a^8} + \frac{1024\,Bb^4x}{35\,\left(bx^2+a\right)^{\frac{3}{2}}a^7} + \frac{768\,Bb^4x}{35\,\left(bx^2+a\right)^{\frac{5}{2}}a^6} + \frac{128\,Bb^4x}{7\,\left(bx^2+a\right)^{\frac{7}{2}}a^5} - \frac{32768\,Ab^5x}{315\,\sqrt{bx^2+a}a^9} - \frac{16384\,Ab^5x}{315\,\left(bx^2+a\right)^{\frac{3}{2}}a^8} - \frac{4096\,Ab^5x}{105\,\left(bx^2+a\right)^{\frac{5}{2}}a^7} - \frac{2048\,Ab^5x}{63\,\left(bx^2+a\right)^{\frac{7}{2}}a^6} + \frac{10\,Db}{3\,\left(bx^2+a\right)^{\frac{7}{2}}a^2x} - \frac{8\,Cb^2}{\left(bx^2+a\right)^{\frac{7}{2}}a^3x} + \frac{16\,Bb^3}{\left(bx^2+a\right)^{\frac{7}{2}}a^4x} - \frac{256\,Ab^4}{9\,\left(bx^2+a\right)^{\frac{7}{2}}a^5x} - \frac{D}{3\,\left(bx^2+a\right)^{\frac{7}{2}}ax^3} + \frac{4\,Cb}{5\,\left(bx^2+a\right)^{\frac{7}{2}}a^2x^3} - \frac{8\,Bb^2}{5\,\left(bx^2+a\right)^{\frac{7}{2}}a^3x^3} + \frac{128\,Ab^3}{45\,\left(bx^2+a\right)^{\frac{7}{2}}a^4x^3} - \frac{C}{5\,\left(bx^2+a\right)^{\frac{7}{2}}ax^5} + \frac{2\,Bb}{5\,\left(bx^2+a\right)^{\frac{7}{2}}a^2x^5} - \frac{32\,Ab^2}{45\,\left(bx^2+a\right)^{\frac{7}{2}}a^3x^5} - \frac{B}{7\,\left(bx^2+a\right)^{\frac{7}{2}}ax^7} + \frac{16\,Ab}{63\,\left(bx^2+a\right)^{\frac{7}{2}}a^2x^7} - \frac{A}{9\,\left(bx^2+a\right)^{\frac{7}{2}}ax^9}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="maxima"
)
```

output
```
256/21*D*b^2*x/(sqrt(b*x^2 + a)*a^6) + 128/21*D*b^2*x/((b*x^2 + a)^(3/2)*a
^5) + 32/7*D*b^2*x/((b*x^2 + a)^(5/2)*a^4) + 80/21*D*b^2*x/((b*x^2 + a)^(7
/2)*a^3) - 1024/35*C*b^3*x/(sqrt(b*x^2 + a)*a^7) - 512/35*C*b^3*x/((b*x^2
+ a)^(3/2)*a^6) - 384/35*C*b^3*x/((b*x^2 + a)^(5/2)*a^5) - 64/7*C*b^3*x/((
b*x^2 + a)^(7/2)*a^4) + 2048/35*B*b^4*x/(sqrt(b*x^2 + a)*a^8) + 1024/35*B*
b^4*x/((b*x^2 + a)^(3/2)*a^7) + 768/35*B*b^4*x/((b*x^2 + a)^(5/2)*a^6) + 1
28/7*B*b^4*x/((b*x^2 + a)^(7/2)*a^5) - 32768/315*A*b^5*x/(sqrt(b*x^2 + a)*
a^9) - 16384/315*A*b^5*x/((b*x^2 + a)^(3/2)*a^8) - 4096/105*A*b^5*x/((b*x^
2 + a)^(5/2)*a^7) - 2048/63*A*b^5*x/((b*x^2 + a)^(7/2)*a^6) + 10/3*D*b/((b
*x^2 + a)^(7/2)*a^2*x) - 8*C*b^2/((b*x^2 + a)^(7/2)*a^3*x) + 16*B*b^3/((b*
x^2 + a)^(7/2)*a^4*x) - 256/9*A*b^4/((b*x^2 + a)^(7/2)*a^5*x) - 1/3*D/((b*
x^2 + a)^(7/2)*a*x^3) + 4/5*C*b/((b*x^2 + a)^(7/2)*a^2*x^3) - 8/5*B*b^2/((
b*x^2 + a)^(7/2)*a^3*x^3) + 128/45*A*b^3/((b*x^2 + a)^(7/2)*a^4*x^3) - 1/5
*C/((b*x^2 + a)^(7/2)*a*x^5) + 2/5*B*b/((b*x^2 + a)^(7/2)*a^2*x^5) - 32/45
*A*b^2/((b*x^2 + a)^(7/2)*a^3*x^5) - 1/7*B/((b*x^2 + a)^(7/2)*a*x^7) + 16/
63*A*b/((b*x^2 + a)^(7/2)*a^2*x^7) - 1/9*A/((b*x^2 + a)^(7/2)*a*x^9)
```

3.168.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. 2(355) = 710.

Time = 0.31 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.96

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((D*x^6+C*x^4+B*x^2+A)/x^10/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
1/105*((x^2*((790*D*a^24*b^8 - 1686*C*a^23*b^9 + 3072*B*a^22*b^10 - 5053*A
*a^21*b^11)*x^2/(a^30*b^3) + 7*(365*D*a^25*b^7 - 768*C*a^24*b^8 + 1386*B*a
^23*b^9 - 2264*A*a^22*b^10)/(a^30*b^3)) + 35*(80*D*a^26*b^6 - 165*C*a^25*b
^7 + 294*B*a^24*b^8 - 476*A*a^23*b^9)/(a^30*b^3))*x^2 + 105*(10*D*a^27*b^5
 - 20*C*a^26*b^6 + 35*B*a^25*b^7 - 56*A*a^24*b^8)/(a^30*b^3))*x/(b*x^2 + a
)^(7/2) - 2/315*(1260*(sqrt(b)*x - sqrt(b*x^2 + a))^16*D*a^3*b^(3/2) - 315
0*(sqrt(b)*x - sqrt(b*x^2 + a))^16*C*a^2*b^(5/2) + 6300*(sqrt(b)*x - sqrt(
b*x^2 + a))^16*B*a*b^(7/2) - 11025*(sqrt(b)*x - sqrt(b*x^2 + a))^16*A*b^(9
/2) - 10710*(sqrt(b)*x - sqrt(b*x^2 + a))^14*D*a^4*b^(3/2) + 27720*(sqrt(b
)*x - sqrt(b*x^2 + a))^14*C*a^3*b^(5/2) - 56700*(sqrt(b)*x - sqrt(b*x^2 +
a))^14*B*a^2*b^(7/2) + 100800*(sqrt(b)*x - sqrt(b*x^2 + a))^14*A*a*b^(9/2)
 + 39270*(sqrt(b)*x - sqrt(b*x^2 + a))^12*D*a^5*b^(3/2) - 105840*(sqrt(b)*
x - sqrt(b*x^2 + a))^12*C*a^4*b^(5/2) + 223020*(sqrt(b)*x - sqrt(b*x^2 + a
))^12*B*a^3*b^(7/2) - 405300*(sqrt(b)*x - sqrt(b*x^2 + a))^12*A*a^2*b^(9/2
) - 81270*(sqrt(b)*x - sqrt(b*x^2 + a))^10*D*a^6*b^(3/2) + 226800*(sqrt(b)
*x - sqrt(b*x^2 + a))^10*C*a^5*b^(5/2) - 495180*(sqrt(b)*x - sqrt(b*x^2 +
a))^10*B*a^4*b^(7/2) + 927360*(sqrt(b)*x - sqrt(b*x^2 + a))^10*A*a^3*b^(9/
2) + 103950*(sqrt(b)*x - sqrt(b*x^2 + a))^8*D*a^7*b^(3/2) - 297108*(sqrt(b
)*x - sqrt(b*x^2 + a))^8*C*a^6*b^(5/2) + 666036*(sqrt(b)*x - sqrt(b*x^2 +
a))^8*B*a^5*b^(7/2) - 1291374*(sqrt(b)*x - sqrt(b*x^2 + a))^8*A*a^4*b^(...
```

3.168.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6}{x^{10}\left(a+bx^2\right)^{9/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+x^6\,D}{x^{10}\,\left(b\,x^2+a\right)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + x^6*D)/(x^10*(a + b*x^2)^(9/2)), x)
```

3.169 $\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$

3.169.1 Optimal result

Integrand size = 33, antiderivative size = 214

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx = \frac{a^2(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^6} - \frac{a(2b^3c-3ab^2d+4a^2be-5a^3f)\left(a+bx^2\right)^{3/2}}{3b^6} + \frac{(b^3c-3ab^2d+6a^2be-10a^3f)\left(a+bx^2\right)^{5/2}}{5b^6} + \frac{(b^2d-4abe+10a^2f)\left(a+bx^2\right)^{7/2}}{7b^6} + \frac{(be-5af)\left(a+bx^2\right)^{9/2}}{9b^6} + \frac{f\left(a+bx^2\right)^{11/2}}{11b^6}$$

output
```
-1/3*a*(-5*a^3*f+4*a^2*b*e-3*a*b^2*d+2*b^3*c)*(b*x^2+a)^(3/2)/b^6+1/5*(-10
*a^3*f+6*a^2*b*e-3*a*b^2*d+b^3*c)*(b*x^2+a)^(5/2)/b^6+1/7*(10*a^2*f-4*a*b*
e+b^2*d)*(b*x^2+a)^(7/2)/b^6+1/9*(-5*a*f+b*e)*(b*x^2+a)^(9/2)/b^6+1/11*f*(
b*x^2+a)^(11/2)/b^6+a^2*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b^6
```

3.169.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.74

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(-1280a^5f+128a^4b(11e+5fx^2)-16a^3b^2(99d+44ex^2+30fx^4)+8a^2b^3(231c+99dx^2+66e}{3465}$$

input
```
Integrate[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-1280*a^5*f + 128*a^4*b*(11*e + 5*f*x^2) - 16*a^3*b^2*(9
9*d + 44*e*x^2 + 30*f*x^4) + 8*a^2*b^3*(231*c + 99*d*x^2 + 66*e*x^4 + 50*f
*x^6) - 2*a*b^4*x^2*(462*c + 297*d*x^2 + 220*e*x^4 + 175*f*x^6) + b^5*x^4*
(693*c + 5*(99*d*x^2 + 77*e*x^4 + 63*f*x^6))))/(3465*b^6)
```

3.169.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2029, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx$$
$$\Big\downarrow \text{2029}$$
$$\int \frac{x^5\left(c+dx^2+ex^4+fx^6\right)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2331}$$
$$\frac{1}{2}\int \frac{x^4\left(fx^6+ex^4+dx^2+c\right)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{2}\int \left(\frac{f\left(bx^2+a\right)^{9/2}}{b^5}+\frac{(be-5af)\left(bx^2+a\right)^{7/2}}{b^5}+\frac{\left(10fa^2-4bea+b^2d\right)\left(bx^2+a\right)^{5/2}}{b^5}+\frac{\left(-10fa^3+6bea^2-3b^2d\right.}{b^5}\right.$$

$$\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{7/2}\left(10a^2f-4abe+b^2d\right)}{7b^6}+\frac{2\left(a+bx^2\right)^{5/2}\left(-10a^3f+6a^2be-3ab^2d+b^3c\right)}{5b^6}-\frac{2a\left(a+bx^2\right)^{3/2}\left(-5a\right.}{}\right.$$

input
```
Int[(c*x^5 + d*x^7 + e*x^9 + f*x^11)/Sqrt[a + b*x^2],x]
```

output
```
((2*a^2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^6 - (2*a*(2
*b^3*c - 3*a*b^2*d + 4*a^2*b*e - 5*a^3*f)*(a + b*x^2)^(3/2))/(3*b^6) + (2*
(b^3*c - 3*a*b^2*d + 6*a^2*b*e - 10*a^3*f)*(a + b*x^2)^(5/2))/(5*b^6) + (2
*(b^2*d - 4*a*b*e + 10*a^2*f)*(a + b*x^2)^(7/2))/(7*b^6) + (2*(b*e - 5*a*f
)*(a + b*x^2)^(9/2))/(9*b^6) + (2*f*(a + b*x^2)^(11/2))/(11*b^6))/2
```

3.169.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.169.4 Maple [A] (verified)

Time = 3.68 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{256\left(-\frac{693\left(\frac{5}{11} f\,x^{6}+\frac{5}{9} e\,x^{4}+\frac{5}{7} d\,x^{2}+c\right) x^{4} b^{5}}{1280}+\frac{231\left(\frac{25}{66} f\,x^{6}+\frac{10}{21} e\,x^{4}+\frac{9}{14} d\,x^{2}+c\right) x^{2} a\,b^{4}}{320}-\frac{231\left(\frac{50}{231} f\,x^{6}+\frac{2}{7} e\,x^{4}+\frac{3}{7} d\,x^{2}+c\right) a^{2} b^{3}}{160}+\frac{99(\ldots}{}\right.}{693 b^{6}}$ …
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,(-315 f\,x^{10} b^{5}+350 a\,b^{4} f\,x^{8}-385 b^{5} e\,x^{8}-400 a^{2} b^{3} f\,x^{6}+440 a\,b^{4} e\,x^{6}-495 b^{5} d\,x^{6}+480 a^{3} b^{2} f\,x^{4}-528 a^{2} b^{3} e\,x^{4}+594\ldots}{346\ldots}$ …
trager	$-\frac{\sqrt{b\,x^{2}+a}\,(-315 f\,x^{10} b^{5}+350 a\,b^{4} f\,x^{8}-385 b^{5} e\,x^{8}-400 a^{2} b^{3} f\,x^{6}+440 a\,b^{4} e\,x^{6}-495 b^{5} d\,x^{6}+480 a^{3} b^{2} f\,x^{4}-528 a^{2} b^{3} e\,x^{4}+594\ldots}{346\ldots}$ …
risch	$-\frac{\sqrt{b\,x^{2}+a}\,(-315 f\,x^{10} b^{5}+350 a\,b^{4} f\,x^{8}-385 b^{5} e\,x^{8}-400 a^{2} b^{3} f\,x^{6}+440 a\,b^{4} e\,x^{6}-495 b^{5} d\,x^{6}+480 a^{3} b^{2} f\,x^{4}-528 a^{2} b^{3} e\,x^{4}+594\ldots}{346\ldots}$ …
default	$e\left(\frac{x^{8} \sqrt{b\,x^{2}+a}}{9 b}-\frac{8 a\left(\frac{x^{6} \sqrt{b\,x^{2}+a}}{7 b}-\frac{6 a\left(\frac{x^{4} \sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a\left(\frac{x^{2} \sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a \sqrt{b\,x^{2}+a}}{3 b^{2}}\right)}{5 b}\right)}{7 b}\right)}{9 b}\right)+d\left(\frac{x^{6} \sqrt{b\,x^{2}+a}}{7 b}-\frac{6 a\left(\frac{x^{4}\ldots}{}\right.}{}\right.$ …

input

```
int((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
-256/693*(-693/1280*(5/11*f*x^6+5/9*e*x^4+5/7*d*x^2+c)*x^4*b^5+231/320*(25
/66*f*x^6+10/21*e*x^4+9/14*d*x^2+c)*x^2*a*b^4-231/160*(50/231*f*x^6+2/7*e*
x^4+3/7*d*x^2+c)*a^2*b^3+99/80*(10/33*f*x^4+4/9*e*x^2+d)*a^3*b^2-11/10*(5/
11*f*x^2+e)*a^4*b+f*a^5)*(b*x^2+a)^(1/2)/b^6
```

3.169.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.83

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a + bx^2}}\,dx = \frac{(315\,b^5fx^{10} + 35\,(11\,b^5e - 10\,ab^4f)x^8 + 5\,(99\,b^5d - 88\,ab^4e + 80\,a^2b^3f)x^6 + 1848\,a^2b^3c - 1584\,a^3b^2d + 14}{}$$

input
```
integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

output
```
1/3465*(315*b^5*f*x^10 + 35*(11*b^5*e - 10*a*b^4*f)*x^8 + 5*(99*b^5*d - 88
*a*b^4*e + 80*a^2*b^3*f)*x^6 + 1848*a^2*b^3*c - 1584*a^3*b^2*d + 1408*a^4*
b*e - 1280*a^5*f + 3*(231*b^5*c - 198*a*b^4*d + 176*a^2*b^3*e - 160*a^3*b^
2*f)*x^4 - 4*(231*a*b^4*c - 198*a^2*b^3*d + 176*a^3*b^2*e - 160*a^4*b*f)*x
^2)*sqrt(b*x^2 + a)/b^6
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(214) = 428$.

Time = 0.46 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.07

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a + bx^2}}\,dx = \begin{cases} -\frac{256a^5f\sqrt{a+bx^2}}{693b^6} + \frac{128a^4e\sqrt{a+bx^2}}{315b^5} + \frac{128a^4fx^2\sqrt{a+bx^2}}{693b^5} - \frac{16a^3d\sqrt{a+bx^2}}{35b^4} - \frac{64a^3ex^2\sqrt{a+bx^2}}{315b^4} - \frac{32a^3fx^4\sqrt{a+bx^2}}{231b^4} + \frac{8a^2c\sqrt{a+bx^2}}{15b^3} \\ \frac{\frac{cx^6}{6} + \frac{dx^8}{8} + \frac{ex^{10}}{10} + \frac{fx^{12}}{12}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**11+e*x**9+d*x**7+c*x**5)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-256*a**5*f*sqrt(a + b*x**2)/(693*b**6) + 128*a**4*e*sqrt(a + b
*x**2)/(315*b**5) + 128*a**4*f*x**2*sqrt(a + b*x**2)/(693*b**5) - 16*a**3*
d*sqrt(a + b*x**2)/(35*b**4) - 64*a**3*e*x**2*sqrt(a + b*x**2)/(315*b**4)
- 32*a**3*f*x**4*sqrt(a + b*x**2)/(231*b**4) + 8*a**2*c*sqrt(a + b*x**2)/(
15*b**3) + 8*a**2*d*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a**2*e*x**4*sqrt(
a + b*x**2)/(105*b**3) + 80*a**2*f*x**6*sqrt(a + b*x**2)/(693*b**3) - 4*a*
c*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*d*x**4*sqrt(a + b*x**2)/(35*b**2)
- 8*a*e*x**6*sqrt(a + b*x**2)/(63*b**2) - 10*a*f*x**8*sqrt(a + b*x**2)/(99
*b**2) + c*x**4*sqrt(a + b*x**2)/(5*b) + d*x**6*sqrt(a + b*x**2)/(7*b) + e
*x**8*sqrt(a + b*x**2)/(9*b) + f*x**10*sqrt(a + b*x**2)/(11*b), Ne(b, 0)),
 ((c*x**6/6 + d*x**8/8 + e*x**10/10 + f*x**12/12)/sqrt(a), True))
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.62

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a + bx^2}}\,dx = \frac{\sqrt{bx^2 + a}fx^{10}}{11\,b} + \frac{\sqrt{bx^2 + a}ex^8}{9\,b} - \frac{10\,\sqrt{bx^2 + a}afx^8}{99\,b^2} + \frac{\sqrt{bx^2 + a}dx^6}{7\,b} - \frac{8\,\sqrt{bx^2 + a}aex^6}{63\,b^2} + \frac{80\,\sqrt{bx^2 + a}a^2fx^6}{693\,b^3} + \frac{\sqrt{bx^2 + a}cx^4}{5\,b} - \frac{6\,\sqrt{bx^2 + a}adx^4}{35\,b^2} + \frac{16\,\sqrt{bx^2 + a}a^2ex^4}{105\,b^3} - \frac{32\,\sqrt{bx^2 + a}a^3fx^4}{231\,b^4} - \frac{4\,\sqrt{bx^2 + a}acx^2}{15\,b^2} + \frac{8\,\sqrt{bx^2 + a}a^2dx^2}{35\,b^3} - \frac{64\,\sqrt{bx^2 + a}a^3ex^2}{315\,b^4} + \frac{128\,\sqrt{bx^2 + a}a^4fx^2}{693\,b^5} + \frac{8\,\sqrt{bx^2 + a}a^2c}{15\,b^3} - \frac{16\,\sqrt{bx^2 + a}a^3d}{35\,b^4} + \frac{128\,\sqrt{bx^2 + a}a^4e}{315\,b^5} - \frac{256\,\sqrt{bx^2 + a}a^5f}{693\,b^6}$$

input
```
integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

output
```
1/11*sqrt(b*x^2 + a)*f*x^10/b + 1/9*sqrt(b*x^2 + a)*e*x^8/b - 10/99*sqrt(b
*x^2 + a)*a*f*x^8/b^2 + 1/7*sqrt(b*x^2 + a)*d*x^6/b - 8/63*sqrt(b*x^2 + a)
*a*e*x^6/b^2 + 80/693*sqrt(b*x^2 + a)*a^2*f*x^6/b^3 + 1/5*sqrt(b*x^2 + a)*
c*x^4/b - 6/35*sqrt(b*x^2 + a)*a*d*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*e*
x^4/b^3 - 32/231*sqrt(b*x^2 + a)*a^3*f*x^4/b^4 - 4/15*sqrt(b*x^2 + a)*a*c*
x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*d*x^2/b^3 - 64/315*sqrt(b*x^2 + a)*a^3*
e*x^2/b^4 + 128/693*sqrt(b*x^2 + a)*a^4*f*x^2/b^5 + 8/15*sqrt(b*x^2 + a)*a
^2*c/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*d/b^4 + 128/315*sqrt(b*x^2 + a)*a^4*e
/b^5 - 256/693*sqrt(b*x^2 + a)*a^5*f/b^6
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.21

$$\int \frac{cx^5 + dx^7 + ex^9 + fx^{11}}{\sqrt{a + bx^2}}\,dx = \frac{(a^2b^3c - a^3b^2d + a^4be - a^5f)\sqrt{bx^2 + a}}{b^6} + \frac{693\,(bx^2 + a)^{\frac{5}{2}}b^3c - 2310\,(bx^2 + a)^{\frac{3}{2}}ab^3c + 495\,(bx^2 + a)^{\frac{7}{2}}b^2d - 2079\,(bx^2 + a)^{\frac{5}{2}}ab^2d + 3465\,(bx^2 + a)^{\frac{3}{2}}}{}$$

input
```
integrate((f*x^11+e*x^9+d*x^7+c*x^5)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(a^2*b^3*c - a^3*b^2*d + a^4*b*e - a^5*f)*sqrt(b*x^2 + a)/b^6 + 1/3465*(69
3*(b*x^2 + a)^(5/2)*b^3*c - 2310*(b*x^2 + a)^(3/2)*a*b^3*c + 495*(b*x^2 +
a)^(7/2)*b^2*d - 2079*(b*x^2 + a)^(5/2)*a*b^2*d + 3465*(b*x^2 + a)^(3/2)*a
^2*b^2*d + 385*(b*x^2 + a)^(9/2)*b*e - 1980*(b*x^2 + a)^(7/2)*a*b*e + 4158
*(b*x^2 + a)^(5/2)*a^2*b*e - 4620*(b*x^2 + a)^(3/2)*a^3*b*e + 315*(b*x^2 +
 a)^(11/2)*f - 1925*(b*x^2 + a)^(9/2)*a*f + 4950*(b*x^2 + a)^(7/2)*a^2*f -
 6930*(b*x^2 + a)^(5/2)*a^3*f + 5775*(b*x^2 + a)^(3/2)*a^4*f)/b^6
```

3.169.9 Mupad [B] (verification not implemented)

Time = 6.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.87

$$\int \frac{cx^5+dx^7+ex^9+fx^{11}}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{x^6\,\left(400\,f\,a^2\,b^3-440\,e\,a\,b^4+495\,d\,b^5\right)}{3465\,b^6} - \frac{1280\,f\,a^5-1408\,e\,a^4\,b+1584\,d\,a^3\,b^2-1848\,c\,a^2\,b^3}{3465\,b^6} + \frac{x^4\,\left(-480\,f\,a^3\,b^2+528\,e\,a^2\,b^3-594\,d\,a\,b^4+693\,c\,b^5\right)}{3465\,b^6} + \frac{f\,x^{10}}{11\,b} + \frac{x^8\,\left(385\,b^5\,e-350\,a\,b^4\,f\right)}{3465\,b^6} - \frac{4\,a\,x^2\,\left(-160\,f\,a^3+176\,e\,a^2\,b-198\,d\,a\,b^2+231\,c\,b^3\right)}{3465\,b^5}\right)$$

input
```
int((c*x^5 + d*x^7 + e*x^9 + f*x^11)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((x^6*(495*b^5*d + 400*a^2*b^3*f - 440*a*b^4*e))/(3465*b
^6) - (1280*a^5*f - 1848*a^2*b^3*c + 1584*a^3*b^2*d - 1408*a^4*b*e)/(3465*
b^6) + (x^4*(693*b^5*c + 528*a^2*b^3*e - 480*a^3*b^2*f - 594*a*b^4*d))/(34
65*b^6) + (f*x^10)/(11*b) + (x^8*(385*b^5*e - 350*a*b^4*f))/(3465*b^6) - (
4*a*x^2*(231*b^3*c - 160*a^3*f - 198*a*b^2*d + 176*a^2*b*e))/(3465*b^5))
```

3.170 $\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$

3.170.1 Optimal result

Integrand size = 33, antiderivative size = 167

$$\begin{aligned}\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx =& -\frac{a(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^5} \\ &+\frac{(b^3c-2ab^2d+3a^2be-4a^3f)\left(a+bx^2\right)^{3/2}}{3b^5} \\ &+\frac{(b^2d-3abe+6a^2f)\left(a+bx^2\right)^{5/2}}{5b^5} \\ &+\frac{(be-4af)\left(a+bx^2\right)^{7/2}}{7b^5}+\frac{f\left(a+bx^2\right)^{9/2}}{9b^5}\end{aligned}$$

output
```
1/3*(-4*a^3*f+3*a^2*b*e-2*a*b^2*d+b^3*c)*(b*x^2+a)^(3/2)/b^5+1/5*(6*a^2*f-
3*a*b*e+b^2*d)*(b*x^2+a)^(5/2)/b^5+1/7*(-4*a*f+b*e)*(b*x^2+a)^(7/2)/b^5+1/
9*f*(b*x^2+a)^(9/2)/b^5-a*(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2+a)^(1/2)/b
^5
```

3.170.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.73

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$$
$$= \frac{\sqrt{a+bx^2}(128a^4f-16a^3b(9e+4fx^2)+24a^2b^2(7d+3ex^2+2fx^4)-2ab^3(105c+42dx^2+27ex^4+20fx}{315b^5}$$

input
```
Integrate[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(128*a^4*f - 16*a^3*b*(9*e + 4*f*x^2) + 24*a^2*b^2*(7*d +
 3*e*x^2 + 2*f*x^4) - 2*a*b^3*(105*c + 42*d*x^2 + 27*e*x^4 + 20*f*x^6) + b
^4*x^2*(105*c + 63*d*x^2 + 45*e*x^4 + 35*f*x^6)))/(315*b^5)
```

3.170.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2029, 2331, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2029}$$
$$\int \frac{x^3(c+dx^2+ex^4+fx^6)}{\sqrt{a+bx^2}}dx$$
$$\Big\downarrow \text{2331}$$
$$\frac{1}{2}\int \frac{x^2(fx^6+ex^4+dx^2+c)}{\sqrt{bx^2+a}}dx^2$$
$$\Big\downarrow \text{2123}$$
$$\frac{1}{2}\int \left(\frac{f(bx^2+a)^{7/2}}{b^4}+\frac{(be-4af)(bx^2+a)^{5/2}}{b^4}+\frac{(6fa^2-3bea+b^2d)(bx^2+a)^{3/2}}{b^4}+\frac{(-4fa^3+3bea^2-2b^2da}{b^4}\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\left(a+bx^2\right)^{5/2}\left(6a^2f-3abe+b^2d\right)}{5b^5}+\frac{2\left(a+bx^2\right)^{3/2}\left(-4a^3f+3a^2be-2ab^2d+b^3c\right)}{3b^5}-\frac{2a\sqrt{a+bx^2}\left(a^3(-f)+\right.}{b^5}\right.$$

input
```
Int[(c*x^3 + d*x^5 + e*x^7 + f*x^9)/Sqrt[a + b*x^2],x]
```

output
```
((-2*a*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^5 + (2*(b^3*
c - 2*a*b^2*d + 3*a^2*b*e - 4*a^3*f)*(a + b*x^2)^(3/2))/(3*b^5) + (2*(b^2*
d - 3*a*b*e + 6*a^2*f)*(a + b*x^2)^(5/2))/(5*b^5) + (2*(b*e - 4*a*f)*(a +
b*x^2)^(7/2))/(7*b^5) + (2*f*(a + b*x^2)^(9/2))/(9*b^5))/2
```

3.170.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

3.170.4 Maple [A] (verified)

Time = 3.51 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$\frac{128\left(\frac{105\left(\frac{1}{3}f\,x^{6}+\frac{3}{7}e\,x^{4}+\frac{3}{5}d\,x^{2}+c\right)x^{2}b^{4}}{128}-\frac{105\left(\frac{4}{21}f\,x^{6}+\frac{9}{35}e\,x^{4}+\frac{2}{5}d\,x^{2}+c\right)a\,b^{3}}{64}+\frac{21\left(\frac{2}{7}f\,x^{4}+\frac{3}{7}e\,x^{2}+d\right)a^{2}b^{2}}{16}-\frac{9\left(\frac{4f\,x^{2}}{9}+e\right)a^{3}b}{8}+a^{4}f\right.}{315b^{5}}$
gosper	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,}{315b^{5}}$
trager	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,}{315b^{5}}$
risch	$\frac{\sqrt{b\,x^{2}+a}\,(35f\,x^{8}b^{4}-40a\,b^{3}f\,x^{6}+45b^{4}e\,x^{6}+48a^{2}b^{2}f\,x^{4}-54a\,b^{3}e\,x^{4}+63b^{4}d\,x^{4}-64a^{3}bf\,x^{2}+72a^{2}b^{2}e\,x^{2}-84a\,b^{3}d\,x^{2}+105b^{4}c\,}{315b^{5}}$
default	$f\left(\frac{x^{8}\sqrt{b\,x^{2}+a}}{9b}-\frac{8a\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5b}-\frac{4a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3b}-\frac{2a\sqrt{b\,x^{2}+a}}{3b^{2}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+e\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7b}-\frac{6a\left(\frac{x^{4}}{}\right.}{}\right.$

input
```
int((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
128/315*(105/128*(1/3*f*x^6+3/7*e*x^4+3/5*d*x^2+c)*x^2*b^4-105/64*(4/21*f*
x^6+9/35*e*x^4+2/5*d*x^2+c)*a*b^3+21/16*(2/7*f*x^4+3/7*e*x^2+d)*a^2*b^2-9/
8*(4/9*f*x^2+e)*a^3*b+a^4*f)*(b*x^2+a)^(1/2)/b^5
```

3.170.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.80

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx$$
$$=\frac{(35\,b^4fx^8+5\,(9\,b^4e-8\,ab^3f)x^6-210\,ab^3c+168\,a^2b^2d-144\,a^3be+128\,a^4f+3\,(21\,b^4d-18\,ab^3e+16\,a}{315\,b^5}$$

input
```
integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/315*(35*b^4*f*x^8 + 5*(9*b^4*e - 8*a*b^3*f)*x^6 - 210*a*b^3*c + 168*a^2*
b^2*d - 144*a^3*b*e + 128*a^4*f + 3*(21*b^4*d - 18*a*b^3*e + 16*a^2*b^2*f)
*x^4 + (105*b^4*c - 84*a*b^3*d + 72*a^2*b^2*e - 64*a^3*b*f)*x^2)*sqrt(b*x^
2 + a)/b^5
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(163) = 326$.

Time = 0.37 (sec) , antiderivative size = 340, normalized size of antiderivative = 2.04

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a + bx^2}}\,dx$$
$$= \begin{cases} \frac{128a^4f\sqrt{a+bx^2}}{315b^5} - \frac{16a^3e\sqrt{a+bx^2}}{35b^4} - \frac{64a^3fx^2\sqrt{a+bx^2}}{315b^4} + \frac{8a^2d\sqrt{a+bx^2}}{15b^3} + \frac{8a^2ex^2\sqrt{a+bx^2}}{35b^3} + \frac{16a^2fx^4\sqrt{a+bx^2}}{105b^3} - \frac{2ac\sqrt{a+bx^2}}{3b^2} - \frac{4ad}{} \\ \frac{\frac{cx^4}{4} + \frac{dx^6}{6} + \frac{ex^8}{8} + \frac{fx^{10}}{10}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**9+e*x**7+d*x**5+c*x**3)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((128*a**4*f*sqrt(a + b*x**2)/(315*b**5) - 16*a**3*e*sqrt(a + b*x
**2)/(35*b**4) - 64*a**3*f*x**2*sqrt(a + b*x**2)/(315*b**4) + 8*a**2*d*sqr
t(a + b*x**2)/(15*b**3) + 8*a**2*e*x**2*sqrt(a + b*x**2)/(35*b**3) + 16*a*
*2*f*x**4*sqrt(a + b*x**2)/(105*b**3) - 2*a*c*sqrt(a + b*x**2)/(3*b**2) -
4*a*d*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*e*x**4*sqrt(a + b*x**2)/(35*b*
*2) - 8*a*f*x**6*sqrt(a + b*x**2)/(63*b**2) + c*x**2*sqrt(a + b*x**2)/(3*b
) + d*x**4*sqrt(a + b*x**2)/(5*b) + e*x**6*sqrt(a + b*x**2)/(7*b) + f*x**8
*sqrt(a + b*x**2)/(9*b), Ne(b, 0)), ((c*x**4/4 + d*x**6/6 + e*x**8/8 + f*x
**10/10)/sqrt(a), True))
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.57

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a + bx^2}}\, dx = \frac{\sqrt{bx^2 + a} f x^8}{9\, b} + \frac{\sqrt{bx^2 + a} e x^6}{7\, b} - \frac{8\,\sqrt{bx^2 + a} a f x^6}{63\, b^2} + \frac{\sqrt{bx^2 + a} d x^4}{5\, b} - \frac{6\,\sqrt{bx^2 + a} a e x^4}{35\, b^2} + \frac{16\,\sqrt{bx^2 + a} a^2 f x^4}{105\, b^3} + \frac{\sqrt{bx^2 + a} c x^2}{3\, b} - \frac{4\,\sqrt{bx^2 + a} a d x^2}{15\, b^2} + \frac{8\,\sqrt{bx^2 + a} a^2 e x^2}{35\, b^3} - \frac{64\,\sqrt{bx^2 + a} a^3 f x^2}{315\, b^4} - \frac{2\,\sqrt{bx^2 + a} a c}{3\, b^2} + \frac{8\,\sqrt{bx^2 + a} a^2 d}{15\, b^3} - \frac{16\,\sqrt{bx^2 + a} a^3 e}{35\, b^4} + \frac{128\,\sqrt{bx^2 + a} a^4 f}{315\, b^5}$$

input
```
integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(b*x^2 + a)*f*x^8/b + 1/7*sqrt(b*x^2 + a)*e*x^6/b - 8/63*sqrt(b*x^
2 + a)*a*f*x^6/b^2 + 1/5*sqrt(b*x^2 + a)*d*x^4/b - 6/35*sqrt(b*x^2 + a)*a*
e*x^4/b^2 + 16/105*sqrt(b*x^2 + a)*a^2*f*x^4/b^3 + 1/3*sqrt(b*x^2 + a)*c*x
^2/b - 4/15*sqrt(b*x^2 + a)*a*d*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*e*x^2/b
^3 - 64/315*sqrt(b*x^2 + a)*a^3*f*x^2/b^4 - 2/3*sqrt(b*x^2 + a)*a*c/b^2 +
8/15*sqrt(b*x^2 + a)*a^2*d/b^3 - 16/35*sqrt(b*x^2 + a)*a^3*e/b^4 + 128/315
*sqrt(b*x^2 + a)*a^4*f/b^5
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.16

$$\int \frac{cx^3 + dx^5 + ex^7 + fx^9}{\sqrt{a + bx^2}}\, dx = -\frac{(ab^3c - a^2b^2d + a^3be - a^4f)\sqrt{bx^2 + a}}{b^5}$$

$$+ \frac{105\,(bx^2 + a)^{\frac{3}{2}} b^3 c + 63\,(bx^2 + a)^{\frac{5}{2}} b^2 d - 210\,(bx^2 + a)^{\frac{3}{2}} a b^2 d + 45\,(bx^2 + a)^{\frac{7}{2}} b e - 189\,(bx^2 + a)^{\frac{5}{2}} a b e + 31}{315\, b^5}$$

input
```
integrate((f*x^9+e*x^7+d*x^5+c*x^3)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
-(a*b^3*c - a^2*b^2*d + a^3*b*e - a^4*f)*sqrt(b*x^2 + a)/b^5 + 1/315*(105*
(b*x^2 + a)^(3/2)*b^3*c + 63*(b*x^2 + a)^(5/2)*b^2*d - 210*(b*x^2 + a)^(3/
2)*a*b^2*d + 45*(b*x^2 + a)^(7/2)*b*e - 189*(b*x^2 + a)^(5/2)*a*b*e + 315*
(b*x^2 + a)^(3/2)*a^2*b*e + 35*(b*x^2 + a)^(9/2)*f - 180*(b*x^2 + a)^(7/2)
*a*f + 378*(b*x^2 + a)^(5/2)*a^2*f - 420*(b*x^2 + a)^(3/2)*a^3*f)/b^5
```

3.170.9 Mupad [B] (verification not implemented)

Time = 5.89 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.87

$$\int \frac{cx^3+dx^5+ex^7+fx^9}{\sqrt{a+bx^2}}\,dx = \sqrt{b\,x^2+a}\left(\frac{128\,f\,a^4-144\,e\,a^3\,b+168\,d\,a^2\,b^2-210\,c\,a\,b^3}{315\,b^5}+\frac{x^4\,(48\,f\,a^2\,b^2-54\,e\,a\,b^3+63\,d\,b^4)}{315\,b^5}+\frac{f\,x^8}{9\,b}+\frac{x^6\,(45\,b^4\,e-40\,a\,b^3\,f)}{315\,b^5}+\frac{x^2\,(-64\,f\,a^3\,b+72\,e\,a^2\,b^2-84\,d\,a\,b^3+105\,c\,b^4)}{315\,b^5}\right)$$

input
```
int((c*x^3 + d*x^5 + e*x^7 + f*x^9)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((128*a^4*f + 168*a^2*b^2*d - 210*a*b^3*c - 144*a^3*b*e)
/(315*b^5) + (x^4*(63*b^4*d + 48*a^2*b^2*f - 54*a*b^3*e))/(315*b^5) + (f*x
^8)/(9*b) + (x^6*(45*b^4*e - 40*a*b^3*f))/(315*b^5) + (x^2*(105*b^4*c + 72
*a^2*b^2*e - 84*a*b^3*d - 64*a^3*b*f))/(315*b^5))
```

3.171 $\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx$

3.171.1 Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{a+bx^2}}{b^4} + \frac{(b^2d-2abe+3a^2f)(a+bx^2)^{3/2}}{3b^4} + \frac{(be-3af)(a+bx^2)^{5/2}}{5b^4} + \frac{f(a+bx^2)^{7/2}}{7b^4}$$

output
```
1/3*(3*a^2*f-2*a*b*e+b^2*d)*(b*x^2+a)^(3/2)/b^4+1/5*(-3*a*f+b*e)*(b*x^2+a)
^(5/2)/b^4+1/7*f*(b*x^2+a)^(7/2)/b^4+(-a^3*f+a^2*b*e-a*b^2*d+b^3*c)*(b*x^2
+a)^(1/2)/b^4
```

3.171.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.74

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}(-48a^3f+8a^2b(7e+3fx^2)-2ab^2(35d+14ex^2+9fx^4)+b^3(105c+35dx^2+21ex^4+15fx^6))}{105b^4}$$

input
```
Integrate[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

output
```
(Sqrt[a + b*x^2]*(-48*a^3*f + 8*a^2*b*(7*e + 3*f*x^2) - 2*a*b^2*(35*d + 14
*e*x^2 + 9*f*x^4) + b^3*(105*c + 35*d*x^2 + 21*e*x^4 + 15*f*x^6)))/(105*b^
4)
```

3.171.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {2029, 2331, 2389, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{cx + dx^3 + ex^5 + fx^7}{\sqrt{a + bx^2}} dx$$

$$\Big\downarrow \ 2029$$

$$\int \frac{x\left(c + dx^2 + ex^4 + fx^6\right)}{\sqrt{a + bx^2}} dx$$

$$\Big\downarrow \ 2331$$

$$\frac{1}{2} \int \frac{fx^6 + ex^4 + dx^2 + c}{\sqrt{bx^2 + a}} dx^2$$

$$\Big\downarrow \ 2389$$

$$\frac{1}{2} \int \left(\frac{f\left(bx^2 + a\right)^{5/2}}{b^3} + \frac{(be - 3af)\left(bx^2 + a\right)^{3/2}}{b^3} + \frac{\left(3fa^2 - 2bea + b^2 d\right)\sqrt{bx^2 + a}}{b^3} + \frac{-fa^3 + bea^2 - b^2 da + b^3 c}{b^3\sqrt{bx^2 + a}}\right)$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}\left(\frac{2\left(a + bx^2\right)^{3/2}\left(3a^2 f - 2abe + b^2 d\right)}{3b^4} + \frac{2\sqrt{a + bx^2}\left(a^3(-f) + a^2 be - ab^2 d + b^3 c\right)}{b^4} + \frac{2\left(a + bx^2\right)^{5/2}(be - 3af)}{5b^4} + \right.$$

input
```
Int[(c*x + d*x^3 + e*x^5 + f*x^7)/Sqrt[a + b*x^2],x]
```

output
```
((2*(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*Sqrt[a + b*x^2])/b^4 + (2*(b^2*d -
 2*a*b*e + 3*a^2*f)*(a + b*x^2)^(3/2))/(3*b^4) + (2*(b*e - 3*a*f)*(a + b*x
^2)^(5/2))/(5*b^4) + (2*f*(a + b*x^2)^(7/2))/(7*b^4))/2
```

3.171.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2029
```
Int[(Fx_.)*((d_.)*(x_)^(q_.) + (a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.) + (c_.)*
(x_)^(t_.))^(p_.), x_Symbol] :> Int[x^(p*r)*(a + b*x^(s - r) + c*x^(t - r)
+ d*x^(q - r))^p*Fx, x] /; FreeQ[{a, b, c, d, r, s, t, q}, x] && IntegerQ[p
] && PosQ[s - r] && PosQ[t - r] && PosQ[q - r] &&  !(EqQ[p, 1] && EqQ[u, 1]
)
```

rule 2331
```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[1/2   S
ubst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
 FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

rule 2389
```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

3.171.4 Maple [A] (verified)

Time = 3.55 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{16\left(\frac{\left(-5 f\,x^{6}-7 e\,x^{4}-\frac{35}{3} d\,x^{2}-35 c\right) b^{3}}{16}+\frac{35\left(\frac{9}{35} f\,x^{4}+\frac{2}{5} e\,x^{2}+d\right) a\,b^{2}}{24}-\frac{7\left(\frac{3 f\,x^{2}}{7}+e\right) a^{2} b}{6}+f\,a^{3}\right)\sqrt{b\,x^{2}+a}}{35 b^{4}}$
gosper	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c\right)}{105 b^{4}}$
trager	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c\right)}{105 b^{4}}$
risch	$-\frac{\sqrt{b\,x^{2}+a}\,\left(-15 f\,x^{6} b^{3}+18 a\,b^{2} f\,x^{4}-21 b^{3} e\,x^{4}-24 a^{2} b f\,x^{2}+28 a\,b^{2} e\,x^{2}-35 b^{3} d\,x^{2}+48 f\,a^{3}-56 a^{2} b e+70 a\,b^{2} d-105 b^{3} c\right)}{105 b^{4}}$
default	$f\left(\frac{x^{6}\sqrt{b\,x^{2}+a}}{7 b}-\frac{6 a\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a\sqrt{b\,x^{2}+a}}{3 b^{2}}\right)}{5 b}\right)}{7 b}\right)+e\left(\frac{x^{4}\sqrt{b\,x^{2}+a}}{5 b}-\frac{4 a\left(\frac{x^{2}\sqrt{b\,x^{2}+a}}{3 b}-\frac{2 a\sqrt{b\,x^{2}-}}{3 b^{2}}\right.}{5 b}\right.$

input
```
int((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16/35*(1/16*(-5*f*x^6-7*e*x^4-35/3*d*x^2-35*c)*b^3+35/24*(9/35*f*x^4+2/5*
e*x^2+d)*a*b^2-7/6*(3/7*f*x^2+e)*a^2*b+f*a^3)*(b*x^2+a)^(1/2)/b^4
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

$$\int \frac{cx + dx^3 + ex^5 + fx^7}{\sqrt{a + bx^2}}\,dx$$
$$= \frac{(15\,b^3fx^6 + 3\,(7\,b^3e - 6\,ab^2f)x^4 + 105\,b^3c - 70\,ab^2d + 56\,a^2be - 48\,a^3f + (35\,b^3d - 28\,ab^2e + 24\,a^2bf)x^2)}{105\,b^4}$$

input
```
integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
1/105*(15*b^3*f*x^6 + 3*(7*b^3*e - 6*a*b^2*f)*x^4 + 105*b^3*c - 70*a*b^2*d
 + 56*a^2*b*e - 48*a^3*f + (35*b^3*d - 28*a*b^2*e + 24*a^2*b*f)*x^2)*sqrt(
b*x^2 + a)/b^4
```

3.171.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. 2(112) = 224.

Time = 0.29 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.97

$$\int \frac{cx + dx^3 + ex^5 + fx^7}{\sqrt{a + bx^2}}\,dx$$
$$= \begin{cases} -\frac{16a^3f\sqrt{a+bx^2}}{35b^4} + \frac{8a^2e\sqrt{a+bx^2}}{15b^3} + \frac{8a^2fx^2\sqrt{a+bx^2}}{35b^3} - \frac{2ad\sqrt{a+bx^2}}{3b^2} - \frac{4aex^2\sqrt{a+bx^2}}{15b^2} - \frac{6afx^4\sqrt{a+bx^2}}{35b^2} + \frac{c\sqrt{a+bx^2}}{b} + \frac{dx^2\sqrt{a+bx^2}}{3b} \\ \frac{\frac{cx^2}{2} + \frac{dx^4}{4} + \frac{ex^6}{6} + \frac{fx^8}{8}}{\sqrt{a}} \end{cases}$$

input
```
integrate((f*x**7+e*x**5+d*x**3+c*x)/(b*x**2+a)**(1/2),x)
```

output
```
Piecewise((-16*a**3*f*sqrt(a + b*x**2)/(35*b**4) + 8*a**2*e*sqrt(a + b*x**
2)/(15*b**3) + 8*a**2*f*x**2*sqrt(a + b*x**2)/(35*b**3) - 2*a*d*sqrt(a + b
*x**2)/(3*b**2) - 4*a*e*x**2*sqrt(a + b*x**2)/(15*b**2) - 6*a*f*x**4*sqrt(
a + b*x**2)/(35*b**2) + c*sqrt(a + b*x**2)/b + d*x**2*sqrt(a + b*x**2)/(3*
b) + e*x**4*sqrt(a + b*x**2)/(5*b) + f*x**6*sqrt(a + b*x**2)/(7*b), Ne(b,
0)), ((c*x**2/2 + d*x**4/4 + e*x**6/6 + f*x**8/8)/sqrt(a), True))
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.49

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{bx^2+a}fx^6}{7\,b}+\frac{\sqrt{bx^2+a}ex^4}{5\,b}-\frac{6\,\sqrt{bx^2+a}afx^4}{35\,b^2}+\frac{\sqrt{bx^2+a}dx^2}{3\,b}-\frac{4\,\sqrt{bx^2+a}aex^2}{15\,b^2}+\frac{8\,\sqrt{bx^2+a}a^2fx^2}{35\,b^3}+\frac{\sqrt{bx^2+a}c}{b}-\frac{2\,\sqrt{bx^2+a}ad}{3\,b^2}+\frac{8\,\sqrt{bx^2+a}a^2e}{15\,b^3}-\frac{16\,\sqrt{bx^2+a}a^3f}{35\,b^4}$$

input
```
integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
1/7*sqrt(b*x^2 + a)*f*x^6/b + 1/5*sqrt(b*x^2 + a)*e*x^4/b - 6/35*sqrt(b*x^
2 + a)*a*f*x^4/b^2 + 1/3*sqrt(b*x^2 + a)*d*x^2/b - 4/15*sqrt(b*x^2 + a)*a*
e*x^2/b^2 + 8/35*sqrt(b*x^2 + a)*a^2*f*x^2/b^3 + sqrt(b*x^2 + a)*c/b - 2/3
*sqrt(b*x^2 + a)*a*d/b^2 + 8/15*sqrt(b*x^2 + a)*a^2*e/b^3 - 16/35*sqrt(b*x
^2 + a)*a^3*f/b^4
```

3.171.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx = \frac{(b^3c-ab^2d+a^2be-a^3f)\sqrt{bx^2+a}}{b^4}+\frac{35\,(bx^2+a)^{\frac{3}{2}}b^2d+21\,(bx^2+a)^{\frac{5}{2}}be-70\,(bx^2+a)^{\frac{3}{2}}abe+15\,(bx^2+a)^{\frac{7}{2}}f-63\,(bx^2+a)^{\frac{5}{2}}af+105\,(bx^2}{105\,b^4}$$

input
```
integrate((f*x^7+e*x^5+d*x^3+c*x)/(b*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
(b^3*c - a*b^2*d + a^2*b*e - a^3*f)*sqrt(b*x^2 + a)/b^4 + 1/105*(35*(b*x^2
 + a)^(3/2)*b^2*d + 21*(b*x^2 + a)^(5/2)*b*e - 70*(b*x^2 + a)^(3/2)*a*b*e
+ 15*(b*x^2 + a)^(7/2)*f - 63*(b*x^2 + a)^(5/2)*a*f + 105*(b*x^2 + a)^(3/2
)*a^2*f)/b^4
```

3.171.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.85

$$\int \frac{cx+dx^3+ex^5+fx^7}{\sqrt{a+bx^2}}\,dx=\sqrt{b\,x^2+a}\,\left(\frac{-48\,f\,a^3+56\,e\,a^2\,b-70\,d\,a\,b^2+105\,c\,b^3}{105\,b^4}+\frac{f\,x^6}{7\,b}+\frac{x^2\,\left(24\,f\,a^2\,b-28\,e\,a\,b^2+35\,d\,b^3\right)}{105\,b^4}+\frac{x^4\,\left(21\,b^3\,e-18\,a\,b^2\,f\right)}{105\,b^4}\right)$$

input
```
int((c*x + d*x^3 + e*x^5 + f*x^7)/(a + b*x^2)^(1/2),x)
```

output
```
(a + b*x^2)^(1/2)*((105*b^3*c - 48*a^3*f - 70*a*b^2*d + 56*a^2*b*e)/(105*b
^4) + (f*x^6)/(7*b) + (x^2*(35*b^3*d - 28*a*b^2*e + 24*a^2*b*f))/(105*b^4)
 + (x^4*(21*b^3*e - 18*a*b^2*f))/(105*b^4))
```

3.172 $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$

3.172.1 Optimal result

Integrand size = 37, antiderivative size = 261

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{(Ab^4-a(b^3B-ab^2C+a^2bD-a^3F))\,x^3}{7ab^4\,(a+bx^2)^{7/2}} + \frac{(4Ab^4+a(3b^3B-10ab^2C+17a^2bD-24a^3F))\,x^3}{35a^2b^4\,(a+bx^2)^{5/2}} + \frac{(8Ab^4+a(6b^3B+15ab^2C-71a^2bD+162a^3F))\,x^3}{105a^3b^4\,(a+bx^2)^{3/2}} - \frac{(bD-4aF)x}{b^5\sqrt{a+bx^2}} + \frac{Fx\sqrt{a+bx^2}}{2b^5} + \frac{(2bD-9aF)\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{2b^{11/2}}$$

output
```
1/7*(A*b^4-a*(B*b^3-C*a*b^2+D*a^2*b-F*a^3))*x^3/a/b^4/(b*x^2+a)^(7/2)+1/35
*(4*A*b^4+a*(3*B*b^3-10*C*a*b^2+17*D*a^2*b-24*F*a^3))*x^3/a^2/b^4/(b*x^2+a
)^(5/2)+1/105*(8*A*b^4+a*(6*B*b^3+15*C*a*b^2-71*D*a^2*b+162*F*a^3))*x^3/a^
3/b^4/(b*x^2+a)^(3/2)+1/2*(2*D*b-9*F*a)*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))
/b^(11/2)-(D*b-4*F*a)*x/b^5/(b*x^2+a)^(1/2)+1/2*F*x*(b*x^2+a)^(1/2)/b^5
```

3.172.2 Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{x(945a^7F+16Ab^7x^6+4ab^6x^4(14A+3Bx^2)-210a^6b(D-15F}{}$$
$$+\frac{(2bD-9aF)\operatorname{arctanh}\left(\frac{\sqrt{b}x}{-\sqrt{a}+\sqrt{a+bx^2}}\right)}{b^{11/2}}$$

input
```
Integrate[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

output
```
(x*(945*a^7*F + 16*A*b^7*x^6 + 4*a*b^6*x^4*(14*A + 3*B*x^2) - 210*a^6*b*(D
 - 15*F*x^2) + a^3*b^4*x^6*(-352*D + 105*F*x^2) + 14*a^5*b^2*x^2*(-50*D +
261*F*x^2) + 4*a^4*b^3*x^4*(-203*D + 396*F*x^2) + 2*a^2*b^5*x^2*(35*A + 21
*B*x^2 + 15*C*x^4)))/(210*a^3*b^5*(a + b*x^2)^(7/2)) + ((2*b*D - 9*a*F)*Ar
cTanh[(Sqrt[b]*x)/(-Sqrt[a] + Sqrt[a + b*x^2])])/b^(11/2)
```

3.172.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.405, Rules used = {2335, 9, 25, 2335, 9, 25, 1586, 9, 27, 360, 25, 27, 299, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}dx$$

$$\Big\downarrow \ 2335$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x\left(7aFx^7+7a\left(D-\frac{aF}{b}\right)x^5+\frac{7a\left(Fa^2-bDa+b^2C\right)x^3}{b^2}+\left(4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)x\right)}{\left(bx^2+a\right)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \ 9$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x^2\left(7aFx^6+7a\left(D-\frac{aF}{b}\right)x^4+7a\left(C-\frac{a(bD-aF)}{b^2}\right)x^2+4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)}{(bx^2+a)^{7/2}}dx}{7ab}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{x^2\left(7aFx^6+7a\left(D-\frac{aF}{b}\right)x^4+7a\left(C-\frac{a(bD-aF)}{b^2}\right)x^2+4Ab+\frac{3a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^3}\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 2335$$

$$\frac{\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x\left(35a^2Fx^5+35a^2\left(D-\frac{2aF}{b}\right)x^3+\left(8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)x\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 9$$

$$\frac{\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}-\frac{\int -\frac{x^2\left(35a^2Fx^4+35a^2\left(D-\frac{2aF}{b}\right)x^2+8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{x^2\left(35a^2Fx^4+35a^2\left(D-\frac{2aF}{b}\right)x^2+8Ab^2+3a\left(\frac{19Fa^3}{b^2}-\frac{12Da^2}{b}+5Ca+2bB\right)\right)}{\left(bx^2+a\right)^{5/2}}dx}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 1586$$

3.172. $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{105x\left(\frac{Fx^3a^3}{b}+\frac{(bD-3aF)xa^3}{b^2}\right)}{\left(bx^2+a\right)^{3/2}}dx}{3a}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 9

$$\frac{\frac{\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{105a^3x^2\left(bFx^2+bD-3aF\right)}{b^2\left(bx^2+a\right)^{3/2}}dx}{3a}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 27

$$\frac{\frac{\frac{35a^2\int \frac{x^2\left(bFx^2+bD-3aF\right)}{\left(bx^2+a\right)^{3/2}}dx}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 360

$$\frac{\frac{\frac{35a^2\left(-\frac{\int -\frac{b\left(bFx^2+bD-4aF\right)}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$\downarrow$ 25

3.172. $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{35a^2\left(\frac{\int \frac{b\left(bFx^2+bD-4aF\right)}{\sqrt{bx^2+a}}dx}{b^2}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{35a^2\left(\frac{\int \frac{bFx^2+bD-4aF}{\sqrt{bx^2+a}}dx}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 299$$

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{1}{2}(2bD-9aF)\int \frac{1}{\sqrt{bx^2+a}}dx+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 224$$

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{1}{2}(2bD-9aF)\int \frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab^2C+3b^3B\right)}{b^3}+4Ab\right)}{5a(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \; 219$$

3.172. $\int \frac{x^2\left(A+Bx^2+Cx^4+Dx^6+Fx^8\right)}{\left(a+bx^2\right)^{9/2}}\,dx$

$$\frac{\frac{\frac{35a^2\left(\frac{\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)(2bD-9aF)}{2\sqrt{b}}+\frac{1}{2}Fx\sqrt{a+bx^2}}{b}-\frac{x(bD-4aF)}{b\sqrt{a+bx^2}}\right)}{b^2}+\frac{x^3\left(a\left(\frac{162a^3F}{b^2}-\frac{71a^2D}{b}+15aC+6bB\right)+8Ab^2\right)}{3a\left(a+bx^2\right)^{3/2}}}{5ab}+\frac{x^3\left(\frac{a\left(-24a^3F+17a^2bD-10ab\right.}{b^3}\right.}{5a(a+bx^2)^5}}{7ab}$$

$$\frac{x^3\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

input
```
Int[(x^2*(A + B*x^2 + C*x^4 + D*x^6 + F*x^8))/(a + b*x^2)^(9/2),x]
```

output
```
((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x^3)/(7*(a + b*x^2)^(7/2)
) + (((4*A*b + (a*(3*b^3*B - 10*a*b^2*C + 17*a^2*b*D - 24*a^3*F))/b^3)*x^3
)/(5*a*(a + b*x^2)^(5/2)) + (((8*A*b^2 + a*(6*b*B + 15*a*C - (71*a^2*D)/b
+ (162*a^3*F)/b^2))*x^3)/(3*a*(a + b*x^2)^(3/2)) + (35*a^2*(-(((b*D - 4*a*
F)*x)/(b*Sqrt[a + b*x^2])) + ((F*x*Sqrt[a + b*x^2])/2 + ((2*b*D - 9*a*F)*A
rcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(2*Sqrt[b]))/b))/b^2)/(5*a*b))/(7*a*b
)
```

3.172.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

3.172. $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$

3.172.4 Maple [A] (verified)

Time = 3.64 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{3(bx^2+a)^{\frac{7}{2}}a^3\left(Db-\frac{9Fa}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx^2+a}}{x\sqrt{b}}\right)+\left(x^2a^2\left(\frac{3}{7}Cx^4+\frac{3}{5}x^2B+A\right)b^{\frac{11}{2}}+\frac{4a\left(\frac{3x^2B}{14}+A\right)x^4b^{\frac{13}{2}}}{5}-3a^6(-15Fx^2+D)b\right.}{3b^{\frac{11}{2}}(bx^2+a)}$
default	$D\left(-\frac{x^7}{7b(bx^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(bx^2+a\right)^{\frac{5}{2}}}+\frac{-\frac{x^3}{3b\left(bx^2+a\right)^{\frac{3}{2}}}+\frac{-\frac{x}{b\sqrt{bx^2+a}}+\frac{\ln\left(x\sqrt{b}+\sqrt{bx^2+a}\right)}{b^{\frac{3}{2}}}}{b}}{b}}{b}\right)+C\left(-\frac{x^5}{2b(bx^2+a)^{\frac{7}{2}}}+\right.$

3.172. $\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx$

input
```
int(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
1/3/b^(11/2)*(3*(b*x^2+a)^(7/2)*a^3*(D*b-9/2*F*a)*arctanh((b*x^2+a)^(1/2)/
x/b^(1/2))+(x^2*a^2*(3/7*C*x^4+3/5*x^2*B+A)*b^(11/2)+4/5*a*(3/14*x^2*B+A)*
x^4*b^(13/2)-3*a^6*(-15*F*x^2+D)*b^(3/2)-10*a^5*(-261/50*F*x^2+D)*x^2*b^(5
/2)-58/5*a^4*(-396/203*F*x^2+D)*x^4*b^(7/2)-176/35*(-105/352*F*x^2+D)*a^3*
x^6*b^(9/2)+8/35*A*b^(15/2)*x^6+27/2*F*b^(1/2)*a^7)*x)/(b*x^2+a)^(7/2)/a^3
```

3.172.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 705, normalized size of antiderivative = 2.70

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \left[-\frac{105\,(9\,Fa^8-2\,Da^7b+(9\,Fa^4b^4-2\,Da^3b^5)x^8+4\,(9\,Fa^5b^3-\right.$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
[-1/420*(105*(9*F*a^8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9
*F*a^5*b^3 - 2*D*a^4*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F
*a^7*b - 2*D*a^6*b^2)*x^2)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b
)*x - a) - 2*(105*F*a^3*b^5*x^9 + 2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*
a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*
B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(45*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*
x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*
a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^6*b^7*x^2 + a^7*b^6), 1/210*(105*(9*F*a^
8 - 2*D*a^7*b + (9*F*a^4*b^4 - 2*D*a^3*b^5)*x^8 + 4*(9*F*a^5*b^3 - 2*D*a^4
*b^4)*x^6 + 6*(9*F*a^6*b^2 - 2*D*a^5*b^3)*x^4 + 4*(9*F*a^7*b - 2*D*a^6*b^2
)*x^2)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + (105*F*a^3*b^5*x^9 +
2*(792*F*a^4*b^4 - 176*D*a^3*b^5 + 15*C*a^2*b^6 + 6*B*a*b^7 + 8*A*b^8)*x^7
 + 14*(261*F*a^5*b^3 - 58*D*a^4*b^4 + 3*B*a^2*b^6 + 4*A*a*b^7)*x^5 + 70*(4
5*F*a^6*b^2 - 10*D*a^5*b^3 + A*a^2*b^6)*x^3 + 105*(9*F*a^7*b - 2*D*a^6*b^2
)*x)*sqrt(b*x^2 + a))/(a^3*b^10*x^8 + 4*a^4*b^9*x^6 + 6*a^5*b^8*x^4 + 4*a^
6*b^7*x^2 + a^7*b^6)]
```

3.172.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6987 vs. 2(253) = 506.

Time = 108.77 (sec) , antiderivative size = 6987, normalized size of antiderivative = 26.77

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**5*x**3/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b*x**2*s
qrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 420*a**(1
3/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1 + b*x**
2/a)) + 63*a**4*b*x**5/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a**(17/2)*b
*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 42
0*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x**8*sqrt(1
+ b*x**2/a)) + 36*a**3*b**2*x**7/(105*a**(19/2)*sqrt(1 + b*x**2/a) + 420*a
**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt(1 + b*x*
*2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2)*b**4*x*
*8*sqrt(1 + b*x**2/a)) + 8*a**2*b**3*x**9/(105*a**(19/2)*sqrt(1 + b*x**2/a
) + 420*a**(17/2)*b*x**2*sqrt(1 + b*x**2/a) + 630*a**(15/2)*b**2*x**4*sqrt
(1 + b*x**2/a) + 420*a**(13/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 105*a**(11/2
)*b**4*x**8*sqrt(1 + b*x**2/a))) + B*(7*a*x**5/(35*a**(11/2)*sqrt(1 + b*x*
*2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) + 105*a**(7/2)*b**2*x**4*sq
rt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 + b*x**2/a)) + 2*b*x**7/(3
5*a**(11/2)*sqrt(1 + b*x**2/a) + 105*a**(9/2)*b*x**2*sqrt(1 + b*x**2/a) +
105*a**(7/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 35*a**(5/2)*b**3*x**6*sqrt(1 +
 b*x**2/a))) + C*x**7/(7*a**(9/2)*sqrt(1 + b*x**2/a) + 21*a**(7/2)*b*x**2*
sqrt(1 + b*x**2/a) + 21*a**(5/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 7*a**(3/2)
*b**3*x**6*sqrt(1 + b*x**2/a)) + D*(105*a**(205/2)*b**45*sqrt(1 + b*x**...
```

3.172.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. 2(235) = 470.

Time = 0.23 (sec) , antiderivative size = 826, normalized size of antiderivative = 3.16

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
1/2*F*x^9/((b*x^2 + a)^(7/2)*b) - 1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*
a*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^
3/((b*x^2 + a)^(7/2)*b^4))*D*x + 9/70*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a
*x^4/((b*x^2 + a)^(7/2)*b^2) + 56*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3
/((b*x^2 + a)^(7/2)*b^4))*F*a*x/b + 3/10*F*a*x*(15*x^4/((b*x^2 + a)^(5/2)*
b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b^2
 - 1/15*D*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^
2) + 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*C*x^5/((b*x^2 + a)^(7/2)*b) +
3/2*F*a*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^3
- 1/3*D*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2
+ 9/2*F*a^2*x^3/((b*x^2 + a)^(5/2)*b^4) - D*a*x^3/((b*x^2 + a)^(5/2)*b^3)
- 5/8*C*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*B*x^3/((b*x^2 + a)^(7/2)*b) -
417/70*F*a*x/(sqrt(b*x^2 + a)*b^5) - 51/70*F*a^2*x/((b*x^2 + a)^(3/2)*b^5)
 + 261/70*F*a^3*x/((b*x^2 + a)^(5/2)*b^5) + 139/105*D*x/(sqrt(b*x^2 + a)*b
^4) + 17/105*D*a*x/((b*x^2 + a)^(3/2)*b^4) - 29/35*D*a^2*x/((b*x^2 + a)^(5
/2)*b^4) + 1/14*C*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*C*x/(sqrt(b*x^2 + a)*a*b
^3) + 3/56*C*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*C*a^2*x/((b*x^2 + a)^(7/2
)*b^3) + 3/140*B*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*B*x/(sqrt(b*x^2 + a)*a^2
*b^2) + 1/35*B*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*B*a*x/((b*x^2 + a)^(7/2)
*b^2) - 1/7*A*x/((b*x^2 + a)^(7/2)*b) + 8/105*A*x/(sqrt(b*x^2 + a)*a^3*...
```

3.172.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.86

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx = \frac{\left(\left(\left(\left(\left(\frac{105\,Fx^2}{b}+\frac{2\,(792\,Fa^4b^7-176\,Da^3b^8+15\,Ca^2b^9+6\,Bab^{10}+8\,Ab^{11})}{a^3b^9}\right)x^2+\frac{14\cdots}{}\right.\right.\right.\right.}{}$$
$$+\frac{(9\,Fa-2\,Db)\log\left(\left|-\sqrt{b}x+\sqrt{bx^2+a}\right|\right)}{2\,b^{\frac{11}{2}}}$$

input
```
integrate(x^2*(F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/210*((((105*F*x^2/b + 2*(792*F*a^4*b^7 - 176*D*a^3*b^8 + 15*C*a^2*b^9 +
6*B*a*b^10 + 8*A*b^11)/(a^3*b^9))*x^2 + 14*(261*F*a^5*b^6 - 58*D*a^4*b^7 +
 3*B*a^2*b^9 + 4*A*a*b^10)/(a^3*b^9))*x^2 + 70*(45*F*a^6*b^5 - 10*D*a^5*b^
6 + A*a^2*b^9)/(a^3*b^9))*x^2 + 105*(9*F*a^7*b^4 - 2*D*a^6*b^5)/(a^3*b^9))
*x/(b*x^2 + a)^(7/2) + 1/2*(9*F*a - 2*D*b)*log(abs(-sqrt(b)*x + sqrt(b*x^2
 + a)))/b^(11/2)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx^2+Cx^4+Dx^6+Fx^8)}{(a+bx^2)^{9/2}}\,dx=\int \frac{x^2\,(A+B\,x^2+C\,x^4+F\,x^8+x^6\,D)}{(b\,x^2+a)^{9/2}}\,dx$$

input
```
int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2),x)
```

output
```
int((x^2*(A + B*x^2 + C*x^4 + F*x^8 + x^6*D))/(a + b*x^2)^(9/2), x)
```

3.173 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}} dx$

3.173.1 Optimal result

Integrand size = 34, antiderivative size = 214

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}} dx = \frac{(Ab^4-a^4F)\,x}{ab^4\,(a+bx^2)^{7/2}} + \frac{(6Ab^4+ab^3B-10a^4F)\,x^3}{3a^2b^3\,(a+bx^2)^{7/2}} + \frac{(24Ab^4+a(4b^3B+3ab^2C-58a^3F))\,x^5}{15a^3b^2\,(a+bx^2)^{7/2}} + \frac{(48Ab^4+a(8b^3B+6ab^2C+15a^2bD-176a^3F))\,x^7}{105a^4b\,(a+bx^2)^{7/2}} + \frac{F\operatorname{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{9/2}}$$

output
```
(A*b^4-F*a^4)*x/a/b^4/(b*x^2+a)^(7/2)+1/3*(6*A*b^4+B*a*b^3-10*F*a^4)*x^3/a
^2/b^3/(b*x^2+a)^(7/2)+1/15*(24*A*b^4+a*(4*B*b^3+3*C*a*b^2-58*F*a^3))*x^5/
a^3/b^2/(b*x^2+a)^(7/2)+1/105*(48*A*b^4+a*(8*B*b^3+6*C*a*b^2+15*D*a^2*b-17
6*F*a^3))*x^7/a^4/b/(b*x^2+a)^(7/2)+F*arctanh(x*b^(1/2)/(b*x^2+a)^(1/2))/b
^(9/2)
```

3.173.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.82

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \frac{x(-105a^7F-350a^6bFx^2-406a^5b^2Fx^4+48Ab^7x^6-176a^4b^3Fx^6+}{}$$
$$-\frac{F\log\left(-\sqrt{b}x+\sqrt{a+bx^2}\right)}{b^{9/2}}$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

output
```
(x*(-105*a^7*F - 350*a^6*b*F*x^2 - 406*a^5*b^2*F*x^4 + 48*A*b^7*x^6 - 176*
a^4*b^3*F*x^6 + 8*a*b^6*x^4*(21*A + B*x^2) + 2*a^2*b^5*x^2*(105*A + 14*B*x
^2 + 3*C*x^4) + a^3*b^4*(105*A + 35*B*x^2 + 21*C*x^4 + 15*D*x^6)))/(105*a^
4*b^4*(a + b*x^2)^(7/2)) - (F*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(9/2)
```

3.173.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2345, 25, 2345, 25, 1471, 25, 27, 298, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{2345}$$

$$\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7(a+bx^2)^{7/2}}-\frac{\int -\frac{\frac{7aFx^6}{b}+\frac{7a(bD-aF)x^4}{b^2}+\frac{7a\left(Fa^2-bDa+b^2C\right)x^2}{b^3}+6A+\frac{a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^4}}{(bx^2+a)^{7/2}}dx}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\frac{7aFx^6}{b}+\frac{7a(bD-aF)x^4}{b^2}+\frac{7a\left(Fa^2-bDa+b^2C\right)x^2}{b^3}+6A+\frac{a\left(-Fa^3+bDa^2-b^2Ca+b^3B\right)}{b^4}}{(bx^2+a)^{7/2}}dx}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 2345}$$

$$\frac{\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}-\frac{\int -\frac{\frac{35a^2Fx^4}{b^2}+\frac{35a^2(bD-2aF)x^2}{b^3}+24A+\frac{a\left(17Fa^3-10bDa^2+3b^2Ca+4b^3B\right)}{b^4}}{\left(bx^2+a\right)^{5/2}}dx}{5a}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{\frac{35a^2Fx^4}{b^2}+\frac{35a^2(bD-2aF)x^2}{b^3}+24A+\frac{a\left(17Fa^3-10bDa^2+3b^2Ca+4b^3B\right)}{b^4}}{\left(bx^2+a\right)^{5/2}}dx}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1471}$$

$$\frac{\frac{\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}-\frac{\int -\frac{-71Fa^4+105bFx^2a^3+15bDa^3+6b^2Ca^2+8b^3Ba+48Ab^4}{b^4\left(bx^2+a\right)^{3/2}}dx}{3a}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\frac{\int \frac{48Ab^4+105a^3Fx^2b+a\left(-71Fa^3+15bDa^2+6b^2Ca+8b^3B\right)}{b^4\left(bx^2+a\right)^{3/2}}dx}{3a}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 27}$$

3.173. $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

$$\frac{\frac{\frac{\int \frac{48Ab^4+105a^3Fx^2b+a\left(-71Fa^3+15bDa^2+6b^2Ca+8b^3B\right)}{\left(bx^2+a\right)^{3/2}}dx}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 298$$

$$\frac{\frac{\frac{105a^3F\int\frac{1}{\sqrt{bx^2+a}}dx+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8ab^3B+48Ab^4\right)}{a\sqrt{a+bx^2}}}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\cdots\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 224$$

$$\frac{\frac{\frac{105a^3F\int\frac{1}{1-\frac{bx^2}{bx^2+a}}d\frac{x}{\sqrt{bx^2+a}}+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8ab^3B+48Ab^4\right)}{a\sqrt{a+bx^2}}}{3ab^4}+\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}}{5a}+\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2\cdots}{b^4}\right)}{5(a+bx^2)^{5/2}}}{7a}+\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow 219$$

$$\frac{x\left(\frac{A}{a}-\frac{a^3(-F)+a^2bD-ab^2C+b^3B}{b^4}\right)}{7\left(a+bx^2\right)^{7/2}}+\frac{\frac{x\left(\frac{-22a^3F+15a^2bD-8ab^2C+b^3B}{b^4}+\frac{6A}{a}\right)}{5(a+bx^2)^{5/2}}+\frac{\frac{x\left(\frac{122a^3F-45a^2bD+3ab^2C+4b^3B}{b^4}+\frac{24A}{a}\right)}{3\left(a+bx^2\right)^{3/2}}+\frac{\frac{105a^3F\text{arctanh}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{\sqrt{b}}+\frac{x\left(-176a^4F+15a^3bD+6a^2b^2C+8\cdots\right)}{a\sqrt{a+bx^2}}}{3ab^4}}{5a}}{7a}$$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(a + b*x^2)^(9/2),x]
```

3.173. $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

output
```
((A/a - (b^3*B - a*b^2*C + a^2*b*D - a^3*F)/b^4)*x)/(7*(a + b*x^2)^(7/2))
+ ((((6*A)/a + (b^3*B - 8*a*b^2*C + 15*a^2*b*D - 22*a^3*F)/b^4)*x)/(5*(a +
 b*x^2)^(5/2)) + ((((24*A)/a + (4*b^3*B + 3*a*b^2*C - 45*a^2*b*D + 122*a^3
*F)/b^4)*x)/(3*(a + b*x^2)^(3/2)) + (((48*A*b^4 + 8*a*b^3*B + 6*a^2*b^2*C
+ 15*a^3*b*D - 176*a^4*F)*x)/(a*Sqrt[a + b*x^2]) + (105*a^3*F*ArcTanh[(Sqr
t[b]*x)/Sqrt[a + b*x^2]])/Sqrt[b])/(3*a*b^4))/(5*a))/(7*a)
```

3.173.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2345
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   In
t[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

3.173.4 Maple [A] (verified)

Time = 3.59 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{x\,a^3\left(\frac{1}{7}Dx^6+\frac{1}{5}C\,x^4+\frac{1}{3}x^2B+A\right)b^{\frac{9}{2}}+2\left(\frac{1}{35}C\,x^4+\frac{2}{15}x^2B+A\right)a^2x^3b^{\frac{11}{2}}+\frac{8\left(\frac{x^2B}{21}+A\right)a\,x^5b^{\frac{13}{2}}}{5}+\frac{16A\,b^{\frac{15}{2}}x^7}{35}+F\,a^4\left(\left(b\,x^2+a\right)^{\frac{7}{2}}a\right.}{b^{\frac{9}{2}}\left(b\,x^2+a\right)^{\frac{7}{2}}a^4}$
default	$A\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+F\left(-\frac{x^7}{7b(b\,x^2+a)^{\frac{7}{2}}}+\frac{-\frac{x^5}{5b\left(b\,x^2+a\right)^{\frac{5}{2}}}+\right.$

3.173. $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/b^(9/2)*(x*a^3*(1/7*D*x^6+1/5*C*x^4+1/3*x^2*B+A)*b^(9/2)+2*(1/35*C*x^4+2
/15*x^2*B+A)*a^2*x^3*b^(11/2)+8/5*(1/21*x^2*B+A)*a*x^5*b^(13/2)+16/35*A*b^
(15/2)*x^7+F*a^4*((b*x^2+a)^(7/2)*arctanh((b*x^2+a)^(1/2)/x/b^(1/2))-176/1
05*x^7*b^(7/2)-58/15*b^(5/2)*a*x^5-10/3*b^(3/2)*a^2*x^3-b^(1/2)*a^3*x))/(b
*x^2+a)^(7/2)/a^4
```

3.173.5 Fricas [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 567, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx = \left[\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{b}\log\left(\right.}{}\right.$$

$$-\frac{105\,(Fa^4b^4x^8+4\,Fa^5b^3x^6+6\,Fa^6b^2x^4+4\,Fa^7bx^2+Fa^8)\sqrt{-b}\arctan\left(\frac{\sqrt{-b}x}{\sqrt{bx^2+a}}\right)+((176\,Fa^4b^4-15\,Da^3b}{1}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="fricas
")
```

output
```
[1/210*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4 + 4*F*a^7*b
*x^2 + F*a^8)*sqrt(b)*log(-2*b*x^2 - 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) - 2*
((176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 +
 7*(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*
a^6*b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt
(b*x^2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2
+ a^8*b^5), -1/105*(105*(F*a^4*b^4*x^8 + 4*F*a^5*b^3*x^6 + 6*F*a^6*b^2*x^4
 + 4*F*a^7*b*x^2 + F*a^8)*sqrt(-b)*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + ((
176*F*a^4*b^4 - 15*D*a^3*b^5 - 6*C*a^2*b^6 - 8*B*a*b^7 - 48*A*b^8)*x^7 + 7
*(58*F*a^5*b^3 - 3*C*a^3*b^5 - 4*B*a^2*b^6 - 24*A*a*b^7)*x^5 + 35*(10*F*a^
6*b^2 - B*a^3*b^5 - 6*A*a^2*b^6)*x^3 + 105*(F*a^7*b - A*a^3*b^5)*x)*sqrt(b
*x^2 + a))/(a^4*b^9*x^8 + 4*a^5*b^8*x^6 + 6*a^6*b^7*x^4 + 4*a^7*b^6*x^2 +
a^8*b^5)]
```

3.173.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5071 vs. $2(211) = 422$.

Time = 74.12 (sec) , antiderivative size = 5071, normalized size of antiderivative = 23.70

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/(b*x**2+a)**(9/2),x)
```

output
```
A*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt
(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2
)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a
) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*
sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 +
 b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b*
*4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) +
 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a**
(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*
a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 +
b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**
5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) +
 429*a**11*b**3*x**7/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x*
*2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a
**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b
*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6
*x**12*sqrt(1 + b*x**2/a)) + 286*a**10*b**4*x**9/(35*a**(37/2)*sqrt(1 + b*
x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**
4*sqrt(1 + b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525...
```

3.173.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(198) = 396$.

Time = 0.24 (sec) , antiderivative size = 597, normalized size of antiderivative = 2.79

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{(a+bx^2)^{9/2}}\,dx =$$

$$-\frac{1}{35}\left(\frac{35\,x^6}{(bx^2+a)^{\frac{7}{2}}b}+\frac{70\,ax^4}{(bx^2+a)^{\frac{7}{2}}b^2}+\frac{56\,a^2x^2}{(bx^2+a)^{\frac{7}{2}}b^3}+\frac{16\,a^3}{(bx^2+a)^{\frac{7}{2}}b^4}\right)Fx$$
$$-\frac{Fx\left(\frac{15\,x^4}{(bx^2+a)^{\frac{5}{2}}b}+\frac{20\,ax^2}{(bx^2+a)^{\frac{5}{2}}b^2}+\frac{8\,a^2}{(bx^2+a)^{\frac{5}{2}}b^3}\right)}{15\,b}-\frac{Dx^5}{2\,(bx^2+a)^{\frac{7}{2}}b}$$
$$-\frac{Fx\left(\frac{3\,x^2}{(bx^2+a)^{\frac{3}{2}}b}+\frac{2\,a}{(bx^2+a)^{\frac{3}{2}}b^2}\right)}{3\,b^2}-\frac{Fax^3}{(bx^2+a)^{\frac{5}{2}}b^3}-\frac{5\,Dax^3}{8\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Cx^3}{4\,(bx^2+a)^{\frac{7}{2}}b}$$
$$+\frac{16\,Ax}{35\,\sqrt{bx^2+a}a^4}+\frac{8\,Ax}{35\,(bx^2+a)^{\frac{3}{2}}a^3}+\frac{6\,Ax}{35\,(bx^2+a)^{\frac{5}{2}}a^2}+\frac{Ax}{7\,(bx^2+a)^{\frac{7}{2}}a}$$
$$+\frac{139\,Fx}{105\,\sqrt{bx^2+a}b^4}+\frac{17\,Fax}{105\,(bx^2+a)^{\frac{3}{2}}b^4}-\frac{29\,Fa^2x}{35\,(bx^2+a)^{\frac{5}{2}}b^4}+\frac{Dx}{14\,(bx^2+a)^{\frac{3}{2}}b^3}$$
$$+\frac{Dx}{7\,\sqrt{bx^2+a}ab^3}+\frac{3\,Dax}{56\,(bx^2+a)^{\frac{5}{2}}b^3}-\frac{15\,Da^2x}{56\,(bx^2+a)^{\frac{7}{2}}b^3}+\frac{3\,Cx}{140\,(bx^2+a)^{\frac{5}{2}}b^2}$$
$$+\frac{2\,Cx}{35\,\sqrt{bx^2+a}a^2b^2}+\frac{Cx}{35\,(bx^2+a)^{\frac{3}{2}}ab^2}-\frac{3\,Cax}{28\,(bx^2+a)^{\frac{7}{2}}b^2}-\frac{Bx}{7\,(bx^2+a)^{\frac{7}{2}}b}$$
$$+\frac{8\,Bx}{105\,\sqrt{bx^2+a}a^3b}+\frac{4\,Bx}{105\,(bx^2+a)^{\frac{3}{2}}a^2b}+\frac{Bx}{35\,(bx^2+a)^{\frac{5}{2}}ab}+\frac{F\,\mathrm{arsinh}\left(\frac{bx}{\sqrt{ab}}\right)}{b^{\frac{9}{2}}}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="maxima
")
```

output
```
-1/35*(35*x^6/((b*x^2 + a)^(7/2)*b) + 70*a*x^4/((b*x^2 + a)^(7/2)*b^2) + 5
6*a^2*x^2/((b*x^2 + a)^(7/2)*b^3) + 16*a^3/((b*x^2 + a)^(7/2)*b^4))*F*x -
1/15*F*x*(15*x^4/((b*x^2 + a)^(5/2)*b) + 20*a*x^2/((b*x^2 + a)^(5/2)*b^2)
+ 8*a^2/((b*x^2 + a)^(5/2)*b^3))/b - 1/2*D*x^5/((b*x^2 + a)^(7/2)*b) - 1/3
*F*x*(3*x^2/((b*x^2 + a)^(3/2)*b) + 2*a/((b*x^2 + a)^(3/2)*b^2))/b^2 - F*a
*x^3/((b*x^2 + a)^(5/2)*b^3) - 5/8*D*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1/4*C
*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*A*x/(sqrt(b*x^2 + a)*a^4) + 8/35*A*x/((
b*x^2 + a)^(3/2)*a^3) + 6/35*A*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*A*x/((b*x^2
 + a)^(7/2)*a) + 139/105*F*x/(sqrt(b*x^2 + a)*b^4) + 17/105*F*a*x/((b*x^2
+ a)^(3/2)*b^4) - 29/35*F*a^2*x/((b*x^2 + a)^(5/2)*b^4) + 1/14*D*x/((b*x^2
 + a)^(3/2)*b^3) + 1/7*D*x/(sqrt(b*x^2 + a)*a*b^3) + 3/56*D*a*x/((b*x^2 +
a)^(5/2)*b^3) - 15/56*D*a^2*x/((b*x^2 + a)^(7/2)*b^3) + 3/140*C*x/((b*x^2
+ a)^(5/2)*b^2) + 2/35*C*x/(sqrt(b*x^2 + a)*a^2*b^2) + 1/35*C*x/((b*x^2 +
a)^(3/2)*a*b^2) - 3/28*C*a*x/((b*x^2 + a)^(7/2)*b^2) - 1/7*B*x/((b*x^2 + a
)^(7/2)*b) + 8/105*B*x/(sqrt(b*x^2 + a)*a^3*b) + 4/105*B*x/((b*x^2 + a)^(3
/2)*a^2*b) + 1/35*B*x/((b*x^2 + a)^(5/2)*a*b) + F*arcsinh(b*x/sqrt(a*b))/b
^(9/2)
```

3.173.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.95

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}}\, dx =$$

$$-\frac{\left(\left(x^2\left(\frac{(176\,Fa^4b^6 - 15\,Da^3b^7 - 6\,Ca^2b^8 - 8\,Bab^9 - 48\,Ab^{10})x^2}{a^4b^7} + \frac{7\,(58\,Fa^5b^5 - 3\,Ca^3b^7 - 4\,Ba^2b^8 - 24\,Aab^9)}{a^4b^7}\right) + \frac{35\,(10\,Fa^6b^4 - Ba^3b^7 - 6\,Aa^2 \cdots}{a^4b^7} \cdots}{105\,(bx^2 + a)^{\frac{7}{2}}}$$

$$-\frac{F\log\left(\left|-\sqrt{b}x + \sqrt{bx^2 + a}\right|\right)}{b^{\frac{9}{2}}}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/(b*x^2+a)^(9/2),x, algorithm="giac")
```

output
```
-1/105*((x^2*((176*F*a^4*b^6 - 15*D*a^3*b^7 - 6*C*a^2*b^8 - 8*B*a*b^9 - 48
*A*b^10)*x^2/(a^4*b^7) + 7*(58*F*a^5*b^5 - 3*C*a^3*b^7 - 4*B*a^2*b^8 - 24*
A*a*b^9)/(a^4*b^7)) + 35*(10*F*a^6*b^4 - B*a^3*b^7 - 6*A*a^2*b^8)/(a^4*b^7
))*x^2 + 105*(F*a^7*b^3 - A*a^3*b^7)/(a^4*b^7))*x/(b*x^2 + a)^(7/2) - F*lo
g(abs(-sqrt(b)*x + sqrt(b*x^2 + a)))/b^(9/2)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{(a + bx^2)^{9/2}} \, dx = \int \frac{A + B\,x^2 + C\,x^4 + F\,x^8 + x^6\,D}{(b\,x^2 + a)^{9/2}} \, dx$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2),x)
```

output
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(a + b*x^2)^(9/2), x)
```

3.174 $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

3.174.1 Optimal result

Integrand size = 37, antiderivative size = 193

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{A}{ax(a+bx^2)^{7/2}} - \frac{(8Ab-aB)x}{a^2(a+bx^2)^{7/2}} - \frac{(48Ab^2-a(6bB+aC))x^3}{3a^3(a+bx^2)^{7/2}} - \frac{(192Ab^3-a(24b^2B+4abC+3a^2D))x^5}{15a^4(a+bx^2)^{7/2}} - \frac{(384Ab^4-a(48b^3B+8ab^2C+6a^2bD+15a^3F))x^7}{105a^5(a+bx^2)^{7/2}}$$

output
```
-A/a/x/(b*x^2+a)^(7/2)-(8*A*b-B*a)*x/a^2/(b*x^2+a)^(7/2)-1/3*(48*A*b^2-a*(
6*B*b+C*a))*x^3/a^3/(b*x^2+a)^(7/2)-1/15*(192*A*b^3-a*(24*B*b^2+4*C*a*b+3*
D*a^2))*x^5/a^4/(b*x^2+a)^(7/2)-1/105*(384*A*b^4-a*(48*B*b^3+8*C*a*b^2+6*D
*a^2*b+15*F*a^3))*x^7/a^5/(b*x^2+a)^(7/2)
```

3.174.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = \frac{-384Ab^4x^8+48ab^3x^6(-28A+Bx^2)+8a^2b^2x^4(-210A+21Bx^2+$$

input
```
Integrate[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
(-384*A*b^4*x^8 + 48*a*b^3*x^6*(-28*A + B*x^2) + 8*a^2*b^2*x^4*(-210*A + 2
1*B*x^2 + C*x^4) + 2*a^3*b*x^2*(-420*A + 105*B*x^2 + 14*C*x^4 + 3*D*x^6) +
 a^4*(-105*A + 105*B*x^2 + 35*C*x^4 + 21*D*x^6 + 15*F*x^8))/(105*a^5*x*(a
+ b*x^2)^(7/2))
```

3.174.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.216, Rules used = {2334, 2344, 1586, 9, 25, 27, 362, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2 \left(a+bx^2\right)^{9/2}} \, dx$$

$$\Big\downarrow \text{2334}$$

$$-\frac{\int \frac{8Ab-a\left(Fx^6+Dx^4+Cx^2+B\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{2344}$$

$$-\frac{\frac{\int \frac{x^2\left(-a^2Fx^4-a^2Dx^2+6b(8Ab-aB)-a^2C\right)}{\left(bx^2+a\right)^{9/2}}dx}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{1586}$$

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a\left(a+bx^2\right)^{7/2}}-\frac{\int -\frac{x\left(\left(192Ab^2-a\left(-\frac{3Fa^3}{b^2}+\frac{3Da^2}{b}+4Ca+24bB\right)\right)x-\frac{7a^3Fx^3}{b}\right)}{\left(bx^2+a\right)^{7/2}}dx}{7a}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$$\Big\downarrow \text{9}$$

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a(a+bx^2)^{7/2}}-\frac{\int -\frac{x^2\left(b\left(192Ab^2-a\left(-\frac{3Fa^3}{b^2}+\frac{3Da^2}{b}+4Ca+24bB\right)\right)-7a^3Fx^2\right)}{b(bx^2+a)^{7/2}}dx}{7a}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 25

$$-\frac{\frac{\frac{\int \frac{x^2\left(-7Fx^2a^3-\left(-\frac{3Fa^3}{b}+3Da^2+4bCa+24b^2B\right)a+192Ab^3\right)}{b(bx^2+a)^{7/2}}dx}{7a}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 27

$$-\frac{\frac{\frac{\int \frac{x^2\left(-7Fx^2a^3-\left(-\frac{3Fa^3}{b}+3Da^2+4bCa+24b^2B\right)a+192Ab^3\right)}{(bx^2+a)^{7/2}}dx}{7ab}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a(a+bx^2)^{7/2}}}{a}+\frac{x(8Ab-aB)}{a(a+bx^2)^{7/2}}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 362

$$-\frac{\frac{\frac{\frac{\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)\int \frac{x^2}{(bx^2+a)^{5/2}}dx}{5ab}+\frac{x^3\left(192Ab^4-a\left(-10a^3F+3a^2bD+4ab^2C+24b^3B\right)\right)}{5ab(a+bx^2)^{5/2}}}{7ab}+\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a(a+bx^2)^{7/2}}}{a}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

$\big\downarrow$ 242

$$-\frac{\frac{\frac{x^3\left(48Ab^2-a\left(\frac{a^3F}{b^2}-\frac{a^2D}{b}+aC+6bB\right)\right)}{7a(a+bx^2)^{7/2}}+\frac{\frac{x^3\left(384Ab^4-a\left(15a^3F+6a^2bD+8ab^2C+48b^3B\right)\right)}{15a^2b(a+bx^2)^{3/2}}+\frac{x^3\left(192Ab^4-a\left(-10a^3F+3a^2bD+4ab^2C+24b^3B\right)\right)}{5ab(a+bx^2)^{5/2}}}{7ab}}{a}+\frac{x(8Ab}{a(a+b}}{a}-\frac{A}{ax\left(a+bx^2\right)^{7/2}}$$

3.174. $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

input
```
Int[(A + B*x^2 + C*x^4 + D*x^6 + F*x^8)/(x^2*(a + b*x^2)^(9/2)),x]
```

output
```
-(A/(a*x*(a + b*x^2)^(7/2))) - (((8*A*b - a*B)*x)/(a*(a + b*x^2)^(7/2)) +
(((48*A*b^2 - a*(6*b*B + a*C - (a^2*D)/b + (a^3*F)/b^2))*x^3)/(7*a*(a + b*
x^2)^(7/2)) + (((192*A*b^4 - a*(24*b^3*B + 4*a*b^2*C + 3*a^2*b*D - 10*a^3*
F))*x^3)/(5*a*b*(a + b*x^2)^(5/2)) + ((384*A*b^4 - a*(48*b^3*B + 8*a*b^2*C
 + 6*a^2*b*D + 15*a^3*F))*x^3)/(15*a^2*b*(a + b*x^2)^(3/2)))/(7*a*b))/a)/a
```

3.174.3.1 Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 362
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*b*e
*(p + 1))), x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(2*a*b*(p + 1))   I
nt[(e*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && N
eQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[p, -5/4]) ||
  !RationalQ[m] || (ILtQ[p + 1/2, 0] && LeQ[-1, m, -2*(p + 1)]))
```

rule 1586
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^
p, d + e*x^2, x], x, 0]}, Simp[(-R)*(f*x)^(m + 1)*((d + e*x^2)^(q + 1)/(2*d
*f*(q + 1))), x] + Simp[f/(2*d*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x^2)^(q
+ 1)*ExpandToSum[2*d*(q + 1)*x*Qx + R*(m + 2*q + 3)*x, x], x], x]] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[q, -1]
&& GtQ[m, 0]
```

rule 2334
```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coef
f[Pq, x, 0], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*
x^(m + 1)*((a + b*x^2)^(p + 1)/(a*(m + 1))), x] + Simp[1/(a*(m + 1))   Int[
x^(m + 2)*(a + b*x^2)^p*(a*(m + 1)*Q - A*b*(m + 2*(p + 1) + 1)), x], x]] /;
 FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && IntegerQ[m/2] && ILtQ[(m + 1)/2 + p,
 0] && LtQ[m + Expon[Pq, x] + 2*p + 1, 0]
```

rule 2344
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{A = Coeff[Pq, x, 0
], Q = PolynomialQuotient[Pq - Coeff[Pq, x, 0], x^2, x]}, Simp[A*x*((a + b*
x^2)^(p + 1)/a), x] + Simp[1/a   Int[x^2*(a + b*x^2)^p*(a*Q - A*b*(2*p + 3)
), x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x^2] && ILtQ[p + 1/2, 0] && LtQ
[Expon[Pq, x] + 2*p + 1, 0]
```

3.174.4 Maple [A] (verified)

Time = 3.61 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{(15F\,x^8+21Dx^6+35C\,x^4+105x^2B-105A)a^4-840b(-\frac{1}{140}Dx^6-\frac{1}{30}C\,x^4-\frac{1}{4}x^2B+A)x^2a^3-1680(-\frac{1}{210}C\,x^4-\frac{1}{10}x^2B+A)b^2}{105(b\,x^2+a)^{\frac{7}{2}}\,x\,a^5}$
gosper	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a}{105x(b\,x^2+a)^{\frac{7}{2}}\,a^5}$
trager	$-\frac{384A\,b^4x^8-48Ba\,b^3x^8-8C\,a^2b^2x^8-6Da^3b\,x^8-15F\,a^4x^8+1344Aa\,b^3x^6-168B\,a^2b^2x^6-28C\,a^3b\,x^6-21Da^4x^6+1680A\,a}{105x(b\,x^2+a)^{\frac{7}{2}}\,a^5}$
default	$B\left(\frac{x}{7a(b\,x^2+a)^{\frac{7}{2}}}+\frac{\frac{6x}{35a\left(b\,x^2+a\right)^{\frac{5}{2}}}+\frac{6\left(\frac{4x}{15a\left(b\,x^2+a\right)^{\frac{3}{2}}}+\frac{8x}{15a^2\sqrt{b\,x^2+a}}\right)}{7a}}{a}\right)+F\left(-\frac{x^5}{2b(b\,x^2+a)^{\frac{7}{2}}}+\frac{5a\left(-\frac{x^3}{4b\left(b\,x^2+a\right)}\right.}{}\right.$

3.174. $\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx$

input
```
int((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x,method=_RETURNVERBOS
E)
```

output
```
1/105*((15*F*x^8+21*D*x^6+35*C*x^4+105*B*x^2-105*A)*a^4-840*b*(-1/140*D*x^
6-1/30*C*x^4-1/4*x^2*B+A)*x^2*a^3-1680*(-1/210*C*x^4-1/10*x^2*B+A)*b^2*x^4
*a^2-1344*b^3*x^6*(-1/28*x^2*B+A)*a-384*A*b^4*x^8)/(b*x^2+a)^(7/2)/x/a^5
```

3.174.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.97

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^2 (a + bx^2)^{9/2}} \, dx = \frac{((15\,Fa^4 + 6\,Da^3b + 8\,Ca^2b^2 + 48\,Bab^3 - 384\,Ab^4)x^8 + 7\,(3\,Da^4 + 4}{10}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="fr
icas")
```

output
```
1/105*((15*F*a^4 + 6*D*a^3*b + 8*C*a^2*b^2 + 48*B*a*b^3 - 384*A*b^4)*x^8 +
 7*(3*D*a^4 + 4*C*a^3*b + 24*B*a^2*b^2 - 192*A*a*b^3)*x^6 - 105*A*a^4 + 35
*(C*a^4 + 6*B*a^3*b - 48*A*a^2*b^2)*x^4 + 105*(B*a^4 - 8*A*a^3*b)*x^2)*sqr
t(b*x^2 + a)/(a^5*b^4*x^9 + 4*a^6*b^3*x^7 + 6*a^7*b^2*x^5 + 4*a^8*b*x^3 +
a^9*x)
```

3.174.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2490 vs. 2(184) = 368.

Time = 98.39 (sec) , antiderivative size = 2490, normalized size of antiderivative = 12.90

$$\int \frac{A + Bx^2 + Cx^4 + Dx^6 + Fx^8}{x^2 (a + bx^2)^{9/2}} \, dx = \text{Too large to display}$$

input
```
integrate((F*x**8+D*x**6+C*x**4+B*x**2+A)/x**2/(b*x**2+a)**(9/2),x)
```

output
```
A*(-35*a**4*b**(33/2)*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8) -
280*a**3*b**(35/2)*x**2*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**
17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
- 560*a**2*b**(37/2)*x**4*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b
**17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8
) - 448*a*b**(39/2)*x**6*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b*
*17*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)
 - 128*b**(41/2)*x**8*sqrt(a/(b*x**2) + 1)/(35*a**9*b**16 + 140*a**8*b**17
*x**2 + 210*a**7*b**18*x**4 + 140*a**6*b**19*x**6 + 35*a**5*b**20*x**8)) +
 B*(35*a**14*x/(35*a**(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqr
t(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1 + b*x**2/a) + 700*a**(31/
2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b**4*x**8*sqrt(1 + b*x**2/
a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a) + 35*a**(25/2)*b**6*x**12
*sqrt(1 + b*x**2/a)) + 175*a**13*b*x**3/(35*a**(37/2)*sqrt(1 + b*x**2/a) +
 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + 525*a**(33/2)*b**2*x**4*sqrt(1
+ b*x**2/a) + 700*a**(31/2)*b**3*x**6*sqrt(1 + b*x**2/a) + 525*a**(29/2)*b
**4*x**8*sqrt(1 + b*x**2/a) + 210*a**(27/2)*b**5*x**10*sqrt(1 + b*x**2/a)
+ 35*a**(25/2)*b**6*x**12*sqrt(1 + b*x**2/a)) + 371*a**12*b**2*x**5/(35*a*
*(37/2)*sqrt(1 + b*x**2/a) + 210*a**(35/2)*b*x**2*sqrt(1 + b*x**2/a) + ...
```

3.174.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(175) = 350$.

Time = 0.21 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.18

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2(a+bx^2)^{9/2}}\,dx = -\frac{Fx^5}{2(bx^2+a)^{\frac{7}{2}}b} - \frac{5\,Fax^3}{8(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Dx^3}{4(bx^2+a)^{\frac{7}{2}}b} + \frac{16\,Bx}{35\sqrt{bx^2+a}a^4} + \frac{8\,Bx}{35(bx^2+a)^{\frac{3}{2}}a^3} + \frac{6\,Bx}{35(bx^2+a)^{\frac{5}{2}}a^2} + \frac{Bx}{7(bx^2+a)^{\frac{7}{2}}a} + \frac{Fx}{14(bx^2+a)^{\frac{3}{2}}b^3} + \frac{Fx}{7\sqrt{bx^2+a}ab^3} + \frac{3\,Fax}{56(bx^2+a)^{\frac{5}{2}}b^3} - \frac{15\,Fa^2x}{56(bx^2+a)^{\frac{7}{2}}b^3} + \frac{3\,Dx}{140(bx^2+a)^{\frac{5}{2}}b^2} + \frac{2\,Dx}{35\sqrt{bx^2+a}a^2b^2} + \frac{Dx}{35(bx^2+a)^{\frac{3}{2}}ab^2} - \frac{3\,Dax}{28(bx^2+a)^{\frac{7}{2}}b^2} - \frac{Cx}{7(bx^2+a)^{\frac{7}{2}}b} + \frac{8\,Cx}{105\sqrt{bx^2+a}a^3b} + \frac{4\,Cx}{105(bx^2+a)^{\frac{3}{2}}a^2b} + \frac{Cx}{35(bx^2+a)^{\frac{5}{2}}ab} - \frac{128\,Abx}{35\sqrt{bx^2+a}a^5} - \frac{64\,Abx}{35(bx^2+a)^{\frac{3}{2}}a^4} - \frac{48\,Abx}{35(bx^2+a)^{\frac{5}{2}}a^3} - \frac{8\,Abx}{7(bx^2+a)^{\frac{7}{2}}a^2} - \frac{A}{(bx^2+a)^{\frac{7}{2}}ax}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="ma
xima")
```

output
```
-1/2*F*x^5/((b*x^2 + a)^(7/2)*b) - 5/8*F*a*x^3/((b*x^2 + a)^(7/2)*b^2) - 1
/4*D*x^3/((b*x^2 + a)^(7/2)*b) + 16/35*B*x/(sqrt(b*x^2 + a)*a^4) + 8/35*B*
x/((b*x^2 + a)^(3/2)*a^3) + 6/35*B*x/((b*x^2 + a)^(5/2)*a^2) + 1/7*B*x/((b
*x^2 + a)^(7/2)*a) + 1/14*F*x/((b*x^2 + a)^(3/2)*b^3) + 1/7*F*x/(sqrt(b*x^
2 + a)*a*b^3) + 3/56*F*a*x/((b*x^2 + a)^(5/2)*b^3) - 15/56*F*a^2*x/((b*x^2
 + a)^(7/2)*b^3) + 3/140*D*x/((b*x^2 + a)^(5/2)*b^2) + 2/35*D*x/(sqrt(b*x^
2 + a)*a^2*b^2) + 1/35*D*x/((b*x^2 + a)^(3/2)*a*b^2) - 3/28*D*a*x/((b*x^2
+ a)^(7/2)*b^2) - 1/7*C*x/((b*x^2 + a)^(7/2)*b) + 8/105*C*x/(sqrt(b*x^2 +
a)*a^3*b) + 4/105*C*x/((b*x^2 + a)^(3/2)*a^2*b) + 1/35*C*x/((b*x^2 + a)^(5
/2)*a*b) - 128/35*A*b*x/(sqrt(b*x^2 + a)*a^5) - 64/35*A*b*x/((b*x^2 + a)^(
3/2)*a^4) - 48/35*A*b*x/((b*x^2 + a)^(5/2)*a^3) - 8/7*A*b*x/((b*x^2 + a)^(
7/2)*a^2) - A/((b*x^2 + a)^(7/2)*a*x)
```

3.174.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.14

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx = \frac{\left(\left(x^2\left(\frac{\left(15\,Fa^{13}b^3+6\,Da^{12}b^4+8\,Ca^{11}b^5+48\,Ba^{10}b^6-279\,Aa^9b^7\right)x^2}{a^{14}b^3}+\frac{7\left(3\,Da^{13}b^3+4\,Ca^{1}\cdots\right.}{\cdots}\right.\right.\right.}{10\cdots} + \frac{2\,A\sqrt{b}}{\left(\left(\sqrt{b}x-\sqrt{bx^2+a}\right)^2-a\right)a^4}$$

input
```
integrate((F*x^8+D*x^6+C*x^4+B*x^2+A)/x^2/(b*x^2+a)^(9/2),x, algorithm="gi
ac")
```

output
```
1/105*((x^2*((15*F*a^13*b^3 + 6*D*a^12*b^4 + 8*C*a^11*b^5 + 48*B*a^10*b^6
- 279*A*a^9*b^7)*x^2/(a^14*b^3) + 7*(3*D*a^13*b^3 + 4*C*a^12*b^4 + 24*B*a^
11*b^5 - 132*A*a^10*b^6)/(a^14*b^3)) + 35*(C*a^13*b^3 + 6*B*a^12*b^4 - 30*
A*a^11*b^5)/(a^14*b^3))*x^2 + 105*(B*a^13*b^3 - 4*A*a^12*b^4)/(a^14*b^3))*
x/(b*x^2 + a)^(7/2) + 2*A*sqrt(b)/(((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a)*a
^4)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx^2+Cx^4+Dx^6+Fx^8}{x^2\left(a+bx^2\right)^{9/2}}\,dx=\int \frac{A+B\,x^2+C\,x^4+F\,x^8+x^6\,D}{x^2\,\left(b\,x^2+a\right)^{9/2}}\,dx$$

input
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)),x)
```

output
```
int((A + B*x^2 + C*x^4 + F*x^8 + x^6*D)/(x^2*(a + b*x^2)^(9/2)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg:  #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
        return max(m1,m2)    #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):    #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()):  #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()):  #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1:  #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```